CS 291
Homework 7

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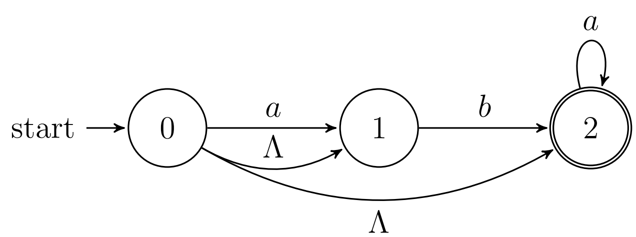
Section 11.3, Exercise 3 Suppose we are given the following NFA over the alphabet $\{a, b\}$:



a. Find a regular expression for the language accepted by the NFA.

Answer:

$$\begin{aligned} \Lambda a^* + (\Lambda + a)ba^* &= \Lambda a^* + (\Lambda ba^* + aba^*) \\ &= a^* + (ba^* + aba^*) \\ &= a^* + ba^* + aba^* \end{aligned}$$

Therefore, the regular expression for the language accepted by the NFA is $a^* + ba^* + aba^*$.

b. Write down the transition table for the NFA.

Answer:

	T	a	b	Λ
Start	0	$\{1\}$	$\varnothing$	$\{1, 2\}$
	1	$\varnothing$	$\{2\}$	$\varnothing$
Final	2	$\{2\}$	$\varnothing$	$\varnothing$

c. Use (11.3.2) to transform the NFA into a DFA.

Answer:

The first state and final state is $\lambda(0) = \{0, 1, 2\}$. So, we have T_D ({0, 1, 2}, a), and T_D({0, 1, 2}, b).

$$
\begin{aligned}
T_D(\{0,1,2\},a) &= \lambda(T_N(0,a)\cup T_N(1,a)\cup T_N(2,a))\\
&= \lambda(\{1\}\cup\{\varnothing\}\cup\{2\})\\
&= \lambda(\{1,2\})\\
&= \lambda(1)\cup\lambda(2)\\
&= \{1\}\cup\{2\}\\
&= \{1,2\},
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{0,1,2\},b) &= \lambda(T_N(0,b)\cup T_N(1,b)\cup T_N(2,b))\\
&= \lambda(\{\varnothing\}\cup\{2\}\cup\{\varnothing\})\\
&= \lambda(\{2\})\\
&= \{2\}.
\end{aligned}
$$

The final state are {1, 2}, {2}. So, we have T_D ({1, 2}, a), T_D({1, 2}, b), T_D ({2}, a), and T_D ({2}, b).

$$
\begin{aligned}
T_D(\{1,2\},a) &= \lambda(T_N(1,a)\cup T_N(2,a))\\
&= \lambda(\{\varnothing\}\cup\{2\})\\
&= \lambda(\{2\})\\
&= \{2\},
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{1,2\},b) &= \lambda(T_N(1,b)\cup T_N(2,b))\\
&= \lambda(\{2\}\cup\{\varnothing\})\\
&= \lambda(\{2\})\\
&= \{2\},
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{2\},a) &= \lambda(T_N(2,a))\\
&= \lambda(\{2\})\\
&= \{2\},
\end{aligned}
$$

$$\begin{aligned} T_D(\{2\}, b) &= \lambda(T_N(2, b)) \\ &= \lambda(\{\varnothing\}) \\ &= \varnothing. \end{aligned}$$

$\varnothing$ is the new enter. So, we have T_D ($\{\varnothing\}$, a) and T_D ($\{\varnothing\}$, b).

$$\begin{aligned} T_D(\{\varnothing\}, a) &= \lambda(T_N(\varnothing, b)) \\ &= \lambda(\{\varnothing\}) \\ &= \varnothing. \end{aligned}$$

$$\begin{aligned} T_D(\{\varnothing\}, b) &= \lambda(T_N(\varnothing, b)) \\ &= \lambda(\{\varnothing\}) \\ &= \varnothing. \end{aligned}$$

Therefore, we have

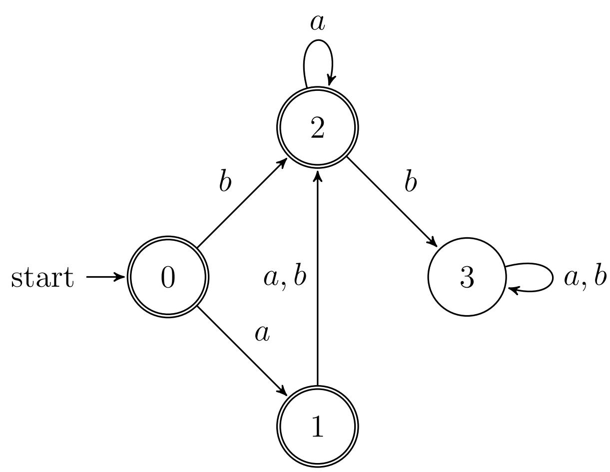
	T	a	b
S	$\{0,1,2\}$	$\{1,2\}$	$\{2\}$
F	$\{1,2\}$	$\{2\}$	$\{2\}$
F	$\{2\}$	$\{2\}$	$\varnothing$
	$\varnothing$	$\varnothing$	$\varnothing$

$\{0,1,2\} = 0$, $\{1,2\} = 1$, $\{2\} = 2$, $\varnothing$ =3.
Therefore, we can get

	T	a	b
S,F	0	1	2
F	1	2	2
F	2	2	3
	3	3	3

d. Draw a picture of the resulting DFA.

Answer:



Section 11.4, Exercise 7.b Show that each of the following languages is not regular by using the pumping lemma (11.4.3).
$\{w|w \in \{a,b\}^* and\ w\ is\ a\ palindrome\ of\ even\ length\}$.

Answer:

Let $L = \{w|w \in \{a,b\}^* and\ w\ is\ a\ palindrome\ of\ even\ length\}$.
Assume, BWOC, the given language is regular. Since w is even length palindrome, we can get

$$\text{s} = \text{a}^m\text{bba}^m = \text{xyz, where y} \neq \Lambda,\ |xy| < \text{m}.$$

It follows that x, y is a string of a's, so we can get y = a^i, i > 0. If we pump up y to y^2, we obtain

$$\begin{aligned} xy^2z &= a^m a^i bba^m \\ &= a^{m+i} bba^m \end{aligned}$$

we could easily find that $\text{xy}^2 \notin L$ because i > 0.
Therefore, according to the pumping lemme this language L is not regular.

Section 11.5, Exercise 1.a Find a context-free grammar for each of the following languages over the alphabet {a, b}.

$\{a^n b^{2n} | n \geq 0\}$

Answer:

We can obtain that {Λ, abb, aabbbb, aaabbbbbb, ...}.

$$S \rightarrow \Lambda | aSbb$$

Section 11.5, Exercise 2.d Find a context-free grammar for each of the following languages.
$\{a^n b^m | n \geq m \geq 0\}$

Answer:

We can obtain that {Λ, a, ab, aa, aab, aabb, ...}.

$$S \rightarrow aS|aSb|\Lambda$$