CS 291

Homework 7

Jingbo Wang
jw6347

Section 11.1, Exercise 2.b Find a regular expression to describe each of the following languages.
$\{aa, ab, ac\}$

Answer:

$$\begin{aligned}\{aa, ab, ac\} &= \{a\}\{a, b, c\} \\ &= a(a + b + c)\end{aligned}$$

Section 11.1, Exercise 2.d Find a regular expression to describe each of the following languages.
$\{a, aaa, aaaaa, ..., a^{2n+1}, ...\}$

Answer:

$$\begin{aligned}\{a, aaa, aaaaa, ..., a^{2n+1}, ...\} &= \{a\}\{\Lambda, aa, aaaa, ..., a^{2n}, ...\} \\ &= \{a\}\{\Lambda, aa, (aa)^2, ..., (aa)^n, ...\} \\ &= \{a\}\{aa\}^* \\ &= a(aa)^*\end{aligned}$$

Section 11.1, Exercise 2.e Find a regular expression to describe each of the following languages.
$\{\Lambda, a, abb, abbbb, ..., ab^{2n}, ...\}$

Answer:

$$\begin{aligned}\{\Lambda, a, abb, abbbb, ..., ab^{2n}, ...\} &= \{\Lambda\} + \{a\}\{\Lambda, bb, ..., b^{2n}, ...\} \\ &= \{\Lambda\} + \{a\}\{\Lambda, bb, ..., (bb)^n, ...\} \\ &= \{\Lambda\} + \{a\}\{bb\}^* \\ &= \Lambda + a(bb)^*\end{aligned}$$

Section 11.1, Exercise 4.b Find a regular expression for each of the following languages over the alphabet {a, b}.
Strings whose length is a multiple of 3.

Answer:

Strings whose length is a multiple of 1 over {a,b} is (a + b).
Strings whose length is a multiple of 3 over {a,b} is (a + b)(a + b)(a + b) = $(a + b)^3$.

Therefore, the regular expression is $((a+b)^3)^*$.

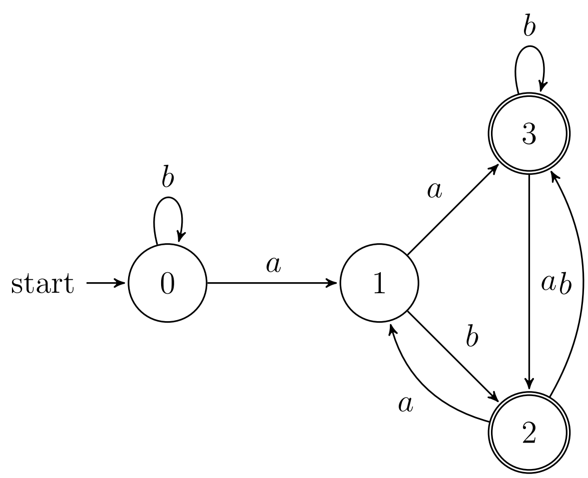
Section 11.1, Exercise 4.c Find a regular expression for each of the following languages over the alphabet {a, b}.
Strings containing the substring *aba*.

Answer:

$(a + b)^*$ describe all the string of one or more a's and one or more b's,
and each string must have substring *aba*.

Therefore, the regular expression is $(a + b)^*aba(a + b)^*$.

Section 11.2, Exercise 1 Write down the transition function for the following DFA.



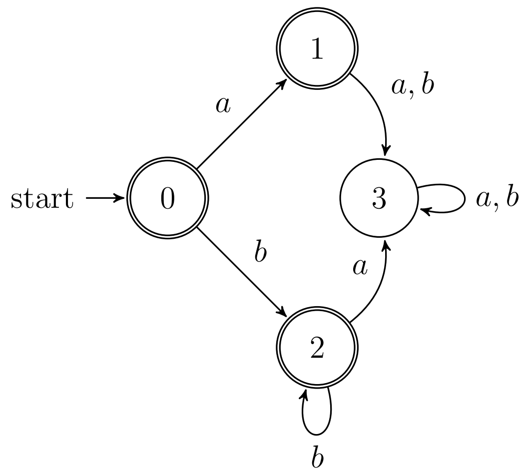
Answer:

$T(0, b) = 0$,
$T(0, a) = T(2, a) = 1$,
$T(1, a) = T(3, a) = 2$,
$T(1, a) = T(3, b) = T(2, b) = 3$,
where 0 is the start and both 2 and 3 are final states.

	T	a	b
Start	0	1	0
	1	3	2
	2	1	3
Final	3	2	3

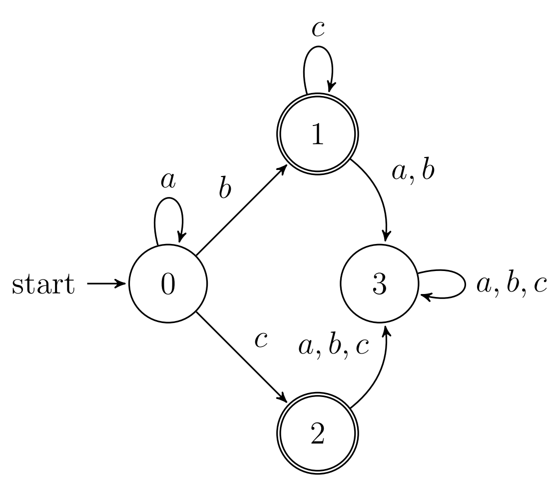
Section 11.2, Exercise 2.c Use your wits to construct a DFA for each of the following regular expressions.
a + b*

Answer:

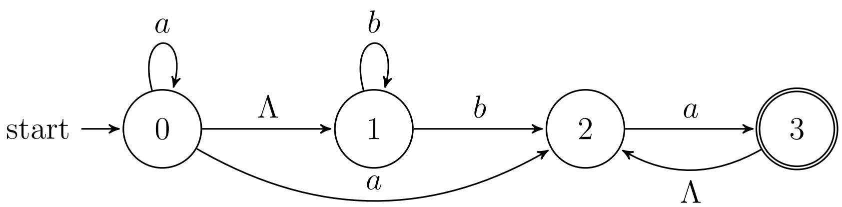


Section 11.2, Exercise 2.f Use your wits to construct a DFA for each of the following regular expressions.
a*bc* + ac

Answer:



Section 11.2, Exercise 4 Write down the transition function for the following NFA:



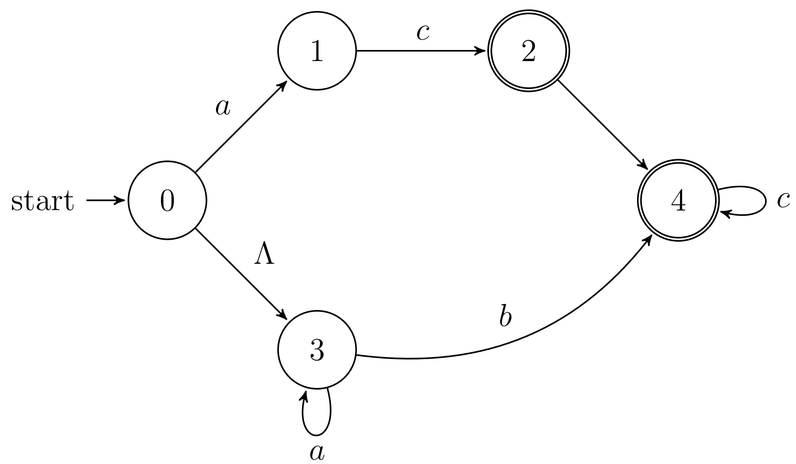
Answer:

$T(0, \text{a}) = 0,$
$T(0, \Lambda) = T(1, \text{b}) = 1,$
$T(1, \text{b}) = T(0, \text{a}) = T(3, \Lambda) = 2,$
$T(2, \text{a}) = 3,$
where 0 is the start and 3 is final state.

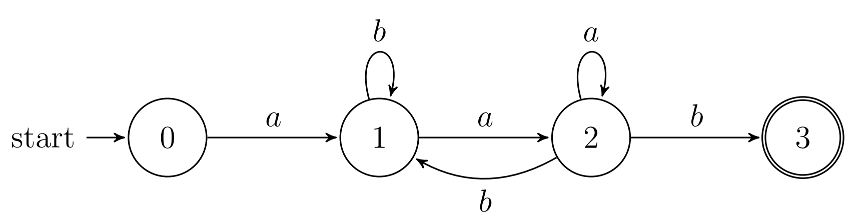
	T	a	b	Λ
Start	0	0	$\varnothing$	1
	1	$\varnothing$	2	$\varnothing$
	2	3	$\varnothing$	$\varnothing$
Final	3	$\varnothing$	$\varnothing$	2

Section 11.2, Exercise 5,a Use your wits to construct an NFA for each of the following regular expressions.
a*bc* + ac

Answer:



Section 11.2, Exercise 8,a Given the following NFA:



Use algorithm (11.2.4) to find two regular expressions for the language accepted by the NFA as follows.
Delete state 1 before deleting state 2.

Answer:

$$
\begin{aligned}
new(0,2) &= old(0,2) + old(0,1)old(1,1)^{*}old(1,2)\\
&= \varnothing + ab^{*}a\\
&= ab^{*}a,\\
new(2,2) &= old(2,2) + old(2,1)old(1,1)^{*}old(1,2)\\
&= a + bb^{*}a,\\
new(0,3) &= old(0,3) + old(0,2)old(2,2)^{*}old(2,3)\\
&= \varnothing + ab^{*}a(a+bb^{*}a)^{*}b\\
&= ab^{*}a(a+bb^{*}a)^{*}b.
\end{aligned}
$$

The final DFA is ab*a(a+bb*a)*b

Section 11.2, Exercise 8.b Given the following NFA:
Delete state 2 before deleting state 1.

Answer:

$$
\begin{aligned}
new(1,3) &= old(1,3) + old(1,2)old(2,2)^{*}old(2,3)\\
&= \varnothing + aa^{*}b\\
&= aa^{*}b,\\
new(1,1) &= old(1,1) + old(1,2)old(2,2)^{*}old(2,1)\\
&= b + aa^{*}b,\\
new(0,3) &= old(0,3) + old(0,1)old(1,1)^{*}old(1,3)\\
&= \varnothing + a(b+aa^{*}b)^{*}aa^{*}b\\
&= a(b+aa^{*}b)^{*}aa^{*}b.
\end{aligned}
$$

The final DFA is a(b+aa*b)aa*b.