EXERCISES AROUND GIRAUD'S AXIOMS

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Exercise 1.

(1.a) Prove that the ω-compact objects of **Set** are the finite sets. Use this to prove that **Set** is ω-presentable.

(1.b) Let C be a small category. Prove that the functor category $\mathbf{Set}^C$ is ω-presentable.

(1.c) Let D be a κ-presentable category and $i\colon D' \hookrightarrow D$ a localization of D with localization functor $L\colon D \to D'$. Prove that if i preserves κ-filtered colimits, then D' is κ-presentable.

Definition. Let D be a category with finite coproducts, and let $\varnothing$ denote the initial object of D (i.e., the empty coproduct). We say that ***coproducts are disjoint*** in D if for any objects $X, Y \in D$, the square

$$\begin{tikzcd} \varnothing \arrow[r] \arrow[d] & Y \arrow[d, "\iota_Y"] \\ X \arrow[r, "\iota_X"'] & X \sqcup Y \end{tikzcd}$$

is a pullback square in D.

Exercise 2.

(2.a) Prove that coproducts in **Set** are disjoint

(2.b) Let I be a small category and D a category admitting finite coproducts and I-shaped colimits. Prove that if coproducts are disjoint in D, then coproducts are disjoint in D^I.

(2.c) Let D be a category admitting finite coproducts and $D' \hookrightarrow D$ a localization of D with localization functor $L\colon D \to D'$. Prove that if coproducts are disjoint in D, then coproducts are disjoint in D'.

Exercise 3. Let C be a category, and $F \in \mathbf{PShv}(C)$. Prove that the slice category $\mathbf{PShv}(C)_{/F}$ is (equivalent to) a presheaf category.

Hint: The category of elements.

Date: January 29, 2018.