

ASSIGNMENT-5

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Let consider a sample dataset have one input (x_i^a) and one output (y_i^a), and number of samples 4. Develop a simple linear regression model using MBGD.

Sample (i)	x_i^a	y_i^a
1	0.2	3.4
2	0.4	3.8
3	0.6	4.2
4	0.8	4.6

Manual Calculations:

Step 1: Read dataset $[x, y]$

$\eta = 0.1$, $m = 1$, $c = -1$, epochs = 2, batch size = 2

Step 2: Split training data into batches

Batch 1

0.2 | 3.4
0.8 | 4.6

Batch-2

0.4 | 3.8
0.6 | 4.2

Step 3: Batch = 1

Step 4: $\eta = 1$

$$\begin{aligned} \frac{\partial E}{\partial m} &= -\frac{1}{2} \left((3.4 - (-1)(0.2) - (-1))(0.2) + \right. \\ &\quad \left. (4.6 - (-1)(0.2) - (-1))(0.2) \right) \\ &= -\frac{1}{2} (4.2)(0.2) + (4.8)(0.2) \\ &= -2.34 \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial c} &= -\frac{1}{2} [4.2 + 4.8] \\ &= -4.5 \end{aligned}$$

step 6:

$$\begin{aligned} \Delta m &= -\eta \frac{\partial E}{\partial m} \\ &= -(0.1)(-2.34) = 0.234 \end{aligned}$$

$$\begin{aligned} \Delta c &= -\eta \frac{\partial E}{\partial c} \\ &= -(0.1)(-4.5) = 0.45 \end{aligned}$$

step 7:

$$\begin{aligned} m &= m + \Delta m \\ &= 1 + 0.234 = 1.234 \end{aligned}$$

$$\begin{aligned} c &= c + \Delta c \\ &= -1 + 0.45 = -0.55 \end{aligned}$$

step 8:

$$\begin{aligned} \text{batch} &= \text{batch} + 1 \\ &= 1 + 1 = 2 \end{aligned}$$

step 9: if batch \geq nb

false

goto step 5

$$\begin{aligned} \text{Step 5: } \frac{\partial E}{\partial m} &= -\frac{1}{2} \left((3.8 - (1.234 \times 0.4) + 0.55)(0.4) \right. \\ &\quad \left. + (4.2 - (1.234 \times 0.6) + 0.55)(0.6) \right) \\ &= -\frac{1}{2} \left((3.8564)(0.4) + (4.0096)(0.6) \right) \\ &= -1.97416 \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial c} &= -\frac{1}{2} \left((3.8564)(0.4) + (4.0096)(0.6) \right) \\ &= -3.933 \end{aligned}$$

$$\begin{aligned} \text{Step 6: } \Delta m &= -\eta \frac{\partial E}{\partial m} = -(0.1)(-1.97416) \\ &= 0.197416 \end{aligned}$$

$$\begin{aligned} \Delta c &= -\eta \frac{\partial E}{\partial c} = -(0.1)(-3.933) \\ &= 0.3933 \end{aligned}$$

$$\begin{aligned} \text{Step 7: } m &= m + \Delta m \\ &= 1.234 + 0.197416 = 1.43414 \end{aligned}$$

$$\begin{aligned} c &= c + \Delta c \\ &= -0.55 + 0.3933 = -0.1567 \end{aligned}$$

$$\begin{aligned} \text{Step 8: } \text{batch} &= \text{batch} + 1 \\ &= 2 + 1 = 3 \end{aligned}$$

$$\begin{aligned} \text{Step 9: } \text{if } \text{batch} &> n_b \\ 3 &> 2 \end{aligned}$$

go to next step

step 10: $96 = 96 + 1$
 $= 141 = 2$

step 11: if $(96 > \text{epochs})$
 $2 > 2$
 false \rightarrow goto step 4.

step 4: batch = 1

step 5: $\frac{\partial E}{\partial m} = -\frac{1}{2} \left((3.4 - (1.4314)(0.2) + 0.1567)(0.2) \right.$
 $\left. + (4.6 - (1.4314)(0.8) + 0.1567)(0.8) \right)$

$$= -\frac{1}{2} \left((3.27042)(0.2) + (3.61158)(0.8) \right)$$

$$= -1.77167$$

$$\frac{\partial E}{\partial c} = -\frac{1}{2} (3.27042 + 3.61158)$$

$$= -3.441$$

step 6: $\Delta m = -\eta \frac{\partial E}{\partial m} = 0.177167$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = 0.3441$$

step 7: $m = m + \Delta m = 1.4314 + 0.177167$
 $= 1.60856$

$$c = c + \Delta c = -0.1567 + 0.3441$$

$$= 0.1874$$

step 8: batch = batch + 1
 $= 1 + 1 = 2$

Step 9: if batch > nb

3 > 2

goto step 5

Step 5: $\frac{\partial E}{\partial m} = -\frac{1}{2} \left((3.8 - (1.60856)(0.4) - 0.1874)(0.4) + (4.2 - (1.60856)(0.6) - 0.1874)(0.6) \right)$

$$= -\frac{1}{2} \left((2.96917)(0.4) + (3.047464)(0.6) \right)$$

$$= -1.50807$$

$$\frac{\partial E}{\partial c} = -\frac{1}{2} (2.96917 + 3.047464)$$

$$= -3.00831$$

Step 6: $\Delta m = -\eta \frac{\partial E}{\partial m} = 0.150807$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = 0.300831$$

Step 7: $m = m + \Delta m$

$$= 1.60856 + 0.150807 = 1.759367$$

$$c = c + \Delta c$$

$$= 0.1874 + 0.300831 = 0.488231$$

Step 8: batch = batch + 1

$$= 2 + 1 = 3$$

Step 9: if (batch > nb)

$$3 > 2$$

goto next step

step 10: $it = it + 1$
 $= 2 + 1 = 3$

step 11: if $(it > epoch)$
 $3 > 2$

goto next step

step 12: print m, c
 $m = 1.759067$
 $c = 0.488231$

step 13: MSE
$$= \frac{(3.4 - 0.84004) + (3.8 - 1.19185) + (4.2 - 1.54367) + (4.6 - 1.89548)}{4}$$

$= 2.63224$