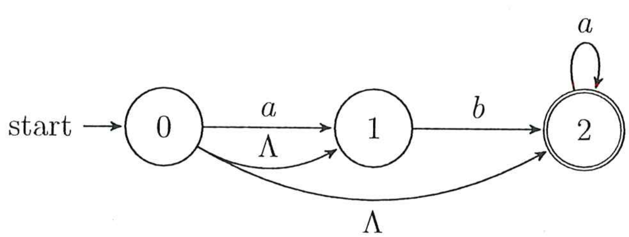
22/22

CS 291
Homework 7

Jingbo Wang
jw6347

Section 11.3, Exercise 3 Suppose we are given the following NFA over the alphabet $\{a, b\}$:



a. Find a regular expression for the language accepted by the NFA.

Answer:

$$\begin{aligned}\Lambda a^* + (\Lambda + a)ba^* &= \Lambda a^* + (\Lambda ba^* + aba^*) \\ &= a^* + (ba^* + aba^*) \\ &= a^* + ba^* + aba^*\end{aligned}$$

3/3

Therefore, the regular expression for the language accepted by the NFA is $a^* + ba^* + aba^*$.

b. Write down the transition table for the NFA.

Answer:

	T	a	b	Λ
Start	0	$\{1\}$	$\varnothing$	$\{1, 2\}$
	1	$\varnothing$	$\{2\}$	$\varnothing$
Final	2	$\{2\}$	$\varnothing$	$\varnothing$

3/3

c. Use (11.3.2) to transform the NFA into a DFA.

Answer:

The first state and final state is $\lambda(0) = \{0, 1, 2\}$. So, we have T_D ({0, 1, 2}, a), and T_D({0, 1, 2}, b).

$$
\begin{aligned}
T_D(\{0,1,2\},a) &= \lambda(T_N(0,a) \cup T_N(1,a) \cup T_N(2,a)) \\
&= \lambda(\{1\} \cup \{\varnothing\} \cup \{2\}) \\
&= \lambda(\{1,2\}) \\
&= \lambda(1) \cup \lambda(2) \\
&= \{1\} \cup \{2\} \\
&= \{1,2\},
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{0,1,2\},b) &= \lambda(T_N(0,b) \cup T_N(1,b) \cup T_N(2,b)) \\
&= \lambda(\{\varnothing\} \cup \{2\} \cup \{\varnothing\}) \\
&= \lambda(\{2\}) \\
&= \{2\}.
\end{aligned}
$$

The final state are {1, 2}, {2}. So, we have T_D ({1, 2}, a), T_D({1, 2}, b), T_D ({2}, a), and T_D ({2}, b).

$$
\begin{aligned}
T_D(\{1,2\},a) &= \lambda(T_N(1,a) \cup T_N(2,a)) \\
&= \lambda(\{\varnothing\} \cup \{2\}) \\
&= \lambda(\{2\}) \\
&= \{2\},
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{1,2\},b) &= \lambda(T_N(1,b) \cup T_N(2,b)) \\
&= \lambda(\{2\} \cup \{\varnothing\}) \\
&= \lambda(\{2\}) \\
&= \{2\},
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{2\},a) &= \lambda(T_N(2,a)) \\
&= \lambda(\{2\}) \\
&= \{2\},
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{2\}, b) &= \lambda(T_N(2, b)) \\
&= \lambda(\{\varnothing\}) \\
&= \varnothing.
\end{aligned}
$$

$\varnothing$ is the new enter. So, we have T_D ($\{\varnothing\}$, a) and T_D ($\{\varnothing\}$, b).

$$
\begin{aligned}
T_D(\{\varnothing\}, a) &= \lambda(T_N(\varnothing, b)) \\
&= \lambda(\{\varnothing\}) \\
&= \varnothing.
\end{aligned}
$$

$$
\begin{aligned}
T_D(\{\varnothing\}, b) &= \lambda(T_N(\varnothing, b)) \\
&= \lambda(\{\varnothing\}) \\
&= \varnothing.
\end{aligned}
$$

Therefore, we have

	T	a	b
S	$\{0, 1, 2\}$	$\{1, 2\}$	$\{2\}$
F	$\{1, 2\}$	$\{2\}$	$\{2\}$
F	$\{2\}$	$\{2\}$	$\varnothing$
	$\varnothing$	$\varnothing$	$\varnothing$

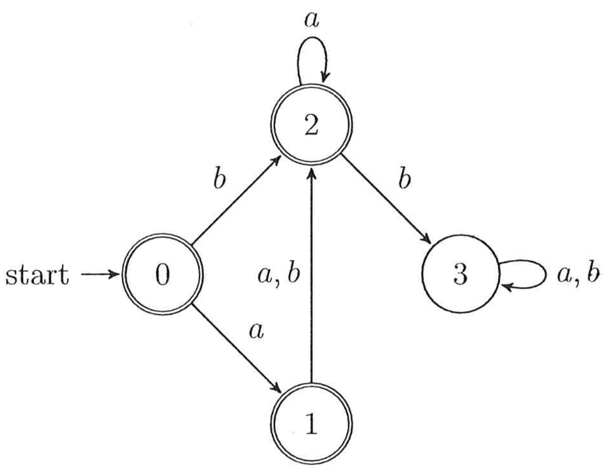
3/3

$\{0, 1, 2\} = 0$, $\{1, 2\} = 1$, $\{2\} = 2$, $\varnothing = 3$.
Therefore, we can get

	T	a	b
S,F	0	1	2
F	1	2	2
F	2	2	3
	3	3	3

d. Draw a picture of the resulting DFA.

Answer:



3/3

Section 11.4, Exercise 7.b Show that each of the following languages is not regular by using the pumping lemma (11.4.3).
$\{w|w \in \{a, b\}^* and\ w\ is\ a\ palindrome\ of\ even\ length\}$.

Answer:

Let $L = \{w|w \in \{a, b\}^* and\ w\ is\ a\ palindrome\ of\ even\ length\}$.
Assume, BWOC, the given language is regular. Since w is even length palindrome, we can get

$$\mathrm{s} = \mathrm{a}^m\mathrm{bba}^m = \mathrm{xyz}, \text{ where } \mathrm{y} \neq \Lambda,\ |xy| < \mathrm{m}.$$

It follows that x, y is a string of a's, so we can get $\mathrm{y} = \mathrm{a}^i$, $\mathrm{i} > 0$. If we pump up y to y^2, we obtain

4/4

$$\begin{aligned} xy^2z &= a^m a^i bba^m \\ &= a^{m+i}bba^m \end{aligned}$$

we could easily find that $\mathrm{xy}^2 \notin L$ because $\mathrm{i} > 0$.
Therefore, according to the pumping lemme this language L is not regular.

Section 11.5, Exercise 1.a Find a context-free grammar for each of the following languages over the alphabet {a, b}.

$\{a^n b^{2n} | n \geq 0\}$

Answer:

3/3

We can obtain that {Λ, abb, aabbbb, aaabbbbbb, ...}.

$$S \to \Lambda | aSbb$$

Section 11.5, Exercise 2.d Find a context-free grammar for each of the following languages.
$\{a^n b^m | n \geq m \geq 0\}$

Answer:

We can obtain that $\{\Lambda$, a, ab, aa, aab, aabb, ...$\}$.

$$S \to aS|aSb|\Lambda$$

3/3