CS 291

Homework 5

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Section 8.1, problem 2.b Prove that each of the following wffs is correct. Assume that the domain is the set of integers.

$\{a > b\}\ x := -a;\ \ y := -b\ \{x < y\}$.

Answer:

1.	$\{x < -b\}\ y := -b\ \{x < y\}$	*AA*
2.	$\{-a < -b\}\ x := -a\ \{x < -b\}$	*AA*
3.	$\quad a > b$	*P* [for $(a > b) \to (-a < -b)$]
4.	$\quad -a < -b$	2, T
5.	$(a > b) \to (-a < -b)$	3, 4, CP
6.	$\{a > b\}\ x := -a;\ \ y := -b\ \{x < y\}$	1, 5, Consequence
	QED.	

Section 8.1, problem 4.a Prove that each of the following wffs is correct. Assume that the domain is the set of integers.

$\{x < 10\}$ **if** $x \geq 5$ **then** $x := 4\ \{x < 5\}$.

Answer:

1.	$\{4 < 5\}\ x := 4\ \{x < 5\}$	*AA*
2.	$\quad (x < 10) \wedge (x \geq 5)$	*P*[for CP]
3.	$\quad 4 < 5$	2, T
4.	$(x < 10) \wedge (x \geq 5) \to (4 < 5)$	2, 3, CP
5.	$\{(x < 10) \wedge (x \geq 5)\}\ x := 4\ \{x < 5\}$	1, 4, Consequence
6.	$\quad (x < 10) \wedge \neg(x \geq 5)$	P[for CP]
7.	$\quad x < 5$	6, Simp
8.	$(x < 10) \wedge \neg(x \geq 5) \to (x < 5)$	6, 7, CP
9.	$\{x < 10\}$ **if** $x \geq 5$ **then** $x := 4\ \{x < 5\}$	5, 8, If-Then
	QED.	

Section 8.1, problem 4.b Prove that each of the following wffs is correct. Assume that the domain is the set of integers.

$\{True\}$ **if** $x \neq y$ **then** $x := y$ $\{x = y\}$.

Answer:

1.	$\{y = y\}\ x := y\ \{x = y\}$	AA
2.	$\quad ((True) \wedge (x \neq y))$	P [for CP]
3.	$\quad y = y$	3, T
4.	$((True) \wedge (x \neq y)) \to (y = y)$	2, 3, CP
5.	$\{(True) \wedge (x \neq y)\}\ x := y\ \{x = y\}$	1, 4, Consequence
6.	$\quad (True) \wedge \neg(x \neq y)$	P [for CP]
7.	$\quad x = y$	6, Simp
8.	$((True) \wedge \neg(x \neq y)) \to (x = y)$	6, 7, CP
9.	$\{True\}$ **if** $x \neq y$ **then** $x := y$ $\{x = y\}$	5, 8, If-Then
	$QED.$	

Section 8.3, problem 1.c Use Skolem's algorithm, if necessary, to transform each of the following wffs into a clausal form.

$\exists y\ \forall x\ (p(x, y) \to q(x))$.

Answer:

$$\begin{aligned} \exists y\ \forall x\ (p(x, y) \to q(x)) &\equiv \exists y\ \forall x\ (\neg p(x, y) \vee q(x)) && remove \to \\ &\equiv \forall x\ (\neg p(x, c) \vee q(x)) && Skolem's rule \end{aligned}$$

Section 8.3, problem 1.d Prove that each of the following wffs is correct. Assume that the domain is the set of integers.

$\exists y\ \forall x\ p(x, y) \to q(x)$.

Answer:

$$\begin{aligned} \exists y\ \forall x\ p(x, y) \to q(x) &\equiv \forall y\ \exists x\ \neg p(x, y) \vee q(x) && remove \to \\ &\equiv \forall y\ \neg p(f(y), y) \vee q(f(y)) && Skolem's rule \end{aligned}$$

Section 8.3, problem 3.c Find a resolution proof to show that each of the following sets of propositional clauses is unsatisfiable.

$\{A \vee B, A \vee \neg C, \neg A \vee C, \neg A \vee \neg B, C \vee \neg B, \neg C \vee B\}$.

Answer:

1.	$A \vee B$	P
2.	$A \vee \neg C$	P
3.	$\neg A \vee C$	P
4.	$\neg A \vee \neg B$	P
5.	$C \vee \neg B$	P
6.	$\neg C \vee B$	P
7.	$B \vee C$	1, 3, R
8.	$B \vee B$	6, 7, R
9.	$\neg A$	4, 8, R
10.	$\neg C$	2, 9, R
11.	$\neg B$	5, 10, R
12.	A	1, 11, R
13.	[ ]	9, 12, R
	QED.	

Section 8.3, problem 5.c Use Robinson's unification algorithm to find a most general unifier for each of the following sets of atoms.

$\{p(f(x,\ g(y)),\ y), p(f(g(a),\ z),\ b)\}$.

Answer:

1. Set $\theta_0 = \in$.
2. $S\theta_0 = S \in = S$, is not a singleton. $D_0 = \{x,\ g(a)\}$.
3. Variable x does not occur in the term g(a) of D_0.
   Put $\theta_1 = \theta_0\{x/g(a)\} = \{x/g(a)\}$.
4. $S\theta_1 = p(f(g(a),\ g(y)),\ y), p(f(g(a),\ z),\ b)$ is not a singleton.$D_1 = \{g(y),\ z\}$.
5. Variable z does not occur in the term *g(y)* of D_1.
   Put $\theta_2 = \theta_1\ \{z/g(y)\} = \{x/g(a)\}\ \{z/g(y)\} = \{x/g(a),\ z/g(y)\}$.
6. $S\theta_2 = \{p(f(g(a), g(y)), y),\ p(f(g(a), g(b)), b)\}$, is not a singleton. $D_2 = \{y, b\}$.
7. Variable y does not occur in the term b of D_2.
   Put $\theta_3 = \theta_2 y/b = \{x/g(a), z/g(y)\}\{y/b\} = \{x/g(a), z/g(b), y/b\}$.
8. $S\theta_3 = p(f(g(a), g(b)), b)$, is a singleton.
   Therefore, the algorithm terminates with the most general unifier $\{x/g(a), z/g(b), y/b\}$ for the given set S.

Section 8.3, problem 5.d Use Robinson's unification algorithm to find a most general unifier for each of the following sets of atoms.

$\{p(x, f(x), y),\;\; p(x, y, z),\;\; p(w, f(a),\;\; b)\}$.

Answer:

1. Set $\theta_0 = \in$.
2. $S\theta_0 = S \in = S$, is not a singleton. $D_0 = \{x,\;\; w\}$.
3. Variable x does not occur in the term g(a) of D_0.
   Put $\theta_1 = \theta_0\{x/w\} = \{x/w\}$.
4. $S\theta_1 = \{p(w, f(w), y), p(w, y, z), p(w, f(a), b)\}$ is not a singleton.$D_1 = \{y,\;\; f(w)\}$.
5. Variable y does not occur in the term *f(w)* of D_1.
   Put $\theta_2 = \theta_1\;\; \{y/f(w)\} = \{x/w\}\;\; \{y/f(w)\} = \{x/w,\;\; y/f(w)\}$.
6. $S\theta_2 = \{p(w, f(w), f(a)), p(w, f(w), z), p(w, f(a), b)\}$, is not a singleton. $D_2 = \{w, a\}$.
7. As we do not have a variable in this disagreement set, the algorithm terminates here, with the conclution that, the given set S is not unifiable.

Section 8.3, problem 8.c Use resolution to show that each of the following sets of clauses is unsatisfiable

$\{p(a) \vee p(x), \neg p(a) \vee \neg p(y)\}$

Answer:

1.	$p(a) \vee p(x)$	P
2.	$\neg p(a) \vee \neg p(y)$	P
3.	[ ]	1, 2, R, $\{x/a,\;\; y/a\}$
	QED.	

Section 8.3, problem 9.c Use Robinson's unification algorithm to find a most general unifier for each of the following sets of atoms.

$(p \vee q) \wedge (q \to r) \wedge (r \to s) \to (p \vee s)$.

Answer:

$$\begin{aligned}
& (p \vee q) \wedge (q \to r) \wedge (r \to s) \to (p \vee s) \\
\equiv\ & \neg((p \vee q) \wedge (q \to r) \wedge (r \to s) \wedge (p \vee s)) \\
\equiv\ & (p \vee q) \wedge (\neg q \vee r) \wedge (\neg r \vee s) \wedge \neg(p \vee s) \\
\equiv\ & (p \vee q) \wedge (\neg q \vee r) \wedge (\neg r \vee s) \wedge \neg p \wedge \neg s
\end{aligned}$$

Giving us five clauses:

$$p \vee q, \quad \neg q \vee r, \quad \neg r \vee s, \quad \neg p, \quad \neg s$$

After negating the statement and putting the result in clausal form, we obtain the following proof:

1.	$p \vee q$	P
2.	$\neg q \vee r$	P
3.	$\neg r \vee s$	P
4.	$\neg p$	P
5.	$\neg s$	P
6.	$\neg r$	3, 5, R
7.	$\neg q$	2, 6, R
8.	q	1, 4, R
9.	[ ]	7, 8, R
	QED.	

Section 8.3, problem 10.d Use Robinson's unification algorithm to find a most general unifier for each of the following sets of atoms.

$\exists x \ \ \forall y \ \ p(x,y) \wedge \forall x \ \ (p(x,x) \rightarrow \exists y \ \ q(y,x)) \rightarrow \exists y \ \ \exists x \ \ q(x,y)$.

Answer:

$$
\begin{aligned}
& \exists x \ \ \forall y \ \ p(x,y) \wedge \forall x \ \ (p(x,x) \rightarrow \exists y \ \ q(y,x)) \rightarrow \exists y \ \ \exists x \ \ q(x,y) \\
\equiv \ & \neg(\exists x \ \ \forall y \ \ p(x,y) \wedge \forall z \ \ (p(z,z) \rightarrow \exists m \ \ q(m,z)) \rightarrow \exists h \ \ \exists n \ \ q(n,h)) \\
& (renamed \ \ variable) \\
\equiv \ & \neg(\exists x \ \ \forall y \ \ p(x,y) \wedge \forall z \ \ (p(z,z) \rightarrow \exists m \ \ q(m,z)) \wedge \exists h \ \ \exists n \ \ q(n,h)) \\
& (removed \ \ outside \ \ \rightarrow) \\
\equiv \ & (\exists x \ \ \forall y \ \ p(x,y) \wedge \forall z \ \ (p(z,z) \rightarrow \exists m \ \ q(m,z))) \wedge \forall h \ \ \forall n \ \ \neg(q(n,h)) \\
& (moved \ \ \neg \ \ inside) \\
\equiv \ & (\exists x \ \ \forall y \ \ p(x,y)) \wedge (\forall z \ \ (\neg p(z,z) \vee \exists m \ \ q(m,z))) \wedge \forall h \ \ \forall n \ \ \neg(q(n,h)) \\
& (removed \ \ inside \ \ \rightarrow) \\
\equiv \ & \exists x \ \ \forall y \ \ \forall z \ \ \forall h \ \ \forall n \ \ (p(x,y) \wedge (\neg p(z,z) \vee \exists m \ \ q(m,z)) \wedge \neg q(n,h)) \\
& (moved \ \ \forall h \ \ \forall n \ \ \forall x \ \ \exists x \ \ \forall y \ \ out) \\
\equiv \ & \exists x \ \ \forall y \ \ \forall z \ \ \exists m \ \ \forall h \ \ \forall n \ \ (p(x,y) \wedge (\neg p(z,z) \vee q(m,z)) \wedge \neg q(n,h)) \\
& (moved \ \ \exists m \ \ out \ \ and \ \ constructed \ \ CNF)
\end{aligned}
$$

Apply Skolem's Rule to eliminate $\exists x, \ \ \exists m$.

$$\forall y \ \ \forall z \ \ \forall h \ \ \forall n \ \ (p(a,y) \wedge (\neg p(z,z) \vee q(f(z),z)) \wedge \neg q(n,h))$$

Giving us three clauses:

$$p(a,y), \ \ \neg p(z,z) \vee q(f(z),z), \ \ \neg q(n,h)$$

After negating the statement and putting the result in clausal form, we obtain the following proof:

1.	$p(a,y)$	P
2.	$\neg p(z,z) \vee q(f(z),z)$	P
3.	$\neg q(n,h)$	P
4.	$q(f(a),a)$	1, 2, R, $\{z/a, y/z\}$
5.	[ ]	3, 4, R, $\{n/f(z), h/z\}$
	QED.	

Section 8.3, problem 10.e Use Robinson's unification algorithm to find a most general unifier for each of the following sets of atoms.

$\forall x\ \ p(x) \vee \forall x\ \ q(x) \to \forall x\ \ (p(x) \vee q(x))$.

Answer:

$$\begin{array}{lll}
 & \forall x\ \ p(x) \vee \forall x\ \ q(x) \to \forall y\ \ (p(y) \vee q(y)) & \\
\equiv & \forall x\ \ p(x) \vee \forall x\ \ q(x) \to \forall z\ \ (p(z) \vee q(z)) & (renamed\ \ variable) \\
\equiv & \neg(\forall x\ \ p(x) \vee \forall y\ \ q(y) \wedge \neg(\forall z\ \ (p(z) \vee q(z))) & (removed \to) \\
\equiv & (\forall x\ \ p(x) \vee \forall y\ \ q(y)) \wedge \exists z\ \ \neg(p(z) \vee q(z))) & (moved\ \neg\ \ inside) \\
\equiv & \forall x\ \ \forall y\ \ (p(x) \vee q(y)) \wedge \exists z\ \ (\neg p(z) \wedge \neg q(z)) & (moved\ \ \ \forall x,\ \ \forall y\ \ out) \\
\equiv & \exists z\ \ \forall x\ \ \forall y\ \ (p(x) \vee q(y) \wedge \neg p(z) \wedge \neg q(z)) & (moved\ \ \ \exists z\ \ out) \\
\equiv & \exists z\ \ \forall x\ \ \forall y\ \ ((p(x) \vee q(y)) \wedge \neg p(z) \wedge \neg q(z)) & (constructed\ \ CNF)
\end{array}$$

Apply Skolem's Rule to eliminate $\exists z$.

$$\forall x\ \ \forall y((p(x) \vee q(y)) \wedge \neg p(a) \wedge \neg q(a))$$

Giving us three clauses:

$$p(x) \vee q(y),\ \ \neg p(a),\ \ \neg q(a)$$

After negating the statement and putting the result in clausal form, we obtain the following proof:

1.	$p(x) \vee q(y)$	P
2.	$\neg p(a)$	P
3.	$\neg q(a)$	P
4.	$q(y)$	1, 2, R, $\{x/a\}$
5.	[ ]	3, 4, R, $\{y/a\}$
	QED.	