微 分 方 程 式 2 ・演習問題

ラプラス積分

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

ラプラス変換の例

(1) $\mathcal{L}\{1\} = \dfrac{1}{s}.$ (2) $\mathcal{L}\{t\} = \dfrac{1}{s^2}.$ (3) $\mathcal{L}\{t^\alpha\} = \dfrac{\Gamma(\alpha+1)}{s^{\alpha+1}}$ $(\mathrm{Re}\, s > 0,\ \alpha > -1).$

(4) $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}.$ （$\mathrm{Re}\,(s-a) > 0$ のとき）

(5) $\mathcal{L}\{\sin \omega t\} = \dfrac{\omega}{s^2+\omega^2}.$ (6) $\mathcal{L}\{\cos \omega t\} = \dfrac{s}{s^2+\omega^2}.$ （$\mathrm{Re}\, s > 0$ のとき）

(7) $\mathcal{L}\{\sinh at\} = \dfrac{a}{s^2-a^2}.$ (8) $\mathcal{L}\{\cosh at\} = \dfrac{s}{s^2-a^2}.$ （$\mathrm{Re}\, s > |a|$ のとき）

(9) $\mathcal{L}\{\log t\} = -\dfrac{\log s + C}{s}.$ （$\mathrm{Re}\, s > 0$ のとき） ただし $C = \displaystyle\lim_{n\to\infty}\left(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log n\right)$

(10) ヘビサイド関数 $Y(t) = \begin{cases} 1 & (t > 0) \\ 0 & (t < 0). \end{cases}$ $a \geqq 0$ のとき $\mathcal{L}\{Y(t-a)\} = \dfrac{e^{-as}}{s}$

ラプラス変換の性質

[線型性] $\mathcal{L}\{c_1 f_1 + c_2 f_2\} = c_1 \mathcal{L}\{f_1\} + c_2 \mathcal{L}\{f_2\}$

[移動性] $\mathcal{L}\{e^{at} f(t)\} = F(s-a).$

[相似性] $\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right),$ $(\mathrm{Re}\, s > \alpha \cdot a).$

[導関数] $f(+0)$ が存在すれば $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f\} - f(+0)$

[積分] $\mathcal{L}\left\{\displaystyle\int_0^t f(u)\,du\right\} = \dfrac{1}{s} F(s).$

[t^n 倍] $\mathcal{L}\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n} F(s)$ $(n = 1, 2, 3, ...)$

[$1/t$ 倍] $\displaystyle\lim_{t\to+0} \frac{f(t)}{t}$ が存在するならば $\mathcal{L}\left\{\dfrac{f(t)}{t}\right\} = \displaystyle\int_s^\infty F(\sigma) d\sigma.$

[合成積] $f * g(t) = \displaystyle\int_0^t f(s) g(t-s)\,ds \to \mathcal{L}\{f * g\} = F(s)G(s).$

[初期値定理] $\displaystyle\lim_{t\to+0} f(t) = a$ が存在すれば $\displaystyle\lim_{s\to\infty} sF(s) = a.$

[最終値定理] $\displaystyle\lim_{t\to\infty} f(t) = a$ が存在すれば $\displaystyle\lim_{s\to+0} sF(s) = a.$

[原関数の移動性] $a \geqq 0$ のとき $\mathcal{L}\{f(t-a)Y(t-a)\} = e^{-as} F(s).$

[周期関数] $f(t)$ が周期 $\omega > 0$ の周期関数のとき $\mathcal{L}\{f(t)\} = \dfrac{1}{1-e^{-\omega s}} \displaystyle\int_0^\omega e^{-st} f(t)\,dt$

定理・部分分数分解

n 次多項式 $P(s)$ が相異なる n 個の零点 $a_1, ..., a_n$ を持つとき

$$\mathcal{L}^{-1}\left\{\frac{Q(s)}{P(s)}\right\} = \sum_{k=1}^n \frac{Q(a_k)}{P'(a_k)} e^{a_k t}.$$

双曲線函数 $\cosh z = \dfrac{e^z + e^{-z}}{2}, \quad \sinh z = \dfrac{e^z - e^{-z}}{2}, \quad \tanh z = \dfrac{\sinh z}{\cosh z} = \dfrac{e^z - e^{-z}}{e^z + e^{-z}}.$

ガンマ函数 $\Gamma(s) = \displaystyle\int_0^\infty e^{-x} x^{s-1}\, dx \quad (s > 0).$

ベータ函数 $B(p, q) = \displaystyle\int_0^1 x^{p-1}(1-x)^{q-1}\, dx = \dfrac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \quad (p, q > 0).$

(1) $\Gamma(s)$ は $s > 0$ で収束して $\Gamma(s) > 0$ (2) $\Gamma(s+1) = s\Gamma(s)$

(3) $\Gamma(1) = 1,\ \Gamma(n) = (n-1)!\ \ n = 1, 2, 3, \ldots$ (4) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

ガウス積分 $\displaystyle\int_0^\infty e^{-x^2} dx = \dfrac{\sqrt{\pi}}{2}$

例題と演習

例 A, ω, θ を定数とすると $\mathcal{L}\{A \sin(\omega t + \theta)\} = \dfrac{A(\omega \cos\theta + s \sin\theta)}{s^2 + \omega^2}.$

例 (1) $\mathcal{L}\{e^{at} t\} = \dfrac{1}{(s-a)^2}$ (2) $\mathcal{L}\{e^{at} \sin \omega t\} = \dfrac{\omega}{(s-a)^2 + \omega^2}$

問題. 次の関数のラプラス変換を求めよ.

(1) $\cos(\omega t + \theta)$ (2) $\sinh(\omega t + \theta)$ (3) $e^{2t} \cos \omega t$ (4) $e^{-t} \sinh t$ (5) $e^t t^2$.

[答] (1) $\dfrac{s \cos\theta - \omega \sin\theta}{\omega^2 + s^2}$ (2) $\dfrac{\omega \cosh\theta + s \sinh\theta}{s^2 - \omega^2}.$ (3) $\dfrac{s-2}{\omega^2 + (s-2)^2}$

(4) $\dfrac{1}{s^2 + 2s}$ (5) $\dfrac{2}{(s-1)^3}$

例. (1) $\mathcal{L}\left\{\displaystyle\int_0^t \sin \omega u\, du\right\} = \dfrac{1}{s} \dfrac{\omega}{s^2 + \omega^2}.$ (2) $\mathcal{L}\left\{\displaystyle\int_0^t \cos \omega u\, du\right\} = \dfrac{1}{s^2 + \omega^2}$

(3) $\mathcal{L}\left\{\displaystyle\int_0^t u e^{-2u}\, du\right\} = \dfrac{1}{s} \dfrac{1}{(s+2)^2}.$

例. (1) $\mathcal{L}\left\{t e^{at}\right\} = \dfrac{1}{(s-a)^2}.$ (2) $\mathcal{L}\{t \sin \omega t\} = \dfrac{2\omega s}{(s^2 + \omega^2)^2}.$

問題. 次の関数のラプラス変換を求めよ

(1) $t \cos \omega t$ (2) $t \cosh at$ (3) $t \sinh at$ (4) $\displaystyle\int_0^t \tau \sin \omega\tau\, d\tau$ (5) $\displaystyle\int_0^t \tau \cos \omega\tau\, d\tau$

[答] (1) $\dfrac{s^2 - \omega^2}{\left(\omega^2 + s^2\right)^2}$ (2) $\dfrac{a^2 + s^2}{\left(s^2 - a^2\right)^2}$ (3) $\dfrac{2as}{\left(a^2 - s^2\right)^2}$ (4) $\dfrac{2\omega}{(\omega^2 + s^2)^2}$ (5) $\dfrac{s^2 - \omega^2}{s\left(\omega^2 + s^2\right)^2}$

例. $s + i\omega = re^{i\theta}$ $(r > 0, 0 < \theta < \pi/2)$ とおく

$$\mathcal{L}\{t^n \cos \omega t\} = \frac{n! \cos(n+1)\theta}{r^{n+1}}, \quad \mathcal{L}\{t^n \sin \omega t\} = \frac{n! \sin(n+1)\theta}{r^{n+1}}.$$

例. $\mathcal{L}\left\{\dfrac{e^{-at}-e^{-bt}}{t}\right\}=\log\dfrac{s+b}{s+a}$.

例. $\mathcal{L}\left\{\dfrac{\sin\omega t}{t}\right\}=\dfrac{\pi}{2}-\tan^{-1}\dfrac{s}{\omega}$, $\quad\displaystyle\int_0^\infty \frac{\sin\omega t}{t}\,dt=\frac{\pi}{2}$.

問題. 次の関数のラプラス変換を求めよ

(1) $\dfrac{1-\cos at}{t}$ (2) $\dfrac{\sinh at}{t}$ (3) $\displaystyle\int_0^t \frac{\sin a\tau}{\tau}\,d\tau \quad (a>0)$

[答] (1) $\dfrac{1}{2}\log\left(\dfrac{a^2}{s^2}+1\right)$ (2) $\dfrac{1}{2}\log\left(\dfrac{s+1}{s-1}\right)$ (3) $\dfrac{1}{s}\tan^{-1}\dfrac{a}{s}$

問題. 積分 $\displaystyle\int_0^\infty te^{-2t}\cos t\,dt$ の値を求めよ.

[答] 3/25

問題. 次の各式を証明せよ

(1) $\mathcal{L}\left\{\dfrac{e^{at}-\cos bt}{t}\right\}=\log\dfrac{\sqrt{s^2+b^2}}{s-a}$. (2) $\mathcal{L}\left\{\displaystyle\int_0^t \frac{\sin^2\tau}{\tau}\,d\tau\right\}=\dfrac{1}{2s}\log\dfrac{\sqrt{s^2+4}}{s}$.

例. (1) $\mathcal{L}\left\{\displaystyle\int_0^t \sin a(t-\tau)\cos b\tau\,d\tau\right\}=\dfrac{a}{s^2+a^2}\dfrac{s}{s^2+b^2}$

(2) $\mathcal{L}\left\{\displaystyle\int_0^t e^{-a(t-\tau)}\sinh\omega\tau\,d\tau\right\}=\dfrac{1}{s+a}\dfrac{\omega}{s^2-\omega^2}$

問題. 次の積分のラプラス変換を求めよ.

(1) $\displaystyle\int_0^t e^{-2(t-\tau)}\cos\omega\tau\,d\tau$ (2) $\displaystyle\int_0^t e^{-a(t-\tau)}\cosh\omega\tau\,d\tau$

[答] (1) $\dfrac{s}{(s+2)(\omega^2+s^2)}$ (2) $\dfrac{s}{(a+s)(s^2-\omega^2)}$

例題. $a>0$ ：(1) $\mathcal{L}\{Y(t-a)\cos\omega(t-a)\}=\dfrac{e^{-as}s}{s^2+\omega^2}$, (2) $\mathcal{L}\left\{Y(t-a)(t-a)^2\right\}=\dfrac{2e^{-as}}{s^3}$

問題. $a>0$ とする。次の関数のラプラス変換を求めよ.

(1) $Y(t-a)(t-a)$, (2) $Y(t-a)\sin\omega(t-a)$

[答] (1) $\dfrac{e^{-as}}{s^2}$ (2) $\dfrac{e^{-as}\omega}{s^2+\omega^2}$

例題. $a>0,\ n=0,1,2,\ldots$ とする. $f(t)=\begin{cases}1 & (2na<t<(2n+1)a)\\ 0 & ((2n+1)a<t<(2n+2)a)\end{cases}$

$$\mathcal{L}\{f(t)\}=\frac{1}{1-e^{-2as}}\frac{1}{s}(1-e^{-as})=\frac{1}{s(1+e^{-as})}.$$

例題. $f(t) = \begin{cases} \cos t & (2n\pi < t < (2n+1)\pi) \\ 0 & ((2n+1)\pi < t < (2n+2)\pi) \end{cases}$

$$\mathcal{L}\{f(t)\} = \frac{s}{(s^2+1)(1+e^{-\pi s})}.$$

問題. 次の関数のラプラス変換を求めよ. $a > 0$, $n = 0, 1, 2, ...$ とする.

(1) $f(t) = \begin{cases} 1 & (2na < t < (2n+1)a) \\ -1 & ((2n+1)a < t < (2n+2)a) \end{cases}$

(2) $f(t) = \begin{cases} \sin t & (2n\pi < t < (2n+1)\pi) \\ 0 & ((2n+1)\pi < t < (2n+2)\pi) \end{cases}$

[答] (1) $\dfrac{1}{s}\tanh\dfrac{as}{2}$ (2) $\dfrac{1}{(s^2+1)(1-e^{-\pi s})}$

例.

$\mathcal{L}^{-1}\left\{\dfrac{1}{(s+1)(s-2)}\right\} = \dfrac{1}{3}(e^{2t} - e^{-t})$ $\mathcal{L}^{-1}\left\{\dfrac{s+3}{s(s^2+4)}\right\} = \dfrac{3}{4} + \dfrac{1}{2}\sin 2t - \dfrac{3}{4}\cos 2t$

$\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s^2+\omega^2)}\right\} = \dfrac{1}{\omega^3}(\omega t - \sin\omega t)$ $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s^2+\omega^2)}\right\} = \dfrac{1}{\omega^3}(\omega t - \sin\omega t)$

$\mathcal{L}^{-1}\left\{\dfrac{s-2}{s^2+4s+5}\right\} = e^{-2t}\cos t - 4e^{-2t}\sin t$

例. $\mathcal{L}^{-1}\left\{\dfrac{e^{-as}}{s^4}\right\} = \dfrac{(t-a)^3}{6}Y(t-a).$ $\mathcal{L}^{-1}\left\{\dfrac{e^{-as}}{s^2+\omega^2}\right\} = \dfrac{1}{\omega}Y(t-a)\sin\omega(t-a).$

問題. 次の関数の逆ラプラス変換を求めよ. $a > 0$ とする

(1) $\dfrac{s+a}{(s+b)^2}$ (2) $\dfrac{1}{(s+3)(s+4)(s+5)}$ (3) $\dfrac{1}{s^2(s^2-\omega^2)}$ (4) $\dfrac{e^{-as}}{s^2}$ (5) $\dfrac{e^{-as}}{s(s-1)}$

[答] (1) $e^{-bt}((a-b)t+1)$ (2) $\dfrac{e^{-5t}}{2} - e^{-4t} + \dfrac{e^{-3t}}{2}$ (3) $\dfrac{\sinh\omega t}{\omega^3} - \dfrac{t}{\omega^2}$
(4) $Y(t-a)(t-a)$ (5) $Y(t-a)(e^{t-a}-1)$

例. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)(s+1)^3}\right\} = \dfrac{1}{8}e^t - \dfrac{1}{8}e^{-t} - \dfrac{1}{4}e^{-t}t - \dfrac{1}{4}e^{-t}t^2$

例. $\mathcal{L}^{-1}\left\{\dfrac{s+2}{s^2(s-1)^2}\right\} = 5 + 2t - 5e^t + 3e^t t$

問題. 次の関数の逆ラプラス変換を求めよ

(1) $\dfrac{s+2}{s^3(s-1)^2}$ (2) $\dfrac{s+2}{(s+1)^2(s+3)}$ (3) $\dfrac{s^3+5}{s^3(s+1)}$

[答] (1) $8 + 5t + t^2 - 8e^t + 3te^t$ (2) $\dfrac{1}{4}e^{-t} + \dfrac{1}{2}te^{-t} - \dfrac{1}{4}e^{-3t}$ (3) $5 - 5t + \dfrac{5}{2}t^2 - 4e^{-t}$

問題. ラプラス変換を用いて次の微分方程式を解け

(1) $y'' + 4y = \sin t,\ y(0) = 0,\ y'(0) = 0$
(2) $y''' + y' = e^t,\ y(0) = 0,\ y'(0) = 0,\ y''(0) = 0$
(3) $y'' + y = t,\ y(\pi) = 0,\ y'(0) = 1$
(4) $y'''' - 2y''' + 5y' = 0,\ y(0) = 0,\ y'(0) = 1,\ y(\pi/4) = 0$

[答] (1) $y = \dfrac{1}{3}\sin t - \dfrac{1}{6}\sin 2t$
(2) $y = -1 + \dfrac{1}{2}\left(e^t + \cos t - \sin t\right)$
(3) $y = \pi \cos t + t$
(4) $y = \dfrac{c-2}{5}\left(1 - \dfrac{1}{5}e^t \cos t\right) + \dfrac{c+3}{10}e^t \sin t \quad \left(c = \dfrac{14 - 3e^{\pi/4}}{2 + e^{\pi/4}}\right)$

問題. 次の微分方程式を解け

(1) $\begin{cases} \dot{y}_1 &= 3y_1 + 4y_2 \\ \dot{y}_2 &= 3y_1 + 2y_2 \end{cases}$ (2) $\begin{cases} \dot{y}_1 &= 4y_1 + \ y_2 \\ \dot{y}_2 &= -y_1 + 2y_2 \end{cases}$ (3) $y''+13y'+42y = 0$ (4) $y''+14y'+49y = 0$

[答] (1) $y_1 = c_1\left(4e^{6x} + 3e^{-x}\right) + 4c_2\left(e^{6x} - e^{-x}\right),\ y_2 = 3c_1\left(e^{6x} - e^{-x}\right) + c_2\left(3e^{6x} + 4e^{-x}\right)$
(2) $y_1 = c_1e^{3x}(x+1) + c_2e^{3x}x,\ y_2 = -c_1xe^{3x} - c_2e^{3x}(x-1)$
(3) $y = c_1e^{-7x} + c_2e^{-6x}$ (4) $y = c_1e^{-7x} + c_2xe^{-7x}$

問題. 次の微分方程式を解け

(1) $y' = z + t,\ z' = -y + t^2 + 1$ (2) $y' = z + e^t(1-t),\ z' = -y + e^t(t+2)$

[答] (1) $y = C_1 \sin t + C_2 \cos t + t^2, z = C_1 \cos t - C_2 \sin t + t$
(2) $y = C_1 \sin t + C_2 \cos t + e^t, z = C_1 \cos t - C_2 \sin t + te^t$

問題. 次の微分方程式を解け

(1) $y' = 2y - z,\ z' = 3y - 2z$
(2) $y' = y - 2z,\ z' = y + 3z$
(3) $y' = z,\ z' = -y + 2z$
(4) $y' = 2y + z + e^t,\ z' = 2y + 3z + 5e^t$
(5) $y' = z + w,\ z' = y + w,\ w' = y + z$

[答] (1) $y = c_1\left(3e^x - e^{-x}\right) - c_2\left(e^x - e^{-x}\right),\ z = 3c_1\left(e^x - e^{-x}\right) - c_2\left(e^x - 3e^{-x}\right)$
(2) $y = c_1e^{2x}(\cos x - \sin x) - 2c_2e^{2x}\sin x,\ z = c_1e^{2x}\sin x + c_2e^{2x}(\sin x + \cos x)$
(3) $y = c_2e^xx - c_1e^x(x-1),\ z = c_2e^x(x+1) - c_1e^x$
(4) $y = c_1\left(e^{4x} + 2e^x\right) + c_2e^{4x} - c_2e^x - (3x+2)e^x,\ z = 2c_1\left(e^{4x} - e^x\right) + 2c_2e^{4x} + c_2e^x + (3x-4)e^x.$
(5) $y = (c_1+c_2+c_3)e^{2x} + (2c_1-c_22-c_3)e^{-x},\ z = (c_1+c_2+c_3)e^{2x} + (-c_1+2c_2-c_3)e^{-x},\ w = (c_1+c_2+c_3)e^{2x} + (-c_1-c_2+2c_3)e^{-x}.$

問題. 次の微分方程式を解け

(1) $\begin{cases}(D^2+2)x+Dy+Dz=0,\\ Dx+\quad y=0,\\ 2x-Dy-Dz=0.\end{cases}$ (2) $\begin{cases}(D+1)x+(D^2+D+1)y=0,\\ Dx+(D^2+1)y=0.\end{cases}$

(3) $\begin{cases}(D^2-2)x+Dy=\cos t,\\ Dx+(D^2-2)y=e^t\sin t.\end{cases}$

[答] (1) $x=c_1e^{-4t},\ y=4c_1e^{-4t}, z=\dfrac{1}{2}c_1e^{-4t}+c_2$
(2) $x=0, y=0$
(3) $x=c_1e^{-2t}+c_2e^{-t}+c_3e^t+c_4e^{2t}+\dfrac{1}{10}\left(e^t\sin t-e^t\cos t-3\cos t\right),$
$y=c_1e^{-2t}-c_2e^{-t}+c_3e^t-c_4e^{2t}+\dfrac{1}{10}\left(-2e^t\sin t+\sin t-2e^t\cos t\right)$

問題. 次の行列の指数関数をもとめよ

(1) $\begin{bmatrix}5&-6\\3&-4\end{bmatrix}$ (2) $\begin{bmatrix}0&1\\8&-2\end{bmatrix}$ (3) $\begin{bmatrix}0&1\\-9&6\end{bmatrix}$ (4) $\begin{bmatrix}0&1\\1&0\end{bmatrix}$

(5) $\begin{bmatrix}0&1&2\\0&0&3\\0&0&0\end{bmatrix}$ (6) $\begin{bmatrix}2&0&0\\0&3&0\\0&1&3\end{bmatrix}$ (7) $\begin{bmatrix}i&0\\0&-i\end{bmatrix}$

[答] (1) $\begin{bmatrix}-\frac{1}{e}+2e^2&\frac{2}{e}-2e^2\\-\frac{1}{e}+e^2&\frac{2}{e}-e^2\end{bmatrix}$ (2) $\dfrac{1}{6}\begin{bmatrix}4e^2+2e^{-4}&e^2-e^{-4}\\8e^2-8e^{-4}&2e^2+4e^{-4}\end{bmatrix}$ (3) $e^3\begin{bmatrix}-2&1\\-9&4\end{bmatrix}$
(4) $\begin{bmatrix}\cosh 1&\sinh 1\\\sinh 1&\cosh 1\end{bmatrix}$ (5) $\begin{bmatrix}1&1&\frac{7}{2}\\0&1&3\\0&0&1\end{bmatrix}$ (6) $\begin{bmatrix}e^2&0&0\\0&e^3&0\\0&e^3&e^3\end{bmatrix}$
(7) $\begin{bmatrix}\cos 1+i\sin 1&0\\0&\cos 1-i\sin 1\end{bmatrix}$

問題. $e^{A+B}\neq e^Ae^B$ となる行列 A,B の例を作れ

[答] $A=\begin{bmatrix}0&0\\1&0\end{bmatrix},\ B=\begin{bmatrix}0&1\\0&0\end{bmatrix}.\ e^A=\begin{bmatrix}1&0\\1&1\end{bmatrix},\ e^B=\begin{bmatrix}1&1\\0&1\end{bmatrix}$ より $e^Ae^B=\begin{bmatrix}1&1\\1&2\end{bmatrix}.$
e^{A+B} は前問 (4).

問題. 3. 次の微分方程式を解け

(1) $x'=2x-y,\ y'=2y$ (2) $x'=2x-y,\ y'=x+2y$

(3) $x'=y,\ y'=x$ (4) $x'=-2x,\ y'=x-2y,\ z'=y-2z$

(5) $x'=y,\ y'=2-x$ (6) $x'=y,\ y'=-4x+\sin 2t$

(7) $x'=x+y+z,\ y'=-2y+t,\ z'=2z+\sin t$

[答] (1) $x = C_1 e^{2t} - C_2 t\, e^{2t},\ y = C_2 e^{2t}$
(2) $x = C_1 e^{2t}\cos t - C_2 e^{2t}\sin t,\ y = C_1 e^{2t}\sin t + C_2 e^{2t}\cos t$
(3) $x = C_1\sinh t + C_2\cosh t,\ y = C_2\sinh t + C_1\cosh t$
(4) $x = C_1 e^{-2t}$, $y = C_1 e^{-2t} t + c_C e^{-2t},\ z = \frac{1}{2} C_1 e^{-2t} t^2 + C_2 e^{-2t} t + C_3 e^{-2t}$
(5) $x = C_2 \sin t + C_1\cos t + 2$, $y = C_2\cos t - C_1 \sin t$
(6) $x = \dfrac{1}{16}(8c_2 + 1)\sin 2t - \dfrac{1}{4}(t - 4c_1)\cos 2t$, $y = \left(c_2 - \dfrac{1}{8}\right)\cos 2t + \dfrac{1}{2}(t - 4c_1)\sin 2t$
(7) $x = -\dfrac{1}{3} c_2 e^{-2t} + \dfrac{1}{3}(3c_1 + c_2 - 3c_3)\, e^t + c_3 e^{2t} + \dfrac{1}{20}(-10t + 2\sin t + 6\cos t - 5)$, $y = c_2 e^{-2t} + \dfrac{t}{2} - \dfrac{1}{4}$, $z = c_3 e^{2t} - \dfrac{2\sin t}{5} - \dfrac{\cos t}{5}$