11.6

1. Find a pushdown automaton for each of the following languages.

a. $\{ab_n cd_n \mid n \geq 0\}$.

(a)

Consider the language $\{ab^n cd^n \mid n \geq 0\}$.

The pushdown automata of this language must have start state 0 and final state 3 and stack can be symbolized as *X*.

Therefore the PDA can be given as

$(0, a, X, \text{nop}, 1)$,
$(1, b, X, \text{push}(b), 1)$,
$(1, b, b, \text{push}(b), 1)$,
$(1, c, X, \text{nop}, 2)$,
$(1, c, b, \text{nop}, 2)$,
$(2, d, b, \text{pop}, 2)$,
$(2, \Lambda, X, \text{nop}, 3)$.

b. All strings over $\{a, b\}$ with the same number of *a*'s and *b*'s.

(b)

Consider the language "All string over the alphabet $\{a, b\}$ with the same number of *a*'s and *b*'s."

This language can be given as $\{a^n b^n \mid n \geq 0\}$.

This PDA for this language must have start state as 0 and final state as 1.

Let the stack is symbolized as *X* then the PDA for this language is given by

$(0, a, X, \text{push}(X), 0)$,
$(0, \Lambda, X, \text{pop}, 1)$,
$(1, b, X, \text{pop}, 1)$.

12.1

1. Construct a Turing machine to recognize the language of all palindromes over $\{a, b\}$.

We need to construct the Turning machine to recognize the language of all palindromes over $\{a,b\}$.

A turning machine consists of two major components, a tape and a control unit. The tape is a sequence of cells that extends to infinity in both directions.

Each cell contains a symbol form a finite alphabet. There is a tape head that reads from a cell and writes into the same cell. The control unit contains a finite set of instructions, which are executed as follows: Each instruction causes the tape head to read the symbol from a cell, to write a symbol into the same cell, and either to move the tape head to an adjacent cell or to leave it at the same cell.

Consider the general algorithm that repeatedly cancels the same letter from each end of the input string be replacing its occurrence by Λ. *A* Turning machine program to accomplish this follows, where the start state is 0.

$(0,a,\Lambda,R,1)$ Replace *a* by Λ

$(0,b,\Lambda,R,4)$ Replace *b* by Λ

$(0,\Lambda,\Lambda,S,\text{Halt})$ It's an even length palindromes.

$(1,a,a,R,1)$ Scan right

$(1,b,b,R,1)$ Scan right

$(1,\Lambda,\Lambda,L,2)$ Found the right end.

$(2,a,\Lambda,L,3)$ Replace rightmost *a* by Λ

$(2,\Lambda,\Lambda,S,\text{Halt})$ It's an odd length palindrome.

$(3,a,a,L,3)$ Scan left.

$(3,b,b,L,3)$ Scan left

$(3,\Lambda,\Lambda,R,0)$ Found left end.

$(4,a,a,R,4)$ Scan right

$(4,b,b,R,4)$ Scan right

$(4,\Lambda,\Lambda,L,5)$ Found the right end.

$(5,b,\Lambda,L,3)$ Replace rightmost *b* by Λ

$(5,\Lambda,\Lambda,S,\text{Halt})$ It's an odd-length palindrome.