

LOCAL SEARCH ALGORITHMS

CHAPTER 4

Outline

- ◇ Hill-climbing
- ◇ Simulated annealing
- ◇ Genetic algorithms (briefly)
- ◇ Local search in continuous spaces (very briefly)

Iterative improvement algorithms

In many optimization problems, **path** is irrelevant;
the goal state itself is the solution

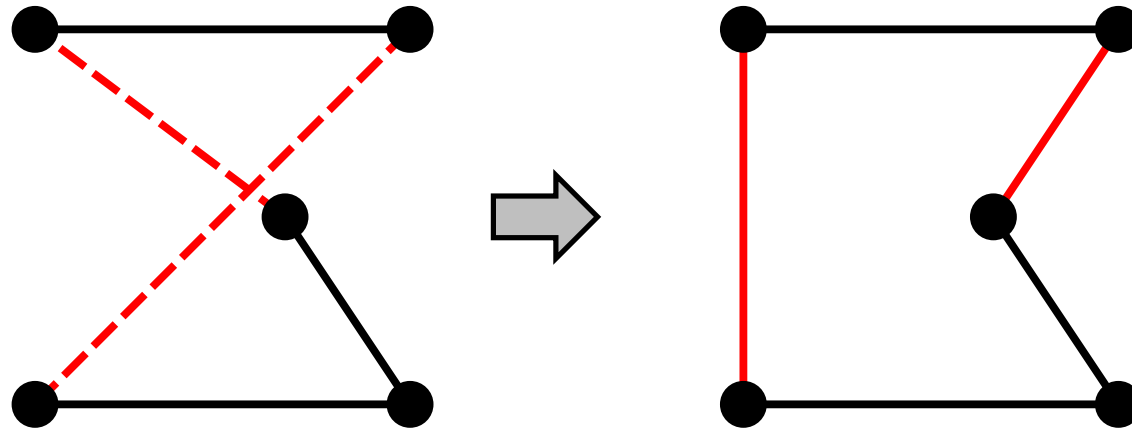
Then state space = set of “complete” configurations;
find **optimal** configuration, e.g., TSP
or, find configuration satisfying constraints, e.g., timetable

In such cases, can use **iterative improvement** algorithms;
keep a single “current” state, try to improve it

Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

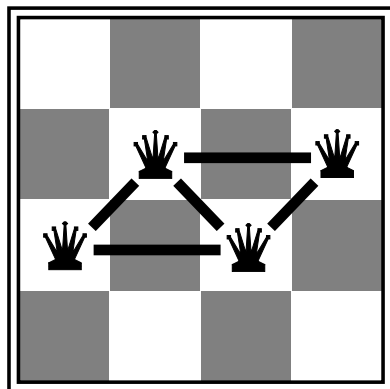


Variants of this approach get within 1% of optimal very quickly with thousands of cities

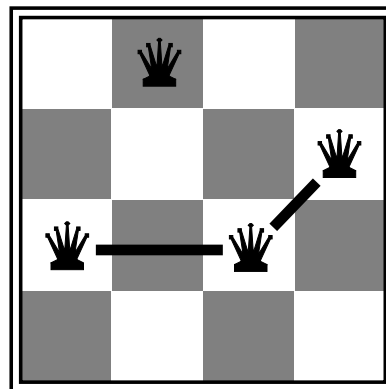
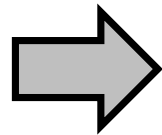
Example: n -queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

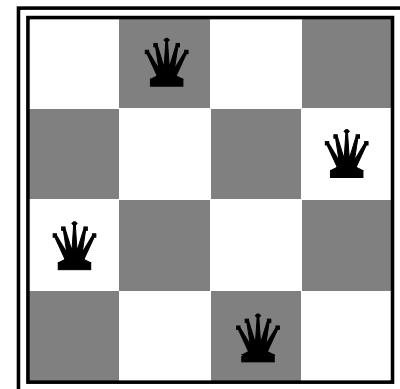
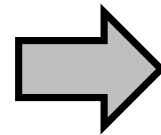
Move a queen to reduce number of conflicts



$h = 5$



$h = 2$



$h = 0$

Almost always solves n -queens problems almost instantaneously for very large n , e.g., $n = 1\text{million}$

Hill-climbing (or gradient ascent/descent)

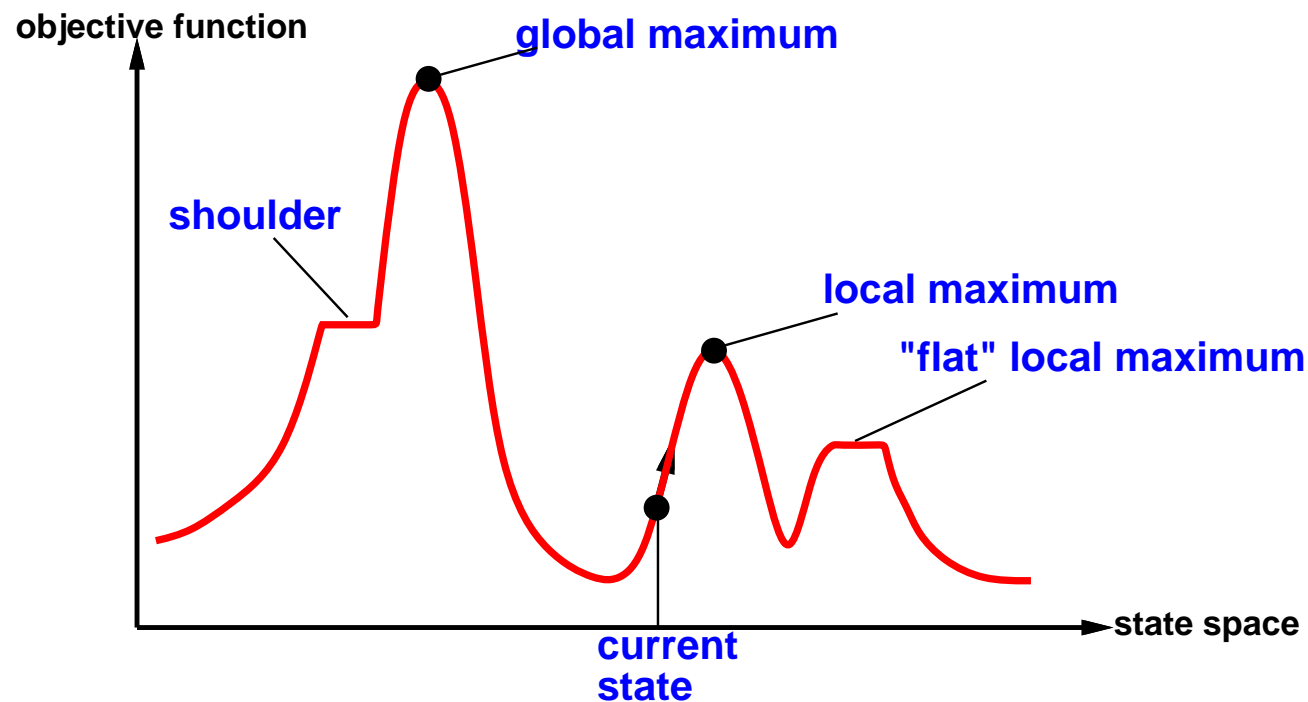
“Like climbing Everest in thick fog with amnesia”

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  inputs: problem, a problem
  local variables: current, a node
                     neighbor, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
    neighbor ← a highest-valued successor of current
    if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
    current ← neighbor
  end
```

Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves 😊 escape from shoulders 😞 loop on flat maxima

Simulated annealing

Idea: escape local maxima by allowing some “bad” moves
but gradually decrease their size and frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
           schedule, a mapping from time to “temperature”
  local variables: current, a node
                   next, a node
                   T, a “temperature” controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
    T ← schedule[t]
    if T = 0 then return current
    next ← a randomly selected successor of current
     $\Delta E \leftarrow \text{VALUE}[\textit{next}] - \text{VALUE}[\textit{current}]$ 
    if  $\Delta E > 0$  then current ← next
    else current ← next only with probability  $e^{\Delta E/T}$ 
```


Properties of simulated annealing

At fixed “temperature” T , state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{-\frac{E(x)}{kT}}$$

T decreased slowly enough \implies always reach best state x^*
because $e^{-\frac{E(x^*)}{kT}} / e^{-\frac{E(x)}{kT}} = e^{\frac{E(x^*) - E(x)}{kT}} \gg 1$ for small T

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Idea: keep k states instead of 1; choose top k of all their successors

Not the same as k searches run in parallel!

Searches that find good states recruit other searches to join them

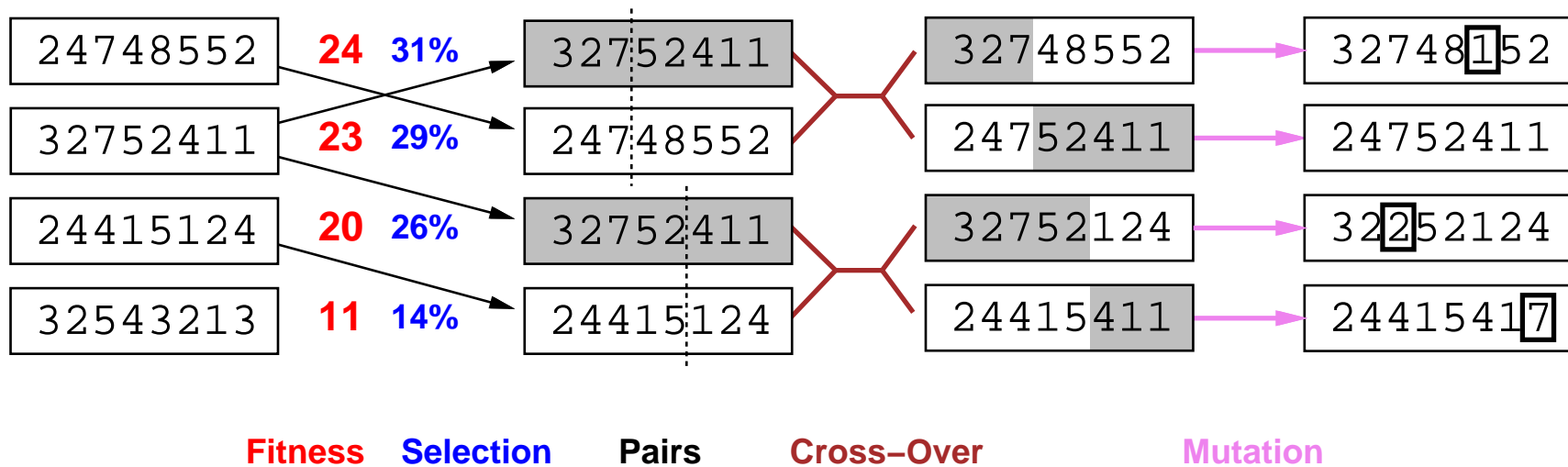
Problem: quite often, all k states end up on same local hill

Idea: choose k successors randomly, biased towards good ones

Observe the close analogy to natural selection!

Genetic algorithms

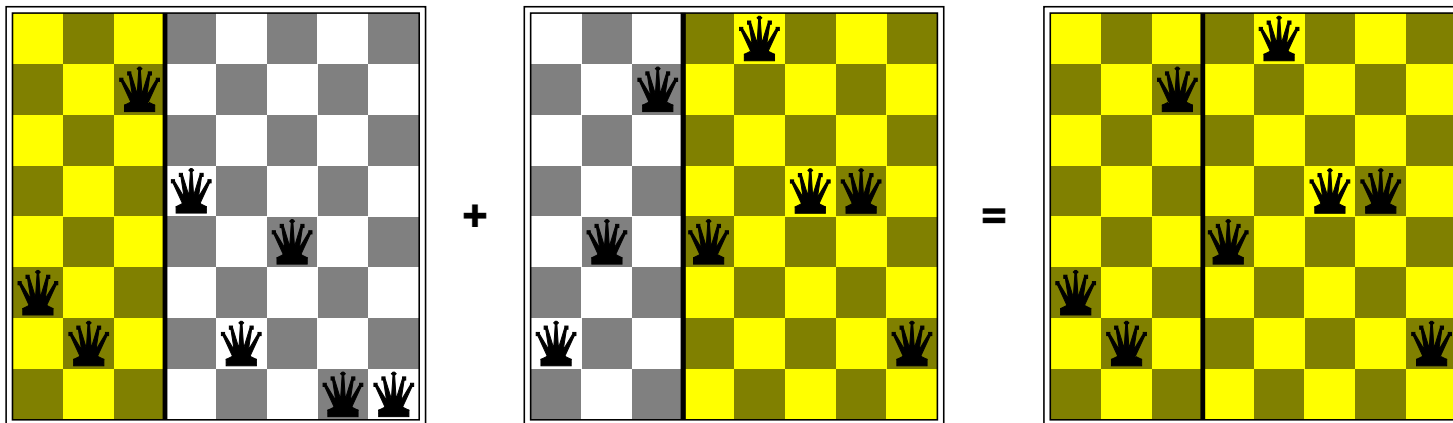
= stochastic local beam search + generate successors from **pairs** of states



Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)

Crossover helps **iff substrings are meaningful components**



GAs \neq evolution: e.g., real genes encode replication machinery!

Continuous state spaces

Suppose we want to site three airports in Romania:

- 6-D state space defined by $(x_1, y_1), (x_2, y_2), (x_3, y_3)$
- objective function $f(x_1, y_1, x_2, y_2, x_3, y_3) =$
sum of squared distances from each city to nearest airport

Discretization methods turn continuous space into discrete space, e.g., **empirical gradient** considers $\pm\delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3} \right)$$

to increase/reduce f , e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city).

Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x}) \nabla f(\mathbf{x})$

to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$