POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Algebra and Geometry

Semester: Spring

Year : 2024
Full Marks: 100
Pass Marks: 45
Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Check whether the given system of linear equations is consistent or not, if consistent then solve the system of linear equations $2x + 5y + 6z = 13,\ 3x + y - 6z = 13,\ x - 3y - 8z = -13.$ 7

   b) i) State Cayley Hamilton theorem. Verify it for the matrix $A = \begin{pmatrix} 1 & -2 \\ 4 & 5 \end{pmatrix}$. 4+4

   ii) Check whether the set of vectors $\{(1,1,0),\ (1,0,1),\ (3,1,1)\}$ form a basis for $\mathbb{R}^3$ or not.

2. a) Define eigen value and eigen vector. Find the eigen values and corresponding eigen vectors of $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. 7

   b) Convert the following primal LPP into dual LPP and solve by using simplex method: Minimize $Z = 8x_1 + 9x_2$ subject to $x_1 + 3x_2 \geq 4,\ 2x_1 + x_2 \geq 5,\ x_1 \geq 0,\ x_2 \geq 0.$ 8

3. a) Find the set of reciprocal system of vectors to $\vec{a} = 2\vec{i} + 3\vec{j} - 2\vec{k}$, $\vec{b} = \vec{i} - \vec{j} - 2\vec{k}$ and $\vec{c} = -\vec{i} + 2\vec{j} + 2\vec{k}$. 7

   b) Find the interval of convergence, centre of convergence and radius of convergence of an infinite series $\sum \frac{n^2}{2^{3n}}(x+4)^n$. 8

4. a) State Cauchy root test. Test the convergence of the following infinite series: 7

   i. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$

   ii. $\sum \left(\frac{2n}{n+1}\right)^n$

   b) Find the vertex, eccentricity, foci and equation of directrix of the ellipse: $3x^2 + 4y^2 - 12x - 8y + 4 = 0.$ 8

5. a) Find the condition for the line $y = mx + c$ to be the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Also, find the point of contact. 7

OR

Sketch and describe the polar conic $r = \frac{12}{3+2\cos\theta}$.

b) Define skew lines. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. Also, find the equation of the line of shortest distance. 8

6. a) i. Find the equation of the cone with vertex at the origin and which passes through the curve of intersection of $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = p$. 2

ii. Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. 5

b) Find the centre and radius of the circle $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0,\ x - 2y + 3z = 3$. 8

OR

Show that the plane $2x - 2y + z = -12$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$. Also, find the point of contact.

7. Attempt all the questions: 4×2.5

a) Check whether the transformation $T : \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = x + y$ is linear or not.

b) Find the volume of the parallelopiped whose concurrent edges are given by: $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{b} = 3\vec{i} + 4\vec{j} - 5\vec{k}$, $\vec{c} = \vec{i} - 2\vec{j} + 3\vec{k}$.

c) Transform the equation $x^2 + 3xy + y^2 = 0$ in which the origin is transformed to (2, 3) with axes remaining parallel to the old axes.

d) Show that the line joining the points (–2, 1, 3) and (1, –3, 4) is parallel to the plane $2x + 3y + 6z + 5 = 0$.