CS 291

Homework 3

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Section 7.1, problem 5.b For each of the following wffs, label each occurrence of the variable as either bound free.

$\forall y \ \ q(y) \wedge \neg p(x, y)$.

Answer:

The one occurrences of x, left to right, is free.
The three occurrences of y, left to right, are bound, bound, and free.

Section 7.1, problem 5.c For each of the following wffs, label each occurrence of the variable as either bound free.

$\neg q(x, y) \vee \exists x \ \ p(x, y)$.

Answer:

The three occurrences of x, left to right, is free, bound, and bound.
The two occurrences of y, left to right, both are free.

Section 7.1, problem 7.e Let isFatherOf(x, y) be "x is the father of y," where the domain is the set of all people now living or who have lived. Find the truth value for each of the following wffs.

$\exists x \ \ \exists y \ \ isFatherOf(x, y)$

Answer:

$\exists x \ \ \exists y \ \ isFatherOf(x, y)$ means some x are the father of some y, so it is True

Section 7.1, problem 7.f Let isFatherOf(x, y) be "x is the father of y," where the domain is the set of all people now living or who have lived. Find the truth value for each of the following wffs.

$\exists y \ \ \exists x \ \ isFatherOf(x, y)$

Answer:

$\exists y \ \ \exists x \ \ isFatherOf(x, y)$ means some y are the son of some x, so it is True

Section 7.1, problem 10.a Let $B(x)$ mean x is a bird, let $W(x)$ mean x is a worm, and let $E(x, y)$ mean x eats y. Find an English sentence to describe each of the following statements.

$\forall x \ \ \forall y \ \ (B(x) \wedge W(y) \to E(x, y))$.

Answer:

Every bird eats every worm.

Section 7.1, problem 10.b Let $B(x)$ mean x is a bird, let $W(x)$ mean x is a worm, and let $E(x,\ y)$ mean x eats y. Find an English sentence to describe each of the following statements.

$\forall x\ \ \forall y\ \ (E(x,y) \to B(x) \wedge W(y))$.

Answer:

If every x eats every y, then x is bird and y is worm.

Section 7.2, problem 4.c Use equivalences to construct a prenex conjunctive normal form for each of the following wffs.

$\forall x\ \ \exists y\ \ p(x,y) \to \exists y\ \ \forall x\ \ p(x,y)$.

Answer: We know that a prenex normal form is called a prenex conjunctive normal form if has the form $Q_1x_1...Q_mx_m[D_1 \wedge ... \wedge D_k]$.

$$\begin{aligned}
\forall x\ \ \exists y\ \ p(x,y) \to \exists y\ \ \forall x\ \ p(x,y) &\equiv \forall x\ \ \exists y\ \ p(x,y) \to \exists z\ \ \forall w\ \ p(w,z) && \text{(rename)}\\
&\equiv \neg(\forall x\ \ \exists y\ \ p(x,y)) \vee \exists z\ \ \forall w\ \ p(w,z) && \text{(remove } \to\text{)}\\
&\equiv \neg\forall x\ \ \exists y\ \ \neg p(x,y) \vee \exists z\ \ \forall w\ \ p(w,z) && \text{(De Morgan's Laws)}\\
&\equiv \exists x\ \ \forall y\ \ \neg p(x,y) \vee \exists z\ \ \forall w\ \ p(w,z) && \text{(1)}\\
&\equiv \exists x\ \ \forall y\ \ \exists z\ \ \forall w\ \ (\neg p(x,y) \vee p(w,z)) && \text{(7b)}
\end{aligned}$$

Section 7.2, problem 4.d Use equivalences to construct a prenex conjunctive normal form for each of the following wffs.

$\forall x\ \ (p(x, f(x)) \to p(x,y))$.

Answer: We know that a prenex normal form is called a prenex conjunctive normal form if has the form $Q_1x_1...Q_mx_m[D_1 \wedge ... \wedge D_k]$.

$$\forall x\ \ (p(x, f(x)) \to p(x,y)) \equiv \forall x\ \ (\neg p(x, f(x)) \vee p(x,y)) \qquad \text{(remove } \to\text{)}$$

Section 7.2, problem 5.d Use equivalences to construct a prenex conjunctive normal form for each of the following wffs.

$\forall x\ \ (p(x, f(x)) \to p(x,y))$.

*Answer:*We know that a prenex normal form is called a prenex disjunctive normal form if has the form $Q_1x_1...Q_mx_m[D_1 \vee ... \vee D_k]$.

$$\forall x\ \ (p(x, f(x)) \to p(x,y)) \equiv \forall x\ \ (\neg p(x, f(x)) \vee p(x,y)) \qquad \text{(remove } \to\text{)}$$

Section 7.2, problem 5.e Use equivalences to construct a prenex conjunctive normal form for each of the following wffs.

$\forall x\ \forall y\ (p(x,y) \to \exists z\ (p(x,z) \wedge p(y,z)))$.

Answer: We know that a prenex normal form is called a prenex disjunctive normal form if has the form $Q_1x_1 ... Q_mx_m[D_1 \vee ... \vee D_k]$.

$$\begin{aligned}
\forall x\ \forall y\ (p(x,y) \to \exists z\ (p(x,z) \wedge p(y,z))) &\equiv \forall x\ \forall y\ (\neg p(x,y) \vee \exists z \\
&\quad (p(x,z) \wedge p(y,z))) && \text{(remove } \to\text{)} \\
&\equiv \forall x\ \forall y\ \exists z\ (\neg p(x,y) \\
&\quad \vee (p(x,z) \wedge p(y,z))) && \text{(7b)}
\end{aligned}$$

Section 7.2, problem 8.b Formalize each of the following statements, where $B(x)$ means x is a bird, $W(x)$ means x is a worm, and $E(x,\ y)$ means x eats y.

Some birds eat worms.

Answer:

$$\exists x(B(x) \to \exists y(W(y) \wedge E(x,y)))$$

Section 7.2, problem 8.c Formalize each of the following statements, where $B(x)$ means x is a bird, $W(x)$ means x is a worm, and $E(x,\ y)$ means x eats y.

Only birds eat worms.

Answer:

$$\forall x\ \forall y\ (W(x) \wedge E(y,x) \to B(y))$$

Section 7.2, problem 8.d Formalize each of the following statements, where $B(x)$ means x is a bird, $W(x)$ means x is a worm, and $E(x,\ y)$ means x eats y.

Not all birds eat worms

Answer:

$$\neg\forall x(B(x) \to \exists y(W(y) \wedge E(x,y)))$$