

CS 291  
Homework 1

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**Section 6.2, problem 7.e** Use Quine's method to show that each wff is a contingency.  
 $(A \rightarrow B) \vee ((C \rightarrow \neg B) \wedge \neg C)$ .

*Answer:* First we compute the wff  $W(A/\text{true})$ .

$$\begin{aligned} W(A/\text{true}) &\equiv (\text{true} \rightarrow B) \vee ((C \rightarrow \neg B) \wedge \neg C) \\ &\equiv B \vee ((C \rightarrow \neg B) \wedge \neg C) \end{aligned}$$

Let  $X = B \vee ((C \rightarrow \neg B) \wedge \neg C)$ . Then we have:

$$\begin{aligned} X(B/\text{true}) &\equiv \text{true} \vee ((C \rightarrow \text{false}) \wedge \neg C) \\ &\equiv \text{true} \vee (\neg C \wedge \neg C) \\ &\equiv \text{true} \vee \neg C \\ &\equiv \text{true} \\ X(B/\text{false}) &\equiv \text{false} \vee ((C \rightarrow \text{true}) \wedge \neg C) \\ &\equiv \text{false} \vee (\text{true} \wedge \neg C) \\ &\equiv \text{false} \vee \neg C \equiv \neg C \end{aligned}$$

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Let  $Y = \text{false} \vee \neg C$ . Then we have:

$$\begin{aligned} Y(C/\text{true}) &\equiv \text{false} \vee \text{false} \\ &\equiv \text{false} \\ Y(C/\text{false}) &\equiv \text{false} \vee \text{true} \\ &\equiv \text{true} \end{aligned}$$

at this point you  
know it's a  
contingency

Therefore  $W(A/\text{true})$  is not a tautology. Next we check  $W(A/\text{false})$ .

$$\begin{aligned} W(A/\text{false}) &\equiv (\text{false} \rightarrow B) \vee ((C \rightarrow \neg B) \wedge \neg C) \\ &\equiv \text{true} \vee ((C \rightarrow \neg B) \wedge \neg C) \end{aligned}$$

Let  $X = \text{true} \vee ((C \rightarrow \neg B) \wedge \neg C)$ . Then we have:

$$\begin{aligned}
 X(C/\text{true}) &\equiv \text{true} \vee (\text{true} \rightarrow \neg B) \wedge \text{false} \\
 &\equiv \text{true} \vee (\neg B \wedge \text{false}) \\
 &\equiv \text{true} \vee \text{false} \\
 &\equiv \text{true} \\
 X(C/\text{false}) &\equiv \text{true} \vee (\text{false} \rightarrow \neg B) \wedge \text{true} \\
 &\equiv \text{true} \vee (\text{true} \wedge \text{true}) \\
 &\equiv \text{true} \vee \text{true} \\
 &\equiv \text{true}
 \end{aligned}$$

This produces a contradiction. Thus the wff is a contingency.

**Section 6.2, problem 8.f** Use Quine's method to show that each wff is a tautology.

$$(A \rightarrow B) \rightarrow (C \vee A \rightarrow C \vee B)$$

*Answer:* First we compute the wff  $W(A/\text{true})$ .

$$\begin{aligned}
 W(A/\text{true}) &\equiv (\text{true} \rightarrow B) \rightarrow (C \vee \text{true} \rightarrow C \vee B) \\
 &\equiv B \rightarrow (\text{true} \rightarrow C \vee B) \\
 &\equiv B \rightarrow (C \vee B)
 \end{aligned}$$

Let  $X = B \rightarrow (C \vee B)$ . Then we have:

$$\begin{aligned}
 X(B/\text{true}) &\equiv \text{true} \rightarrow (C \vee \text{true}) \\
 &\equiv \text{true} \rightarrow \text{true} \\
 &\equiv \text{true} \\
 X(B/\text{false}) &\equiv \text{false} \rightarrow (C \vee \text{false}) \\
 &\equiv \text{false} \rightarrow C \\
 &\equiv \text{true}
 \end{aligned}$$

Therefore  $W(A/\text{true})$  is a tautology. Next we check  $W(A/\text{false})$ .

$$\begin{aligned}
 W(A/\text{false}) &\equiv (\text{false} \rightarrow B) \rightarrow (C \vee \text{false} \rightarrow C \vee B) \\
 &\equiv \text{true} \rightarrow (C \rightarrow C \vee B) \\
 &\equiv C \rightarrow C \vee B
 \end{aligned}$$

Let  $X = \text{true} \rightarrow (C \rightarrow C \vee B)$ . Then we have:

$$\begin{aligned} X(C/\text{true}) &\equiv \text{true} \rightarrow \text{true} \vee B \\ &\equiv \text{true} \\ X(C/\text{false}) &\equiv \text{false} \rightarrow \text{false} \vee B \\ &\equiv \text{false} \rightarrow \text{false} \\ &\equiv \text{true} \end{aligned}$$

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Thus the wff is a tautology.

**Section 6.2, problem 9.b** Verify each of the following equivalences by writing an equivalence proof. That is, start on one side and use known equivalences to the other side.

$$(A \wedge B) \rightarrow C \equiv (A \rightarrow C) \vee (B \rightarrow C).$$

*Answer:* We start with the left hand side and derive the right, using the equivalences given in Table 6.2.5 on page 475.

$$\begin{aligned} (A \wedge B) \rightarrow C &\equiv \neg(A \wedge B) \vee C && \text{1st conversion} \\ &\equiv (\neg A \vee \neg B) \vee C && \text{De Morgan's Laws} \\ &\equiv (\neg A \vee C) \vee (\neg B \vee C) && \text{distribute property} \\ &\equiv (A \rightarrow C) \vee (B \rightarrow C) && \text{1st conversion} \end{aligned}$$

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**Section 6.2, problem 11.e** Use equivalences to transform each of the following wffs into a DNF.

$$P \rightarrow Q \wedge R.$$

$$\begin{aligned} P \rightarrow Q \wedge R &\equiv \neg P \vee Q \wedge R \\ &\equiv \neg P \vee (Q \wedge R) \end{aligned}$$

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**Section 6.2, problem 12.d** Use equivalences to transform each of the following wffs into a CNF.

$$(P \vee Q) \wedge R.$$

$$(P \vee Q) \wedge R \equiv (P \vee Q) \wedge R$$

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