CS 291

Homework 4

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Section 7.3, problem 6.d Give a formal proof for each of the following tautologies by using the CP rule. Do not use the IP rule.

$\forall x\ (p(x) \to q(x)) \to (\exists x\ p(x) \to \exists x\ q(x))$.

Answer:

1.	$\forall x\ (p(x) \to q(x))$	P
2.	$\quad \exists x\ p(x)$	P [for $\exists x\ p(x) \to \exists x\ q(x)$]
3.	$\quad p(x) \to q(x)$	1, UI
4.	$\quad p(x)$	2, EI
5.	$\quad q(x)$	3, 4, MP
6.	$\quad \exists x\ q(x)$	5, EG
7.	$\exists x\ p(x) \to \exists x\ q(x)$	2-6, CP
	QED	1,7, CP

Section 7.3, problem 6.f Give a formal proof for each of the following tautologies by using the CP rule. Do not use the IP rule.

$\forall x\ (p(x) \to q(x)) \to (\forall x\ p(x) \to \forall x\ q(x))$.

Answer:

1.	$\forall x\ (p(x) \to q(x))$	P
2.	$\quad \forall x\ p(x)$	P [for $\forall x\ p(x) \to \forall x\ q(x)$]
3.	$\quad p(x) \to q(x)$	1, UI
4.	$\quad p(x)$	2, UI
5.	$\quad q(x)$	3, 4, MP
6.	$\quad \forall x\ q(x)$	5, UG
7.	$\forall x\ p(x) \to \forall x\ q(x)$	2-6, CP
	QED	1, 7, CP

Section 7.3, problem 7.c Give a formal proof that each of the following wffs is valid by using the CP rule and by using the IP rule in each proof.

$\exists y \ \forall x \ p(x, y) \to \forall x \ \exists y \ p(x, y)$.

Answer:

1.	$\exists y \ \forall x \ p(x, y)$	P
2.	$\neg(\forall x \ \exists y \ p(x, y))$	P[for$\forall x \ \exists y \ p(x, y)$]
3.	$\exists x \ \forall y \ \neg p(x, y)$	2, T
4.	$\forall x \ p(x, c)$	1, EI
5.	$p(d, c)$	4, UI
6.	$\forall y \ \neg p(d, y)$	3, EI
7.	$\neg p(d, c)$	6, UI
8.	$False$	5, 7, Contr
9.	$\forall x \ \exists y \ p(x, y)$	2-8, IP
	QED	1, 9, CP

Section 7.3, problem 8.d Transform each informal argument into a formalized wff. Then give a formal proof of the wff.

Every rational number is a real number. There is a rational number.
Therefore, there is a real number.

Answer:

Let D(x) mean that x is rational number, L(x) mean that x is real number.
Every rational number is a real number: $\forall x(D(x) \to L(x))$
There is a rational number: $\exists x \ D(x)$
Therefore, there is a real number: $\exists x \ L(x)$

The argument can be written as the follow wff:

$\forall x(D(x) \to L(x)) \wedge \exists x \ D(x) \to \exists x \ L(x)$.

1.	$\forall x(D(x) \to L(x))$	P
2.	$\exists x \ D(x)$	P
3.	$D(b)$	2, EI
4.	$D(b) \to L(d)$	1, UI
5.	$L(d)$	4, MP
6.	$\exists x \ L(x)$	5, EG
	QED	1, 2, 6, CP

Section 7.3, problem 8.e Transform each informal argument into a formalized wff. Then give a formal proof of the wff.

Some freshmen like all sophomores. No freshman likes any junior.
Therefore, no sophomore is a junior.

Answer:

Let F(x) mean that x is freshmen, S(x) mean that x is sophomore,
J(x) mean that x is junior, and L(x,y) mean that x likes y.
A: Some freshmen like all sophomores: $\exists x \ (F(x) \wedge \forall y \ (S(y) \rightarrow L(x,y)))$
No freshman likes any junior: $\forall x \ (F(x) \rightarrow \forall y \ (J(y) \rightarrow \neg L(x,y)))$
B: Therefore, no sophomore is a junior: $\forall x \ (S(x) \rightarrow \neg J(x))$

Then the argument can be formalized as $A \rightarrow B$, where

$$\exists x \ (F(x) \wedge \forall y \ (S(y) \rightarrow L(x,y))) \wedge \forall x \ (F(x) \rightarrow \forall y \ (J(y) \rightarrow \neg L(x,y)))$$
$$\rightarrow \forall x \ (S(x) \rightarrow \neg J(x)).$$

1.	$\exists x \ (F(x) \wedge \forall y \ (S(y) \rightarrow L(x,y)))$	P
2.	$\forall x \ (F(x) \rightarrow \forall y \ (J(y) \rightarrow \neg L(x,y)))$	P
3.	$F(c) \wedge \forall y \ (S(y) \rightarrow L(c,y))$	1, EI
4.	$\forall y \ (S(y) \rightarrow L(c,y))$	3, Simp
5.	$S(x) \rightarrow L(c,x)$	4, UI
6.	$\quad S(x)$	P[for $S(x) \rightarrow \neg J(x)$]
7.	$\quad L(c,x)$	5, 6, MP
8.	$\quad F(c) \rightarrow \forall y \ (J(y) \rightarrow \neg L(c,y))$	2, UI
9.	$\quad F(c)$	3, Simp
10.	$\quad \forall y \ (J(y) \rightarrow \neg L(c,y))$	8, 9, MP
11.	$\quad J(x) \rightarrow \neg L(c,x)$	10, UI
12.	$\quad \neg J(x)$	7, 11 , MT
13.	$\quad S(x) \rightarrow \neg J(x)$	6, 12, CP
14.	$\forall x \ (S(x) \rightarrow \neg J(x))$	13, UG
	QED	1-5, 13, 14, CP