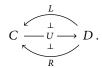
EXERCISES ON ADJUNCTIONS

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Notation. We denote an adjunction by an opposing pair of arrows: $F: C \rightleftharpoons D: G$. In this notation, F will always be the left adjoint and G the right adjoint.

Exercise 1. Let $F\colon C\rightleftarrows D\colon G$ be an adjunction. Prove that the right adjoint G is fully faithful if and only if the counit $\varepsilon\colon FG\Rightarrow \operatorname{id}_D$ is a natural isomorphism. Notice that in this case the left adjoint F is essentially surjective on objects. Dually, F is fully faithful if and only if the unit $\eta\colon \operatorname{id}_C\Rightarrow GF$ is a natural isomorphism.

Exercise 2. Let *C* and *D* be categories and suppose that we are given adjunctions



Prove that L is fully faithful if and only if R is fully faithful.

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