ASSIGNMENT - 3

Let consider a sample dataset have one input (x_i^a) and one output $(y_i)^a$, and number of samples 4. Develop a simple linear regression model using stochastic gradient descent optimizer.

Sample (i)	X_i^a	Y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

Manual Calculations:

Step 1: [x, y], epochs = 2, $\eta = 0.1$, $m = 1$, $c = -1$.

Step 2: Iter = 1

Step 3: Sample = 1

Step 4: $E = \frac{1}{2}(y_i - mx_i + c)^2$

$= \frac{1}{2}(3.4 - (1)(0.2) - 1)^2$

$= 0.5(3.4 + 0.8)^2$

$= 8.82$

$$\frac{\partial E}{\partial m} = -(y_i - mx_i - c)\, x_i$$

$$= -\{3.4 - (1)(0.2) - (-1)\}(0.2)$$

$$= -0.84$$

$$\frac{\partial E}{\partial c} = -(y_i - mx_i - c)$$

$$= -(3.4 - (1)(0.2) - (-1))$$

$$= -4.2$$

step5: $\Delta m = -\eta \frac{\partial E}{\partial m} = (-0.1)(-0.84) = 0.084$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = (-0.1)(-4.2) = 0.42$$

step6: $m = m + \Delta m = 1 + 0.084 = 1.084$

$$c = c + \Delta c = -1 + 0.42 = -0.58$$

step7: sample = sample + 1

$$= 1 + 1 = 2$$

step8: if (sample > ns)

false → goto step 4

step4: $y = (1.084)(0.4) - 0.58 = -0.1464$

$$E = (+0.5)(3.8 + 0.1464)^2 = 7.79$$

$$\frac{\partial E}{\partial m} = -(y_i - mx_i - c)\, x_i$$

$$= -(3.8 - (1.084)(0.4) + 0.58)\, 0.4$$

$$= -1.58$$

$$\frac{\partial E}{\partial c} = -(y_i - mx_i - c)$$

$$= -(3.8 - (1.084)(0.4) + 0.98)$$

$$= -3.94$$

Step 5: $\Delta m = -\eta \frac{\partial E}{\partial m}$

$$= (-0.1)(-1.58) = 0.158$$

$$\Delta c = -\eta \frac{\partial E}{\partial c}$$

$$= (-0.1)(-3.94) = 0.394$$

Step 6: $m = m + \Delta m$

$$= 1.084 + 0.158 = 1.242$$

$$c = c + \Delta c$$

$$= -0.58 + 0.394 = -0.186$$

Step 7: Sample = Sample + 1

$$= 2 + 1 = 3$$

Step 8: if (Sample > ns)

goto next step

Step 9: ifr = ifr + 1

$$= 1 + 1 = 2$$

Step 10: if ifr < epochs

goto step 3

step 3: sample = 1

step 4: $y = (1.242)(0.2) + (-0.186) = 0.0624$

$E = \frac{1}{2}(3.4 - 0.0624) = 1.6688$

$\frac{\partial E}{\partial m} = -(3.4 - 0.0624)\,0.2 = -0.66752$

$\frac{\partial E}{\partial c} = -(3.4 - 0.0624) = -3.3376$

step 5: $\Delta m = -\eta \frac{\partial E}{\partial m}$

$= (-0.1)(-0.66752) = 0.066752$

$\Delta c = -\eta \frac{\partial E}{\partial c}$

$= (-0.1).(-3.3376) = 0.33376$

step 6: $m = m + \Delta m = 1.242 + 0.66752$

$= 1.90952$

$c = c + \Delta c = -0.186 + 0.33376$

$= 0.14776$

step 7: sample = sample + 1

$= 1 + 1 = 2$

step 8: if (sample > ns)

goto ~~next~~ step 4

step 4: $\frac{\partial E}{\partial m} = -(3.8 - (1.90952)(0.4) - (0.14776))\,(0.4)$

$= -(2.888432)(0.4)$

$= -1.155372$

$$\frac{\partial E}{\partial c} = -(3.8 - (1.909582)(0.4) - (0.14716))$$

$$= -2.888432$$

Step5: $\Delta m = -\eta \frac{\partial E}{\partial m} = (-0.1)(-1.155372)$

$$= 0.1155372$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = (-0.1)(-2.888432)$$

$$= 0.2888432$$

Step6: $m = m + \Delta m = 2.025057$

$$c = c + \Delta c = 0.4366032$$

Step7: sample = sample + 1

$= 2 + 1 = 3$

Step8: if (sample > ns)

true → goto next step

Step9: ptr = ptr + 1

$= 2 + 1 = 3$

step10: if (itr > epochs)

go to next step

step11: print m, c

$m = 2.025057$

$c = 0.4366032$

step12: Compute MSE

$$= \frac{(3.4 - 0.841614) + (3.8 - 1.246626)}{2}$$

$$= 2.556063$$