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1. $A(x) \land \forall x(B(x) \rightarrow \exists y\, C(x,y) \lor \neg\exists y\, A(y))$

$\equiv A(x) \land \forall x(B(x) \rightarrow \exists y\, C(x,y) \lor \neg\exists z\, A(z))$ (renamed)
$\equiv A(x) \land \forall x(\neg B(x) \lor \exists y\, C(x,y) \lor \neg\exists z\, A(z))$ (removed $\rightarrow$)
$\equiv A(x) \land \forall x(\neg B(x) \lor \exists y\, C(x,y) \lor \forall z\, A(z))$ (De Morgan's Laws)
$\equiv \forall x\, \exists y\, \forall z\, (A(x) \land (\neg B(x) \lor C(x,y) \lor A(z)))$ (conjunctive)
$\equiv \forall x\, \exists y\, \forall z\, ((A(x) \land \neg B(x)) \lor (A(x) \land (C(x,y) \lor A(z))$ (disconjunctive)

2. $\forall x(p(x) \rightarrow q(x)) \rightarrow (\exists x\, p(x) \rightarrow \exists x\, q(x))$

1.	$\forall x(p(x) \rightarrow q(x))$	P
2.	$\exists x\, p(x)$	P (Need: $\exists x\, p(x) \rightarrow \exists x\, q(x)$)
3.	$p(d) \rightarrow q(d)$	1, UI
4.	$p(d)$	2, EI
5.	$q(d)$	3, 4, MP
6.	$\exists x\, q(x)$	5, EG
7.	$\exists x\, p(x) \rightarrow \exists x\, q(x)$	2-6, CP
	QED	1, 7, CP

3. $S = \{p(f(x, g(y)), y), p(f(g(a), z), b)\}$

1. Set $\theta_0 = \epsilon$.
2. $S\theta_0 = S\epsilon = S$, is not a singleton. $D_0 = \{x, g(a)\}$
3. Variable x does not occur in the term $g(a)$ of D_0.
   Put $\theta_1 = \theta_0\{x/g(a)\} = \{x/g(a)\}$.
4. $S\theta_1 = p(f(g(a), g(y)), y), p(f(g(a), z), b)$ is not singletion.
   $D_1 = \{g(y), z\}$.
5. Variable z does not occur in the term $g(y)$ of D_1.
   Put $\theta_2 = \theta_1\{z/g(y)\} = \{x/g(a)\}\{z/g(y)\} = \{x/g(x), z/g(y)\}$.
6. $S\theta_2 = \{p(f(g(a), g(y)), y), p(f(g(a), g(b)), b)\}$, is not singleton.
   $D_2 = \{y, b\}$
7. Variable y does not occur in the term b of D_2.
8. $S\theta_3 = p(f(g(a), g(b)), b)$ is a singleton.

Therefore, the algorithm terminates with most general unifier $\{x/g(a), z/g(b), y/b\}$ for the give set S.

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4. $\forall x A(x) \vee \forall y B(y) \rightarrow \forall x (C(x) \vee D(x))$

$\equiv \forall x A(x) \vee \forall y B(y) \rightarrow \forall z (C(z) \vee D(z))$ (renamed)

$\equiv \neg(\forall x A(x) \vee \forall y B(y) \wedge \forall z (C(z) \vee D(z))$ (removed $\rightarrow$)

$\equiv \forall x A(x) \vee \forall y B(y) \wedge \exists z \neg(C(z) \vee D(z))$ (moved $\neg$ inside)

$\equiv \forall x \forall y (A(x) \vee B(y) \wedge \exists y (\neg C(z) \wedge \neg D(z))$ (moved $\forall x$, $\forall y$ outside)

$\equiv \exists z \forall x \forall y (A(x) \vee B(y) \wedge \neg C(z) \wedge \neg D(z))$ (moved $\exists z$ outside)

$\equiv \exists z \forall x \forall y ((A(x) \vee B(y)) \wedge \neg C(z) \wedge \neg D(z))$ (constructed CNF)

Apply Skolem's Rule to eliminate $\exists z$.

$\forall x \forall y ((A(x) \vee B(y) \wedge \neg C(a) \wedge \neg D(a))$

Giving us three clauses:

$A(x) \vee B(y)$, $\neg C(a)$, $\neg D(a)$

proof:

1. $A(x) \vee B(y)$ — P
2. $\neg C(a)$ — P
3. $\neg D(a)$ — P
4. $B(y)$ — 1, 2, R $\{x/a\}$
5. $[\,]$ — 3, 4, R, $\{y/a\}$

QED

5. $\{x < y\}$ $temp := x; x := y; y := temp$ $\{y < x\}$

1. $\{temp < x\}$ $y := temp$ $\{y < x\}$ — AA
2. $\{temp < y\}$ $x := y$ $\{temp < x\}$ — AA
3. $\{x < \cancel{x}y\}$ $temp := x$ $\{temp < \cancel{x}y\}$ — AA
4. $x < y$ — $P[(x<y) \rightarrow (x < \cancel{x})]$
5. $x < \cancel{x}y$ — 3, T
6. $(x < y) \rightarrow (x < \cancel{x}y)$ — 4, 5, CP
7. $\{x<y\}$ $temp := x; x := y; y := temp$ $\{y<x\}$ — 1, 6, Consequence

QED

6. Valid:

$\forall x(p(x) \rightarrow p(x))$ is valid because $p(x) \rightarrow p(x)$ is true for all interpretations.

statistiable: $F = TRUE$

$F = A \land \neg B$ is statistiable, because $A = True$, $B = False$ makes

unsatistiable:

$\forall x(p(x) \land \neg p(x))$ is unstatistiable because $p(x) \land \neg p(x)$ is always false.

7. $\neg \forall x(C(x) \rightarrow T(x))$