Jingbo Wang

1. $\neg(A \wedge B) \equiv \neg A \vee \neg B$

Answer:

A	B	$\neg(A \wedge B)$	$\neg A \vee \neg B$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

2. $A \to (C \to B) \equiv C \to (A \to B)$

Answer: $A \to (C \to B) \equiv A \to (\neg C \vee B)$

$\equiv \neg A \vee (\neg C \vee B)$

$\equiv \neg C \vee (\neg A \vee B)$

$\equiv C \to (\neg A \vee B)$

$\equiv C \to (A \to B)$

QED.

3. $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$

Answer: First we compute the wff $W(A/true)$.

$W(A/true) = (true \to (B \to C)) \to ((true \to B) \to (true \to C))$

$= (B \to C) \to (B \to C)$

$= true$

Therefore, $W(A/true)$ is a tautology. Next we check $W(A/false)$.

$W(A/false) = (false \to (B \to C)) \to ((false \to B) \to (false \to C))$

$= true \to (true \to true)$

$= true \to true$

$= true$

Thus the wff is a tautology.

4. $(A \vee B) \wedge (C \to D)$

$$(A \vee B) \wedge (C \to D) \equiv (A \vee B) \wedge (\neg C \vee D)$$
$$\equiv ((A \vee B) \wedge \neg C) \vee ((A \vee B) \wedge D)$$
$$\equiv (A \wedge \neg C) \vee (B \wedge \neg C) \vee (A \wedge D) \vee (B \wedge D)$$

5. $(A \wedge B) \vee E \vee F$

$$(A \wedge B) \vee E \vee F \equiv (A \vee E) \wedge (B \vee E) \vee F$$
$$\equiv ((A \vee E) \vee F) \wedge ((B \vee E) \vee F)$$
$$\equiv ((A \vee F) \vee (E \vee F)) \wedge ((B \vee F) \vee (E \vee F))$$

6. $(A \to (B \to C)) \to (B \to (A \to C))$

$$(A \to (B \to C)) \to (B \to (A \to C))$$

1.	$A \to (B \to C)$	P
2.	B	P[for $B \to (A \to C)$]
3.	A	P[for $A \to C$]
4.	$B \to C$	1, 3, MP
5.	C	2, 4, MP
6.	$A \to C$	3, 5, CP
7.	$B \to (A \to C)$	2, 6, CP
	QEP	1-3, 7, CP

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7. $(B \to C) \to (A \wedge B \to A \wedge C)$

$(B \to C) \to (A \wedge B \to A \wedge C)$

1.	$B \to C$	P
2.	$A \wedge B$	P[for $A \wedge B \to A \wedge C$]
3.	B	2, Simp
4	C	1, 3, MP
5.	$\neg(A \wedge C)$	P[for $A \wedge C$]
6.	A	2, Simp
7.	$A \wedge C$	4, 6, Conj
8.	False	5, 7, Contr
9.	$A \wedge C$	$5 \to 8$, IP
10.	$A \wedge B \to A \wedge C$	2, 9, CP
	QED	1, 10, CP

8.

Soundness: All proofs yield theorems that are tautologies. This means that everything we can prove is fact true.

Completeness: All tautolgies are provable as theorems. This means that everything that is true can be proven.