

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Algebra and Geometry

Semester: Fall

Year : 2024
Full Marks : 100
Pass Marks : 45
Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Check the given system of linear equations is consistent or not, if consistent then solve: 7
$$x + y + z = 1 \quad x + 2y + 3z = -4 \quad x + 3y + 7z = 13$$
- b) Find eigenvalue and eigenvector of the matrix, $\begin{bmatrix} 2 & 1 & 2 \\ 0 & -1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. 8
2. a) i. Let $V = \mathbb{R}^2$ be a vector space. Show that 3
 $W = \{(x, y) : x + 2y = 0\}$ is a vector sub-space of V .
ii. Check the vectors $(1, 2, 1), (2, 1, 0), (1, -1, 2)$ form a basis for \mathbb{R}^3 or not. 4
- b) Solve the given LPP by using appropriate simplex method. 8
$$\text{Max } Z = 2x_1 + x_2$$
$$\text{S.t. } 2x_1 - x_2 \geq 2 ; x_1 - x_2 \leq 2 ; x_1 + x_2 \leq 4 ; x_1 \geq 0, x_2 \geq 0.$$

OR

Construct the dual problem and solve by simplex method.

Minimize $Z = 8x_1 + 9x_2$ subject to constraints $x_1 + x_2 \geq 6,$
 $3x_1 + x_2 \geq 21 \quad x_1, x_2 \geq 0.$
3. a) State and prove p-series test. 7
b) Find the interval and radius of convergence of the series, 8
$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}.$$
4. a) Find the value of λ when the line $y = 2x + \lambda$ is tangent to the hyperbola $3x^2 - y^2 = 3$. 7

- b) Show that the equations $9x^2 - 24xy + 16y^2 - 50x - 100y + 225 = 0$ represents a parabola. Also, determine its equation of axis, vertex, latus rectum, focus, and directrix. 8

5. a) Examine, whether the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar or not. If so, find the equation of plane containing them. 7

OR

Find the length and equation of the shortest distance between the lines

$$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1} \text{ and } \frac{x-2}{3} = \frac{1-y}{5} = \frac{z+2}{2}.$$

- b) Find the centre and radius of circle $x^2 + y^2 + z^2 + 12x - 13y - 16z + 111 = 0, 2x + 2y + z = 17$. 8
6. a) Find the equation of right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 - x - y - z = 0, x + y + z = 1$. 7

- b) i. If $\vec{a}, \vec{b}, \vec{c}$ be any three vectors. 4

Show that $[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$.

- ii. Define reciprocal system of vectors. Find the reciprocal of \vec{a} for $\vec{a} = (-1, 2, 2), \vec{b} = (2, 3, 1)$ and $\vec{c} = (1, -1, -2)$. 4

7. Attempt all the questions 4×2.5

- a) Find the area of the triangle whose vertices are $(-1, 1, 2), (2, 5, -1), (1, 2, 3)$.
- b) Define cross product of four vectors.
- c) Find the equation of plane through the origin and having direction cosines proportional to $(5, 2, -3)$ and $(2, 1, -2)$.
- d) For what value of 'k' the plane $x - 2y - 2z = k$ touches the sphere $x^2 + y^2 + z^2 + 4y - 2x - 6z = -5$.