CS 310

Assignment 113

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**Problem 1** — Algorithms often have the following properties:

- the steps are stated *unambiguously* so that there is no question how the algorithm proceeds
- the algorithm is *deterministic* so that repeating the algorithm on the same input produces the same output
- the algorithm is *finite* because it terminates after a finite number of steps have been performed
- the algorithm produces *correct* output for a given input

For the following algorithm, for each property listed above, determine whether the algorithm exhibits this property:

```
unsigned max3(unsigned a, unsigned b, unsigned c)
{
  unsigned result = a;
  if (b > result)
  {
    result = b;
  }
  if (c > result)
  {
    result = c;
  }
  return result;
}
```

*Answer:* This algorithm is unambiguous because the syntax for the operations is well-understood. It is deterministic because it always produces the same output for a given input. It is finite because the number of lines of code executed (including the header) is strictly between 3 and 7 inclusive. It is correct because for all possible valid input combinations it does in fact return a value equal to the maximum input value.

**Problem 2** — Repeat problem 1 for the following algorithm. This algorithm empirically checks the correctness of Goldbach's conjecture, which states (in a modern interpretation) that every even number greater than 2 is the sum of two prime numbers. Assume `has_prime_addends` is a valid function that correctly determines whether its argument has two prime addends.

```
bool goldbach()
{
  unsigned value = 4;
  bool ok = true;
  while (ok)
  {
    if (!has_prime_addends(value))
```

```
8        {
9          ok = false;
10       }
11       else
12       {
13         value += 2;
14       }
15     }
16     return ok;
17   }
```

*Answer:* This algorithm is unambiguous because the syntax for the operations is well-understood. It is not deterministic because it does not produce any output for a given input. It is not finite because the algorithm only runs lines 11 - 14 in the while loop, and ”ok” value is always true, it cannot stop. It is not correct because this algorithm cannot give any Boolean value.

**Problem 3 —** What is the hexadecimal representation of $724_{10}$?

*Answer:* The first few powers of 16 are:

$$\begin{aligned} 16^0 &= 1 \\ 16^1 &= 16 \\ 16^2 &= 256 \\ 16^3 &= 4096 \end{aligned}$$

Thus we have:

$$\begin{array}{r} 724 \\ \underline{-2 \times 256 = 512} \\ 212 \\ \underline{-13 \times 16 = 208} \\ 4 \\ \underline{-4 \times 1 = 4} \\ 0 \end{array}$$

And thus we have $724_{10} = 2d4_{16}$.

**Problem 4 —** Based on the hexadecimal value found in the previous solution, what is the binary representation of $724_{10}$?

*Answer:* Because we have $2d4_{16}$, it is easy to know that:

$$
\begin{aligned}
2_{16} &= 0010_2 \\
d_{16} &= 1101_2 \\
4_{16} &= 0100_2
\end{aligned}
$$

Thus we have:

$$2d4_{16} = 001011010100_2$$

And thus we have $724_{10} = 001011010100_2$.

**Problem 5 —** What is the decimal representation of `0x2b3a`?

*Answer:* It is easy to know that:

$$
\begin{aligned}
a_{16} &= 10_{10} \\
b_{16} &= 11_{10}
\end{aligned}
$$

Thus we have:

$$
\begin{aligned}
&10 \times 16^0 + 3 \times 16^1 + 11 \times 16^2 + 2 \times 16^3 \\
=&10 + 48 + 2816 + 8192 \\
=&11066
\end{aligned}
$$