ASSIGNMENT - II

18K41A0231

Let consider a sample dataset have one input (x_i^2) and one output (y_i^2) and number of samples 4. Develop a simple linear regression model using Nesterov Accelerated Gradient (NAG) optimizer.

Sample (i)	x_i^2	y_i^2
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

Manual Calculations:

step 1: $[x, y]$, $m=1$, $c=-1$, $\eta=0.1$, $\gamma=0.9$, $v_m=v_c=0$,

epochs = 2, no. of samples = 2

x	y
0.2	3.4
0.4	3.8

step 2: itr = 1

step 3: sample = 1

step 4: $g_m = -\left(y_i - (m+\gamma v_m)x_i - (c+\gamma v_c)\right)x_i$

$= -\left(3.4 - (1+(0.9)(0))0.2 - ((-1)+0)\right)(0.2)$

$= -0.84$

$g_c = -4.2$

step 5: $v_m = \gamma v_m - \eta g_m$

$= (0.9)(0) - (0.1)(-0.84) = 0.084$

$v_c = \gamma v_c - \eta g_c$

$= (0) - (0.1)(-4.2) = 0.42$

step-6: $m = m + v_m$

$= 1 + 0.084 = 1.084$

$c = c + v_c$

$= -1 + 0.42 = -0.58$

step 7: sample $= 1+1 = 2$

step 8: if (sample > no. of samples)

$2 > 2$

goto step 4

step 4: $g_m = -(3.8 - (1.084 + (0.9)(0.084)) (0.4) - (-0.58 + (0.9)(0.42))) 0.4$

$= -1.717664$

$g_c = -4.29416$

step 5: $v_m = \gamma v_m - \eta g_m$

$= (0.9)(0.084) - (0.1)(-1.717664)$

$= 0.2473664$

$v_c = \gamma v_c - \eta g_c$

$= (0.9)(0.42) - (0.1)(-4.29416)$

$= 0.807416$

Step 6: $m = m + V_m$

$= 1.084 + 0.24736 = 1.33136$

$c = c + V_c$

$= -0.58 + 0.807416 = 0.227416$

Step 7: Sample $= 2+1 = 3$

Step 8: if (sample > no. of samples)

$3 > 2$

goto next step

Step 9: itr = itr + 1

$= 1+1 = 2$

Step 10: if (itr > epochs)

$2 > 2$

false → goto step 3

Step 3: sample $= 1$

Step 4: $g_m = -(y_i - (m + \gamma V_m) x_i - (c + \gamma V_c)) x_i$

$= -(3.4 - (1.33136 + (0.9)(0.24736)) 0.2 - (0.227416 + (0.9)(0.807416))$

$= -2.13571$

$g_c = -0.891926$

Step 5: $V_m = \gamma V_m - \eta g_m$

$= (0.9)(0.2473664) - (0.1)(-2.13571)$

$= 0.43614$

$v_c = \gamma v_c - \eta g_c$

$= (0.9)(0.807416) - (0.1)(-0.891926)$

$= 0.815867$

step 6: $m = m + v_m$

$= 1.3316 + 0.43614 = 1.76774$

$c = c + v_c$

$= 0.227416 + 0.815867 = 1.043283$

step 7: sample = sample + 1

$= 1+1 = 2$

step 8: if (sample > no. of samples)

$2 > 2$

false → goto ~~next~~ step 4

step 4: $g_m = -(y_i - (m + \gamma v_m)x_i - (c + \gamma v_c))x_i$

$= -(3.8 - (1.76774 + (0.9)(0.43614))0.4 - (1.043283 + (0.9)(0.815867)))0.4$

$= -0.463332$

$g_c = -(3.8 - (2.160266)(0.4) - 1.7775633)$

$= -1.1583303$

step 5: $v_m = \gamma v_m - \eta \frac{\partial E}{\partial m}$

$= (0.9)(0.43614) - (0.1)(-0.463332)$

$= 0.4388592$

$$v_c = \gamma v_c - \eta \frac{\partial C}{\partial c}$$

$$= (0.9)(0.815867) - (0.1)(-1.1583303)$$

$$= 0.8501133$$

step 6: $m = 1.76774 + 0.4388592 = 2.2065992$

$c = 1.043283 + 1.1583303 = 2.2016133$

step 7: sample = 2+1 = 3

step 8: if (sample > no. of samples)

3 > 2

true → goto next step

step 9: itr = 2+1 = 3

step 10: if (itr > epochs)

3 > 2

true → goto next step

step 11: print m, c

$m = 2.2065992$

$c = 2.2016133$

step 12: MSE

$$= \frac{(3.4-(2.2065992\times0.2) - 2.2016133)^2 + (3.8-(2.2065992\times0.4) - 2.2016133)^2}{2}$$

$$= \frac{1.085443}{2} = 0.54271$$