IAP DRP CATEGORY THEORY & TOPOS THEORY READING PLAN

1. Logistics

Mentor: Peter Haine
Mentee: Diego Roque Montoya
Texts:
- *Topoi* by Robert Goldblatt [1]
- *Sheaves in Geometry and Logic* by Saunders MacLane and Ieke Moerdijk [2]
- *Category Theory in Context* by Emily Riehl [3]

Goals: To thoroughly cover the foundations of category theory (chapters 1–4 of [3]), keeping examples from sheaf theory in mind (chapter II of [2]). Then go through the basics of topos theory, discussing examples from algebra, geometry, and logic (chapter III of [2] as well as selected material from: chapters IV–VII of [2] and chapters 10–14 of [1]). We'll use [2, 3] as main texts, and will reference [1] occasionally, mostly for examples.

Meetings: Typically for 60 minutes on weekdays
- *Location:* 36-372
- *Time:* 3:30PM

2. Schedule

The term *Exposé* means that Peter will talk about a topic, usually at the beginning of a chapter to collect ideas and put them in a framework that will hopefully make things easier to understand. The term *Presentation* means that Diego will prepare a short 30 minute (approximately) presentation on the topic, and we'll discuss it more after the presentation. The term *Discussion* means that we'll talk over a topic as a group. The term *Problem Session* means that we'll discuss the exercises and Diego will try to present his solutions.

The exercises aren't problem sets or anything — they're for your health, and you should try to do them. They also won't be particularly hard, rather, they will be chosen to (hopefully) be maximally enlightening. We'll discuss them as together so don't worry if you get stuck on them.

As a general piece of advice, you should try to keep a notebook (either handwritten or electronic) where you write notes on your reading, new insights, and do your exercises. (Most professional mathematicians keep a "research notebook", "idea journal", or something akin.)

2.1. **Before IAP.** Read chapter 1 of [3] as well as §§II.1–II.3 of [2] and complete the following activities:
- Work on the following exercises: 1.1.ii, 1.2.i–1.2.iii, 1.2.v–1.2.vii, 1.3.i–1.3.v, 1.3.x, 1.4.i–1.4.iv, 1.5.i, 1.5.iii–1.5.vii, 1.5.ix–1.5.xi (this last question is tricky, but good to think about — feel free to Google things), 1.6.i–1.6.vi, 1.7.i–1.7.vii
- Read the proof of the Snake Lemma in the diagram chasing notes and watch the proof of the Snake Lemma from *It's My Turn*.

> ➤ To get used to diagram-chasing arguments, do the exercises from the diagram chasing notes (i.e., prove the splitting lemma and the 3×3 lemma).

2.2. **Week of 8 January.** *Basic category theory, Representability, the Yoneda lemma, and Basic sheaf theory*

8 January: No meeting

9 January: Discussion of the basic category theory in [3, Ch. 1]; first part of problem session on the problems from [3, Ch. 1]
> ➤ Continue working on exercises from [3, Ch. 1]
> ➤ Review diagram chasing notes
> ➤ Complete any "before IAP" activities that you haven't completed

10 January: Exposé on representability & universal properties; finish problem session on the problems from [3, Ch. 1]
> ➤ Read [3, §§2.1–2.2]
> ➤ Prepare a presentation on the Yoneda lemma and the Yoneda embedding
> ➤ Work on the following exercises from [3]: 2.1.i–2.1.v, 2.2.i, 2.2.ii, 2.2.iv–2.2.vii

11 January: Presentation on the Yoneda Lemma & discussion of the motivation for sheaf theory
> ➤ Read [3, §§2.3–2.4]
> ➤ Work on the following exercises from [3]: 2.3.i–2.3.iii, 2.4.i–2.4.x

12 January: Discussion on the category of elements and problem session for [3, Ch. 2]
> ➤ Review any material from [3, Ch. 2] that you need reviewing
> ➤ Sit down without the text and try to state the Yoneda Lemma and Yoneda Embedding. Repeat this exercise until you can do it without thinking.
> ➤ Continue working on the exercises from [3, Ch. 2]

2.3. **Week of 15 January.** *Limits & colimits and Introduction to adjunctions*

15 January: No meeting

16 January: Continuation of the problem session for [3, Ch. 2] & Exposé on limits and colimits
> ➤ Read [3, §§3.1–3.2]
> ➤ Prepare a presentaion on limits and colimits in **Set**

17 January: Presentation on limits and colimits in **Set** with examples & Discussion of sieves
> ➤ Read [3, §§3.3–3.4] (and reread §§3.1–3.2 if necessary to solidify your understanding)
> ➤ Carefully read through [2, §§II.1–II.3]
> ➤ Work on the following exercises from [3]: 3.1.i–3.1.ix, 3.1.xii, 3.1.xiii, 3.2.ii–3.2.v
> ➤ Work on the exercise sheet on Limits & Colimits

18 January: Problem Session on limits & colimits
> ➤ Read [3, §§3.5–3.8]
> ➤ Work on the following exercises from [3]: 3.3.i–3.3.vi, 3.4.ii–3.4.iv, 3.5.i–3.5.v
> ➤ Work on the following exercises from [2, Ch. II]: 1, 2, 4, 5, 10
> ➤ Think through exercise 3.5.vii of [3].
> ➤ Skim through [2, §§II.4–II.5]; don't worry about the details, but try to internalize the main results

19 January: Problem Session on limits & colimits and Exposé on adjunctions, using sheafification as motivation
- ➤ Continue working on the exercises and reading
- ➤ Read [2, §§ɪɪ.6]
- ➤ Read [3, §§4.1–4.2]
- ➤ Prepare a presentation on [3, §3.4] (the representable nature of limits & colimits)

2.4. **Week of 22 January.** *Sheaves on a site & topoi*

22 January: Presentation on the representable nature of limits & colimits; Éxpose on Grothendieck sites
- ➤ Read [3, §§4.3–4.6]
- ➤ Read [2, §§ɪɪɪ.1 & ɪɪɪ.2]

23 January: Exposé on sheafification & Inverse image sheaves
- ➤ Read [2, §§ɪɪɪ.3–ɪɪɪ.5]
- ➤ Read [2, §§ɪɪɪ.6–ɪɪɪ.8]
- ➤ Work on the following exercises from [3]: 4.1.i–4.1.iii, 4.2.i–4.2.iv, 4.3.i–4.3.iv, 4.4.i–4.4.iii, 4.5.i–4.5.iii, 4.5.v–4.5.vii, 4.6.i–4.6.iii.

24 January: Problem Session on Adjunction & Discussion of [2, Ch. ɪɪɪ]
- ➤ Read [2, §ɪɪɪ.9]
- ➤ Continue working on the exercises assigned on 23 January
- ➤ Work on the following exercises from [2, Ch. ɪɪɪ]: 1–4, 6(a), 8, 9, 12–6

25 January: Problem Session
- ➤ Read [2, §§ɪv.1–ɪv.3]

26 January: Giraud's axioms ɪ: presentability & coproducts are disjoint
- ➤ Read [2, §§ɪv.4–ɪv.7]
- ➤ Work more on the exercises

2.5. **Week of 29 January.** *Giraud's Axioms*

29 January: Giraud's axioms ɪɪ: colimits are universal
- ➤ Read [2, App. §§1 & 2]
- ➤ Read [2, §§vɪ.1]
- ➤ Read [2, §§vɪɪ.1–vɪɪ.3]

30 January: Giraud's axioms ɪɪɪ: equivalence relations are effective & constructing the equivalence
- ➤ Read [2, App. §§3 & 4]
- ➤ Read [2, §§vɪ.2–vɪ.4]
- ➤ Read [2, §§vɪɪ.4–vɪɪ.6]

<div align="center">Rᴇꜰᴇʀᴇɴᴄᴇꜱ</div>

1. Robert Goldblatt, *Topoi: The categorical analysis of logic*, Dover Books on Mathematics, Dover Publications, 2006.
2. Saunders MacLane and Ieke Moerdijk, *Sheaves in geometry and logic*, Universitext, Springer-Verlag New York, 1992.
3. Emily Riehl, *Category theory in context*, Aurora: Dover Modern Math Originals, Dover Publications, 2016.