POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Applied Mathematics

Semester: Spring

Year : 2024
Full Marks: 100
Pass Marks: 45
Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) Define analyticity of a function f(z). Show that the necessary condition for the function $f(z) = u(x,y) + iv(x,y)$ to be analytic on a domain D is $u_x = v_y$ and $u_y = -v_x$ at each point (x,y) of D. 7

   b) State and prove Cauchy integral formula. Evaluate the integral $\oint_C \frac{e^{5z}}{(z+i)^4} dz$, where c: $|z| = 2$. 8

OR

State Cauchy's residue theorem and using it, evaluate:
$\oint_C \frac{2z}{(z+1)(z-1)^3(z+3)} dz$ where c:$|z| = 2$ counter clockwise.

2. a) Find the expansion of $\frac{7z-2}{z(z+1)(z-2)}$ in the region given by 7
   i. $0 < |z+1| < 1$.
   ii. $1 < |z+1| < 3$.

   b) Define bilinear transformation. Find the bilinear transformation which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$. Also, find the image of the unit circle $|z| = 1$. 8

3. a) State and prove first shifting theorem on Z-transform. Find Z-transform of $e^{\frac{in\pi}{2}}$ and then find $Z(\cos\frac{n\pi}{2})$ and $Z(\sin\frac{n\pi}{2})$. 7

   b) Solve the difference equation $y_{n+2} - 7y_{n+1} + 12y_n = 2n$, $y_0 = 0$, $y_1 = 0$ by using Z-transform. 8

4. a) Show that $Z[nf(t)] = -z\frac{d}{dz}[F(z),]$ where F(z) = $Z[f(t)]$. 7
   Find $Z^{-1}\left[\frac{z}{(z+1)^2(z-1)}\right]$.

   b) Find the solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ with initial velocity $g(x)$, initial deflection $f(x)$ and boundary condition u(0, L) = 0 = u(L, t). 8

5. a) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar coordinates. 7

OR

Derive two-dimensional heat equation completely with necessary assumptions.

b) Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is $f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$. 8

6. a) Find the Fourier cosine transform of $f(x) = e^{-mx}$ for $m > 0$, and then show that $\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$. 7

b) Show that $\int_0^\infty \frac{\cos wx + \dot{w} \sin wx}{1+w^2} dw = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$. 8

7. Attempt all the questions: 4×2.5

a) Check analyticity of $f(z) = z^2$.

b) Find $Z(a^n)$.

c) Find the solution of the partial differential equation $u_{xx} + 9u = 0$.

d) Define linear partial differential equation with suitable example.