Pokhara Engineering College

Level : Bachelor **Semester : 2024 Spring**

Programme : BE Computer **Full Mark : 100**

Course : Applied Maths **Pass Mark :40**

Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable. Figures in margin indicate full marks.

Attempt all the questions

1) a) Define harmonic function. Verify that $u = \cos x \cosh y$ is harmonic and its harmonic conjugate. (8)

b) Integrate the function $f(z) = \frac{1}{z^2+4}$ over the given contour c counterclockwise where c is the ellipse $4x^2 + (y-2)^2 = 4$. (7)

2) a) State Cauchy Residue theorem. Evaluate $\int_c \frac{z+1}{z^4-2z^3}dz$ where $c: |z| = \frac{1}{2}$ counterclockwise. (8)

b) Find the Taylor and Laurent's series of $f(z) = \frac{2z-3i}{z^2-3iz-2}$ in the region

(i) $0 < |z| < 1$ (ii) $|z| > 2$ (7)

3) a) Find the Z-transform of $e^{\frac{in\pi}{2}}$ and then deduce the value of $Z\left(\cos\frac{n\pi}{2}\right)$ and $Z\left(\sin\frac{n\pi}{2}\right)$. (8)

b) State and prove first shifting theorem of Z-transform. Use it to evaluate $Z(na^n)$ and $Z(e^{-at})$. (7)

4) a) Find the inverse z-transform of $F(z) = \frac{3z^2+2z+1}{z^2+3z+2}$ (8)

OR

Solve using Z-transform

$y_{n+2} - 3y_{n+1} + 2y_n = 4^n, y_0 = 0, y_1 = 1$

b) Derive one dimensional wave equation of a string of length L which is fixed in two end points with necessary assumptions. (7)

OR

Find the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with initial temperature $f(x)$ and boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$.

5) a) Solve by using separation of variables

(i) $u_x + u_y = 0$ (ii) $u_{xy} - u = 0$. (8)

b) Express the Laplacian in polar coordinate system from cartesian co-ordinate system. (7)

OR

Find the temperature distribution in a laterally insulated thin copper bar $(c^2 = 1.158\ cm^2/sec)$. 100cm long and of constant cross section whose end points at $x = 0$ and $x = 100$ are kept at $0°C$ and whose initial temperature is

(i) $f(x) = \sin(0.01)\pi x$ (ii) $f(x) = \sin^3(0.01)\pi x$

6) a) Show that:

$$\int_0^\infty \left[\frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2}\right] d\omega = \begin{cases} 0 \text{ if } x < 0 \\ \frac{\pi}{2} \text{ if } x = 0 \\ \pi e^{-x} \text{ if } x > 0 \end{cases}$$ (8)

b) Find the Fourier cosine transform of $f(x) = e^{-mx}; m > 0$ and then show that $\int_0^\infty \frac{\cos kx}{1+x^2}dx = \frac{\pi}{2}e^{-k}$ (7)

7) Attempt any two (2×5=10)

a) Show that $z\bar{z}$ is not an analytic function.

b) Find the location & order of zeros of $(z^2+1)(e^z-1)$.

c) Verify $u = e^{-t}\sin x$ to satisfy one dimensional heat equation.