CS 291

Homework 1

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Section 6.2, problem 7.e Use Quine's method to show that each wff is a contingency. $(A \to B) \vee ((C \to \neg B) \wedge \neg C)$.

Answer: First we compute the wff $W(A/\text{true})$.

$$\begin{aligned} W(A/\text{true}) &\equiv (\text{true} \to B) \vee ((C \to \neg B) \wedge \neg C) \\ &\equiv B \vee ((C \to \neg B) \wedge \neg C) \end{aligned}$$

Let $X = B \vee ((C \to \neg B) \wedge \neg C)$. Then we have:

$$\begin{aligned} X(B/\text{true}) &\equiv \text{true} \vee ((C \to \text{false}) \wedge \neg C) \\ &\equiv \text{true} \vee (\neg C \wedge \neg C) \\ &\equiv \text{true} \vee \neg C \\ &\equiv \text{true} \\ X(B/\text{false}) &\equiv \text{false} \vee ((C \to \text{true}) \wedge \neg C) \\ &\equiv \text{false} \vee (\text{true} \wedge \neg C) \\ &\equiv \text{false} \vee \neg C \end{aligned}$$

Let $Y = \text{false} \vee \neg C$. Then we have:

$$\begin{aligned} Y(C/\text{true}) &\equiv \text{false} \vee \text{false} \\ &\equiv \text{false} \\ Y(C/\text{false}) &\equiv \text{false} \vee \text{true} \\ &\equiv \text{true} \end{aligned}$$

Therefore $W(A/\text{true})$ is not a tautology. Next we check $W(A/\text{false})$.

$$\begin{aligned} W(A/\text{false}) &\equiv (\text{false} \to B) \vee ((C \to \neg B) \wedge \neg C) \\ &\equiv \text{true} \vee ((C \to \neg B) \wedge \neg C) \end{aligned}$$

Let $X = \text{true} \vee ((C \to \neg B) \wedge \neg C)$. Then we have:

$$\begin{aligned}
X(C/\text{true}) &\equiv \text{true} \vee (\text{true} \to \neg B) \wedge \text{false}) \\
&\equiv \text{true} \vee (\neg B \wedge \text{false}) \\
&\equiv \text{true} \vee \text{false} \\
&\equiv \text{true} \\
X(C/\text{false}) &\equiv \text{true} \vee (\text{false} \to \neg B) \wedge \text{true}) \\
&\equiv \text{true} \vee (\text{true} \wedge \text{true}) \\
&\equiv \text{true} \vee \text{true} \\
&\equiv \text{true}
\end{aligned}$$

This produces a contradiction. Thus the wff is a contingency.

Section 6.2, problem 8.f Use Quine's method to show that each wff is a tautology.
$(A \to B) \to (C \vee A \to C \vee B)$

Answer: First we compute the wff $W(A/\text{true})$.

$$\begin{aligned}
W(A/\text{true}) &\equiv (\text{true} \to B) \to (C \vee \text{true} \to C \vee B) \\
&\equiv B \to (true \to C \vee B) \\
&\equiv B \to (C \vee B)
\end{aligned}$$

Let $X = B \to (C \vee B)$. Then we have:

$$\begin{aligned}
X(B/\text{true}) &\equiv \text{true} \to (C \vee \text{true}) \\
&\equiv \text{true} \to \text{true} \\
&\equiv \text{true} \\
X(B/\text{false}) &\equiv \text{false} \to (C \vee \text{fale}) \\
&\equiv \text{false} \to C \\
&\equiv \text{true}
\end{aligned}$$

Therefore $W(A/\text{true})$ is a tautology. Next we check $W(A/\text{false})$.

$$\begin{aligned}
W(A/\text{false}) &\equiv (\text{false} \to B) \to (C \vee \text{false} \to C \vee B) \\
&\equiv \text{true} \to (C \to C \vee B) \\
&\equiv C \to C \vee B
\end{aligned}$$

Let $X = \text{true} \to (C \to C \vee B)$. Then we have:

$$\begin{aligned}
X(C/\text{true}) &\equiv \text{true} \to \text{true} \vee B \\
&\equiv \text{true} \\
X(C/\text{false}) &\equiv \text{false} \to \text{false} \vee B \\
&\equiv \text{false} \to \text{false} \\
&\equiv \text{true}
\end{aligned}$$

Thus the wff is a tautology.

Section 6.2, problem 9.b Verify each of the following equivalences by writing an equivalence proof. That is, start on one side and use known equivalences to the other side.
$(A \wedge B) \to C \equiv (A \to C) \vee (B \to C)$.

Answer: We start with the left hand side and derive the right, using the equivalences given in Table 6.2.5 on page 475.

$$\begin{aligned}
(A \wedge B) \to C &\equiv \neg(A \wedge B) \vee C && \text{1st conversion} \\
&\equiv (\neg A \vee \neg B) \vee C && \text{De Morgan's Laws} \\
&\equiv (\neg A \vee C) \vee (\neg B \vee C) && \text{distribute property} \\
&\equiv (A \to C) \vee (B \to C) && \text{1st conversion}
\end{aligned}$$

Section 6.2, problem 11.e Use equivalences to transform each of the following wffs into a DNF.
$P \to Q \wedge R$.

$$\begin{aligned}
P \to Q \wedge R &\equiv \neg P \vee Q \wedge R \\
&\equiv \neg P \vee (Q \wedge R)
\end{aligned}$$

Section 6.2, problem 12.d Use equivalences to transform each of the following wffs into a CNF.
$(P \vee Q) \wedge R$.

$$(P \vee Q) \wedge R \equiv (P \vee Q) \wedge R$$