CS 291
Homework 6

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Section 3.3, Exercise 4.d Find a grammar for each language.
$\{a^m b^n | m,\ n \in N,\ where\ n > 0\}$.

Answer:

let $\boldsymbol{L} = \{a^m b^n | m,\ n \in N,\ where\ n > 0\}$.
We notice that $\boldsymbol{L}$ can be written as a product $\boldsymbol{L} = \boldsymbol{MN}$, where $\boldsymbol{M} = \{a^m | m \in N\}$, and $\mathbf{N} = \{b^n | n \in N\}$ where $n > 0$.
Thus we can write the following grammar for$\boldsymbol{L}$:

$$S \to AB\ product\ rule,$$
$$A \to \wedge | aA\ grammar\ for\ \mathbf{M},$$
$$B \to b | bB\ grammar\ for\ \mathbf{N}.$$

Section 3.3, Exercise 4.e Find a grammar for each language.
$\{a^m b^n | m,\ n \in N,\ where\ m > 0\ and\ n > 0\}$

Answer:

let $\boldsymbol{L} = \{a^m b^n | m,\ n \in N,\ where\ n > 0\}$.
We notice that $\boldsymbol{L}$ can be written as a product $\boldsymbol{L} = \boldsymbol{MN}$, where $\boldsymbol{M} = \{a^m | m \in N\}$ where $m > 0$, and $\mathbf{N} = \{b^n | n \in N\}$ where $n > 0$.
Thus we can write the following grammar for$\boldsymbol{L}$:

$$S \to AB \text{ product rule},$$
$$A \to a | aA \text{ grammar for } \mathbf{M},$$
$$B \to b | bB \text{ grammar for } \mathbf{N}.$$

Section 3.3, Exercise 5.a Find a grammar for each language.
The even palindromes over $\{a, b, c\}$.

Answer:

We can use Closure Rule for this question, here even palindromes over $\{a, b, c\}$ contains a string of from $\wedge$ or aSa or bSb or cSc.
Therefore, we can get grammaras follow by:

$$S \to \wedge | ASA$$
$$A \to a | b | c$$

On simplifying by substitution for A, we can get:

$$S \to \wedge | aSa | bSb | cSc.$$

Section 3.3, Exercise 5.b Find a grammar for each language. The odd palindromes over $\{a, b, c\}$.

We can use Closure Rule for this question, here odd palindromes over $\{a, b, c\}$ contains a string of from a or b or c or $\wedge$ or aSa or bSb or cSc.
Therefore, we can get grammaras follow by:

$$S \to a|b|c|ASA$$
$$A \to aSa|bSb|cSc$$

On simplifying by substitution for A, we can get:

$$S \to a|b|c|aSa|bSb|cSc.$$

Section 3.3, Exercise 6.b Find a grammar for each of the following languages.
The set of binary numerals that represent even natural numbers

Answer:

we know that:

$$0 \to 0000$$
$$2 \to 0010$$
$$4 \to 0100$$
$$6 \to 0110$$
$$8 \to 1000$$

If E is the start symbol of even natural number, then the grammar is

$$E \to B0 \text{ and } B \to \wedge|B0|B1$$

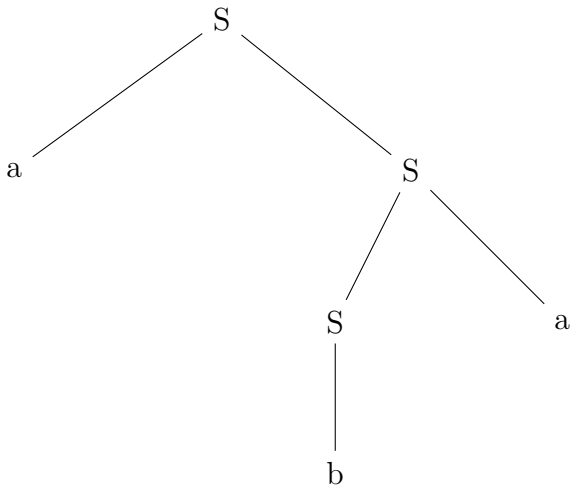
Section 3.3, Exercise 11.d Show that each of the following grammars is ambiguous. In otherwords, find a string that has two different parse trees (equivalently, two different leftmost derivations or two different rightmost derivations).
$S \to aS|Sa|b$.

Answer:

There are two different left most derivations to get the language of aba.

$$S \Rightarrow aS \Rightarrow aSa \Rightarrow aba$$
$$S \Rightarrow Sa \Rightarrow aSa \Rightarrow aba$$

First tree:



Second tree: