CS 310

Recurrence Relations Master Theorem

Let a, b, and d be integers such that $a > 0$, $b > 1$, and $d \geq 0$.

Upper bound:

If

$$T(n) \leq aT\left(\frac{n}{b}\right) + O(n^d)$$

then

$$T(n) \in \begin{cases} O(n^d) & \text{if } a < b^d \text{ (or } d > \log_b a) \\ O(n^d \log_b n) & \text{if } a = b^d \text{ (or } d = \log_b a) \\ O(n^{\log_b a}) & \text{if } a > b^d \text{ (or } d < \log_b a) \end{cases}$$

Lower bound:

If

$$T(n) \geq aT\left(\frac{n}{b}\right) + \Omega(n^d)$$

then

$$T(n) \in \begin{cases} \Omega(n^d) & \text{if } a < b^d \text{ (or } d > \log_b a) \\ \Omega(n^d \log_b n) & \text{if } a = b^d \text{ (or } d = \log_b a) \\ \Omega(n^{\log_b a}) & \text{if } a > b^d \text{ (or } d < \log_b a) \end{cases}$$

Tight bound:

If

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^d)$$

then

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \text{ (or } d > \log_b a) \\ \Theta(n^d \log_b n) & \text{if } a = b^d \text{ (or } d = \log_b a) \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \text{ (or } d < \log_b a) \end{cases}$$