CS 291 Homework 4 13/16

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Section 7.3, problem 6.d Give a formal proof for each of the following tautologies by using the CP rule. Do not use the IP rule.

$$\forall x \ (p(x) \to q(x)) \to (\exists x \ p(x) \to \exists x \ q(x)).$$

## Answer:

1. 2. 3. 4. 5. 6. 7.	$\forall x \ (p(x) \to q(x))$ $\exists x \ p(x)$ $p(x) \to q(x)$ $p(x)$ $q(x)$ $\exists x \ q(x)$	shuld all be knts	P P [for $\exists x \ p(x) \rightarrow \exists x \ q(x)$ ] 1, UI 2, EI EI always instantintes 3, 4, MP 5, EG 2-6, CP
	QED		1,7, CP

Section 7.3, problem 6.f Give a formal proof for each of the following tautologies by using the CP rule. Do not use the IP rule.

$$\forall x \ (p(x) \to q(x)) \to (\forall x \ p(x) \to \forall x \ q(x)).$$

## Answer:

1.	$\forall x \ (p(x) \to q(x))$		P
2.	$\forall x \ p(x)$	1	$P [for \forall x \ p(x) \rightarrow \forall x \ q(x)]$
3.	$p(x) \to q(x)$	2/2	1, UI
4.	p(x)	$\supset \setminus \supset$	2, UI
5.	q(x)		3, 4, MP
6.	$\forall x \ q(x)$		5, UG
7.	$\forall x \ p(x) \to \forall x \ q(x)$		2-6, CP
	QED		1, 7, CP

Section 7.3, problem 7.c Give a formal proof that each of the following wffs is valid by using the CP rule and by using the IP rule in each proof.

$$\exists y \ \forall x \ p(x,y) \to \forall x \ \exists y \ p(x,y).$$

Answer:

1.	$\exists y \ \forall x \ p(x,y)$	P
2.	$\neg(\forall x \exists y \ p(x,y))$	$P[for \forall x \exists y \ p(x,y)]$
3.	$\exists x \ \forall y \ \neg p(x,y)$	2, T 1, EI
4.	$\forall x \ p(x,c)$	1, EI
5.	p(d,c)	4, UI
6.	$\forall y \ \neg p(d,y)$	3, EI
7.	eg p(d,c)	6, UI
8.	False	5, 7, Contr
9.	$\forall x \; \exists y \; p(x,y)$	2-8, IP
	QED	1, 9, CP

Section 7.3, problem 8.d Transform each informal argument into a formalized wff. Then give a formal proof of the wff.

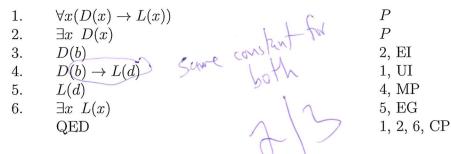
Every rational number is a real number. There is a rational number. Therefore, there is a real number.

Answer:

Let D(x) mean that x is rational number, L(x) mean that x is real number. Every rational number is a real number:  $\forall x(D(x) \to L(x))$ There is a rational number:  $\exists x \ D(x)$ Therefore, there is a real number:  $\exists x \ L(x)$ 

The argument can be written as the follow wff:

$$\forall x (D(x) \to L(x)) \land \exists x \ D(x) \to \exists x \ L(x).$$



Section 7.3, problem 8.e Transform each informal argument into a formalized wff. Then give a formal proof of the wff.

Some freshmen like all sophomores. No freshman likes any junior. Therefore, no sophomore is a junior.

Answer:

Let F(x) mean that x is freshmen, S(x) mean that x is sophomore, J(x) mean that x is junior, and L(x,y) mean that x likes y. A: Some freshmen like all sophomores:  $\exists x \ (F(x) \land \forall y \ (S(y) \to L(x,y)))$ No freshman likes any junior:  $\forall x \ (F(x) \to \forall y \ (J(y) \to \neg L(x,y)))$ B: Therefore, no sophomore is a junior:  $\forall x \ (S(x) \to \neg J(x))$ 

Then the argument can be formalized as  $A \rightarrow B$ , where

$$\exists x \ (F(x) \land \forall y \ (S(y) \to L(x,y))) \land \forall x \ (F(x) \to \forall y \ (J(y) \to \neg L(x,y))) \\ \to \forall x \ (S(x) \to \neg J(x)).$$