18/18

CS 291
Homework 2

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Section 6.3, problem 5.d Give a formal proof for each of the following tautologies by using the CP rule. Do not use the IP rule.

Prove $(B \to C) \to (A \wedge B \to A \wedge C)$ is a tautology.

Answer:

1.	$B \to C$	P
2.	$\quad A \wedge B$	P [for $A \wedge B \to A \wedge C$]
3.	$\quad A$	2, Simp
4.	$\quad B$	2, Simp
5.	$\quad C$	1, 4, MP
6.	$\quad A \wedge C$	3, 5, Conj
7.	$A \wedge B \to A \wedge C$	2-6, CP
	QED	1,7, CP

3/3

Section 6.3, problem 5.e Give a formal proof for each of the following tautologies by using the CP rule. Do not use the IP rule.

Prove $(A \vee B \to C \wedge D) \to (B \to D)$ is a tautology.

Answer:

1.	$A \vee B \to C \wedge D$	P
2.	$\quad B$	P [for$B \to D$]
3.	$\quad A \vee B$	2, Add
4.	$\quad C \wedge D$	1, 3, MP
5.	$\quad D$	4, Simp
6.	$B \to D$	2-5, CP
	QED	1,6, CP

3/3

Section 6.3, problem 6.d Give a formal proof for each of the following tautologies by using the CP rule and by using the IP rule at least once in each proof.

Prove $(A \to C) \to (A \to B \vee C)$ is a tautology.

Answer:

1.	$A \to C$	P
2.	$\quad A$	P [for $A \to B \vee C$]
3.	$\quad\quad \neg C$	P [for C]
4.	$\quad\quad C$	1, 2, MP
5.	$\quad\quad False$	3, 4, Contr
6.	$\quad C$	3-5, IP
7.	$\quad B \vee C$	6, Add
8.	$A \to B \vee C$	2, 6, 7, CP
	QED	1, 8, CP

weird but okay

3/3

Section 6.3, problem 6.e Give a formal proof for each of the following tautologies by using the CP rule and by using the IP rule at least once in each proof.

Prove $(A \to B) \to ((A \to \neg B) \to \neg A)$ is a tautology.

Answer:

1.	$A \to B$	P
2.	$\quad A \to \neg B$	P [for $(A \to \neg B) \to \neg A$]
3.	$\quad\quad \neg\neg A$	P [for $\neg A$]
4.	$\quad\quad A$	3, DN
5.	$\quad\quad B$	1, 4, MP
6.	$\quad\quad \neg B$	2, 4, MP
7.	$\quad\quad False$	5, 6, Contr
8.	$\quad \neg A$	3-7, IP
9.	$(A \to \neg B) \to \neg A$	2, 8, CP
	QED	1, 9, CP

3/3

Section 6.3, problem 7.c Give a formal proof for each of the following tautologies by using the CP rule and by using the IP rule at least once in each proof.

Prove $(A \vee B \to C \wedge D) \to (B \to D)$ is a tautology.

Answer:

1.	$A \vee B \to C \wedge D$	P
2.	$\quad B$	P [for $B \to D$]
3.	$\quad A \vee B$	2, Add
4.	$\quad C \wedge D$	1, 3, MP
5.	$\quad\quad \neg D$	P [for D]
6.	$\quad\quad D$	4, Simp
7.	$\quad\quad False$	5, 6, Contr
8.	$\quad D$	5-7, IP
9.	$B \to D$	2, 9, CP
	QED	1, 9, CP

3/3

Section 6.3, problem 7.d Give a formal proof for each of the following tautologies by using the CP rule and by using the IP rule at least once in each proof.

Prove $(A \vee B \to C) \wedge (C \to D \wedge E) \to (A \to D)$ is a tautology.

Answer:

1.	$A \vee B \to C$	P
2.	$C \to D \wedge E$	P
3.	$\quad A$	P [for $A \to D$]
4.	$\quad A \vee B$	3, Add
5.	$\quad C$	1, 4, MP
6.	$\quad D \wedge E$	2, 5, MP
7.	$\quad\quad \neg D$	P [for D]
8.	$\quad\quad D$	6, Simp
9.	$\quad\quad False$	7, 8 Contr
10.	$\quad D$	7-9, IP
11.	$A \to D$	3,10, CP
	QED	1, 2, 11, CP

3/3