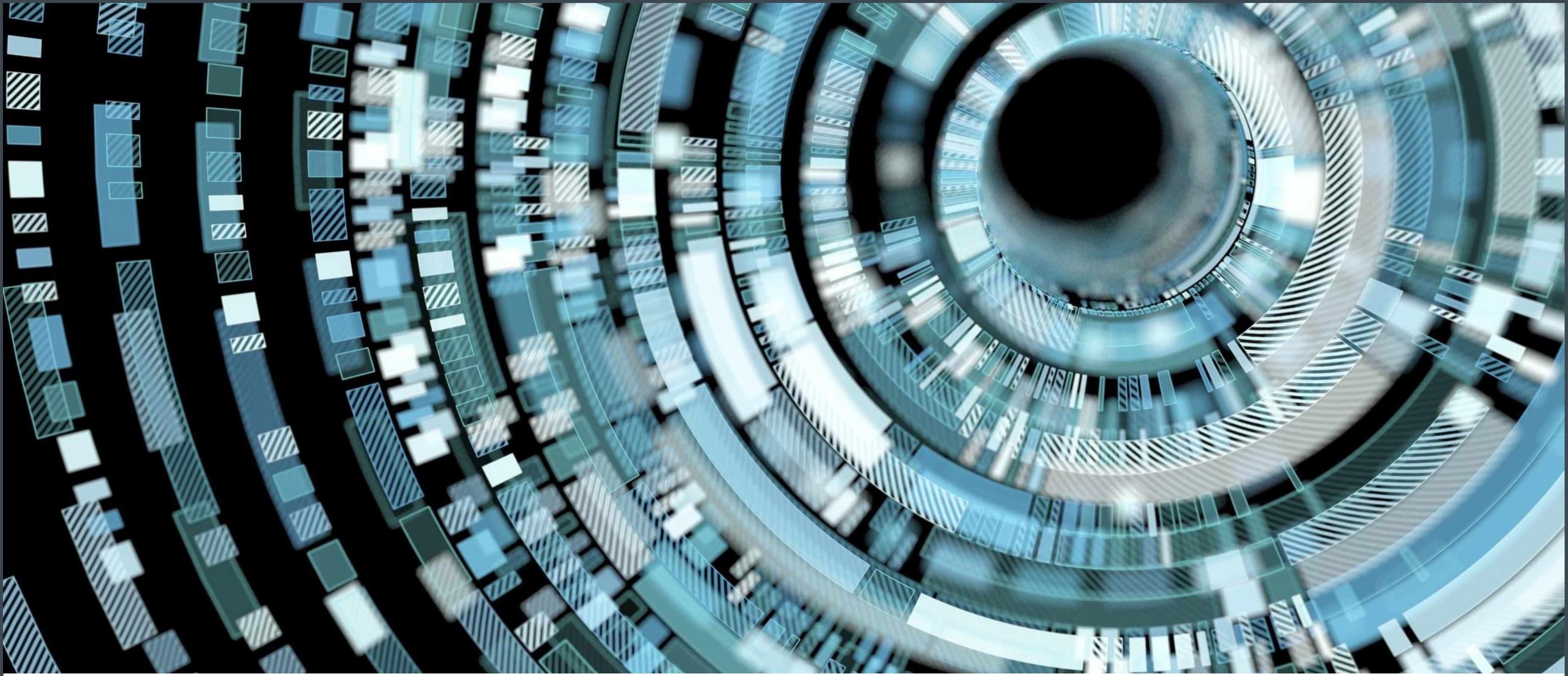
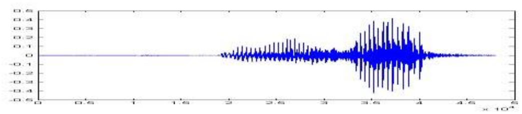


Introduction of Machine Learning

Machine Learning
≈ Looking for Function

- Speech Recognition

$f($  $) =$ "How are you"

- Image Recognition

$f($  $) =$ "Cat"

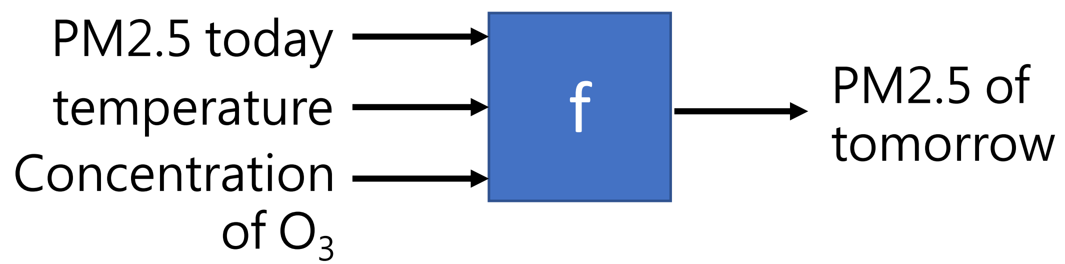
- Playing Go

$f($  $) =$ "5-5" (next move)

Different types of Functions

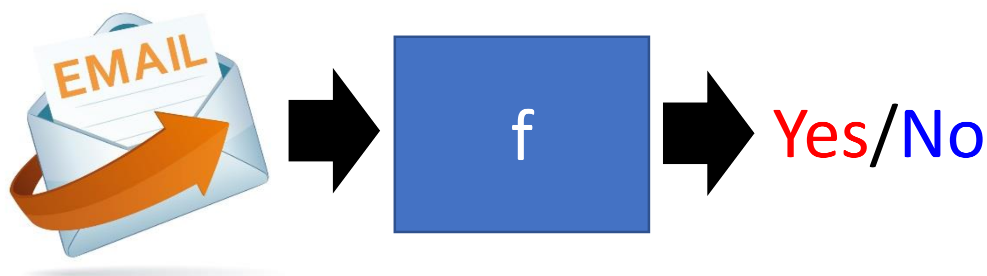
<u>Regression</u>: The function outputs a scalar.

Predict PM2.5



<u>Classification</u>: Given options (**classes**), the function outputs the correct one.

Spam filtering



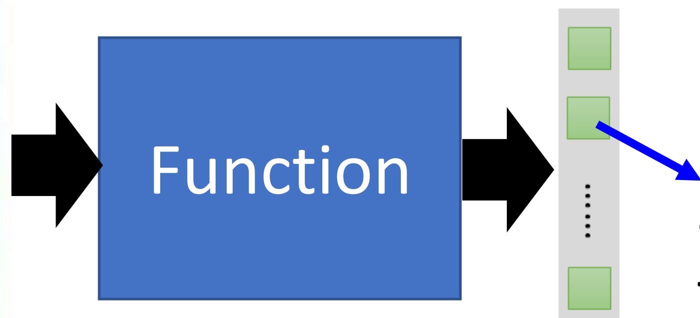
Different types of Functions

<u>**Classification:**</u> Given options (**classes**), the function outputs the correct one.

Each position is a class

(19 x 19 classes)

<u>Playing GO</u>



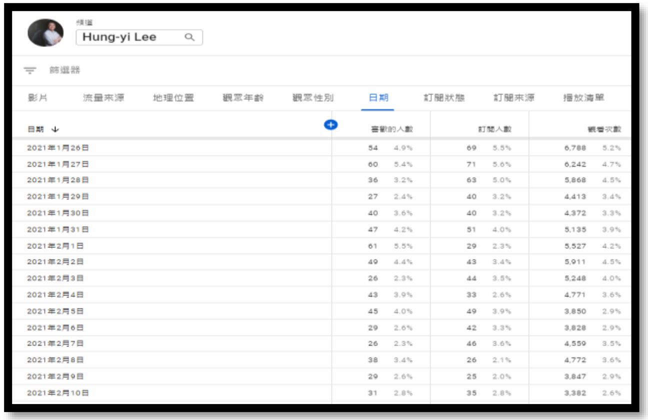
a position on the board

Next move

How to find a function?
A Case Study

The function we want to find …

$$y = f(\quad)$$



頻道

Hung-yi Lee

篩選器

影片 流量來源 地理位置 觀眾年齡 觀眾性別 日期 訂閱狀態 訂閱來源 播放清單

日期	喜歡的人數		訂閱人數		觀看次數	
2021年1月26日	54	4.9%	69	5.5%	6,788	5.2%
2021年1月27日	60	5.4%	71	5.6%	6,242	4.7%
2021年1月28日	36	3.2%	63	5.0%	5,868	4.5%
2021年1月29日	27	2.4%	40	3.2%	4,413	3.4%
2021年1月30日	40	3.6%	40	3.2%	4,372	3.3%
2021年1月31日	47	4.2%	51	4.0%	5,135	3.9%
2021年2月1日	61	5.5%	29	2.3%	5,527	4.2%
2021年2月2日	49	4.4%	43	3.4%	5,911	4.5%
2021年2月3日	26	2.3%	44	3.5%	5,248	4.0%
2021年2月4日	43	3.9%	33	2.6%	4,771	3.6%
2021年2月5日	45	4.0%	49	3.9%	3,850	2.9%
2021年2月6日	29	2.6%	42	3.3%	3,828	2.9%
2021年2月7日	26	2.3%	46	3.6%	4,559	3.5%
2021年2月8日	38	3.4%	26	2.1%	4,772	3.6%
2021年2月9日	29	2.6%	25	2.0%	3,847	2.9%
2021年2月10日	31	2.8%	35	2.8%	3,382	2.6%

no. of views
on 2/26

1. Function with Unknown Parameters

$y = f($  $)$

Model $y = b + wx_1$ based on domain knowledge

feature

y: no. of views on 2/26, x_1: no. of views on 2/25

w and b are unknown parameters (learned from data)

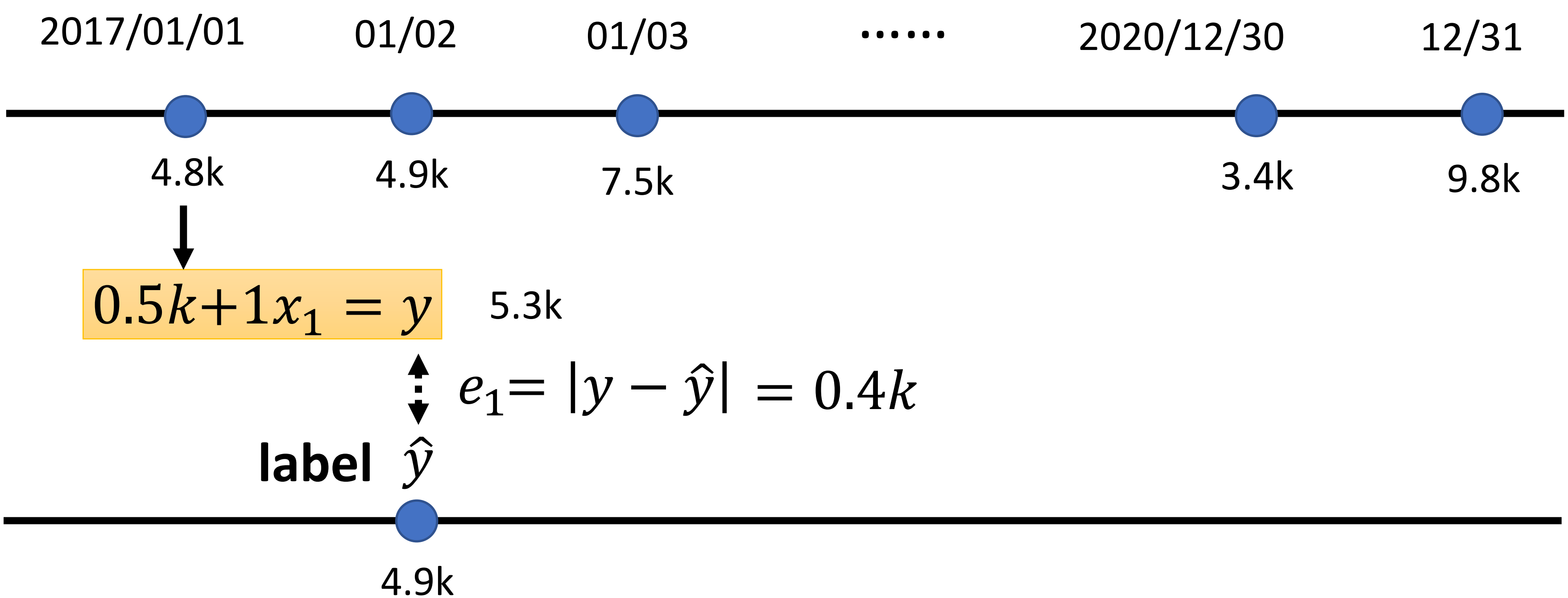
weight **bias**

2. Define Loss from Training Data

- Loss is a function of parameters $L(b, w)$
- Loss: how good a set of values is.

$L(0.5k, 1)$ $\quad y = b + wx_1 \longrightarrow y = 0.5k + 1x_1$ How good it is?

Data from 2017/01/01 – 2020/12/31



2. Define Loss from Training Data

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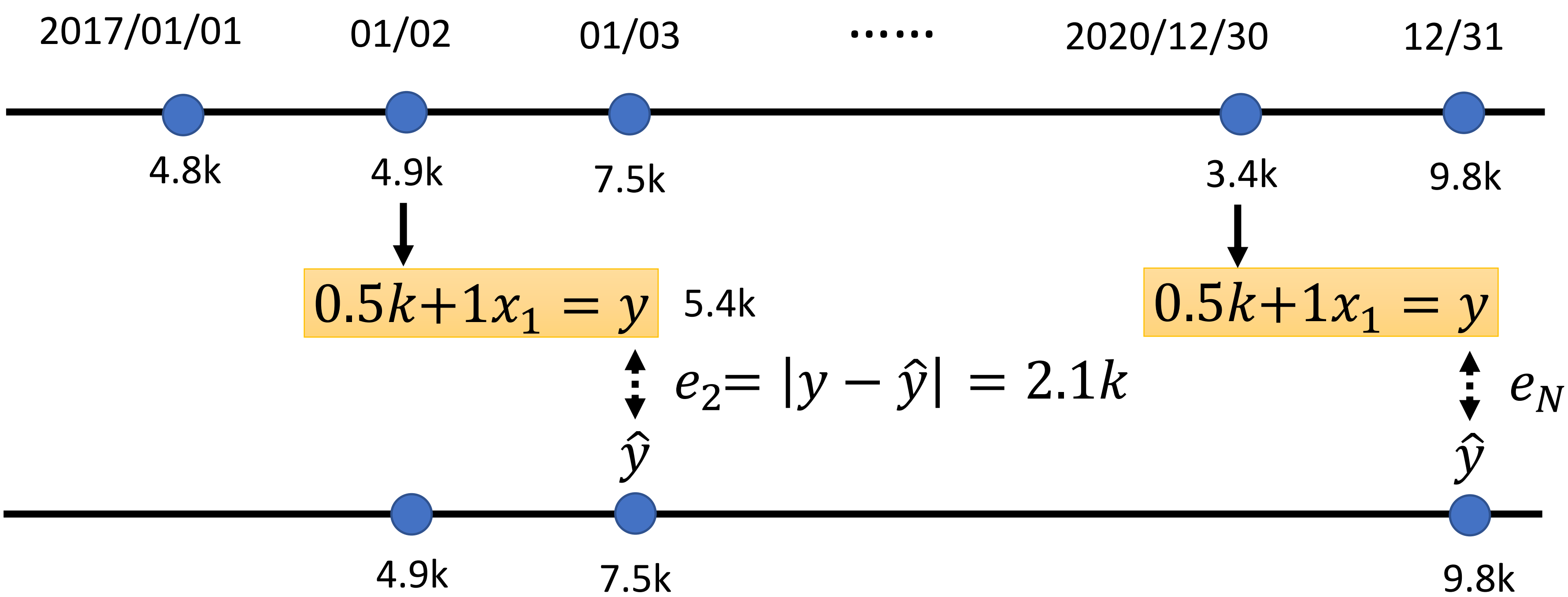
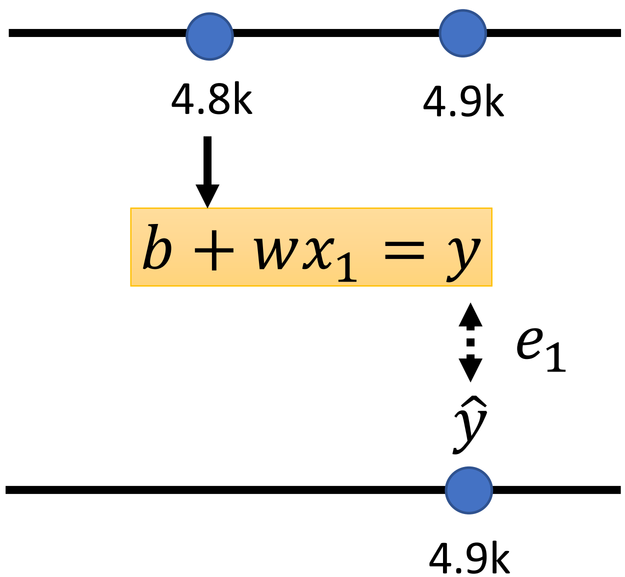
Data from 2017/01/01 – 2020/12/31



2. Define Loss from Training Data

- Loss is a function of parameters $L(b, w)$
- Loss: how good a set of values is.



Loss: $$L = \frac{1}{N}\sum_{n} e_n$$

$e = |y - \hat{y}|$ L is mean absolute error (MAE)

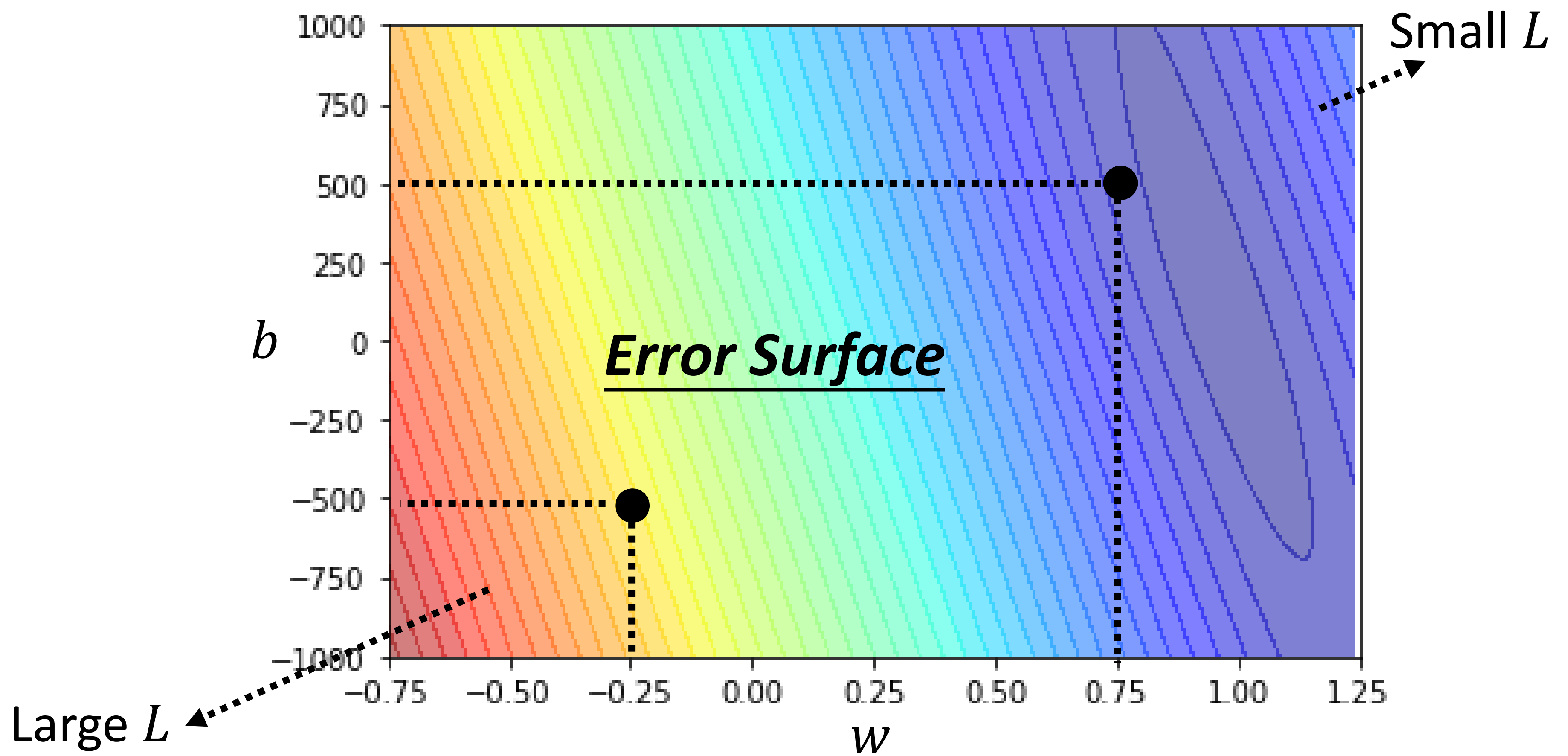
$e = (y - \hat{y})^2$ L is mean square error (MSE)

If y and $\hat{y}$ are both probability distributions → Cross-entropy

2. Define Loss from Training Data

- Loss is a function of parameters $L(b, w)$
- Loss: how good a set of values is.

Model $y = b + wx_1$

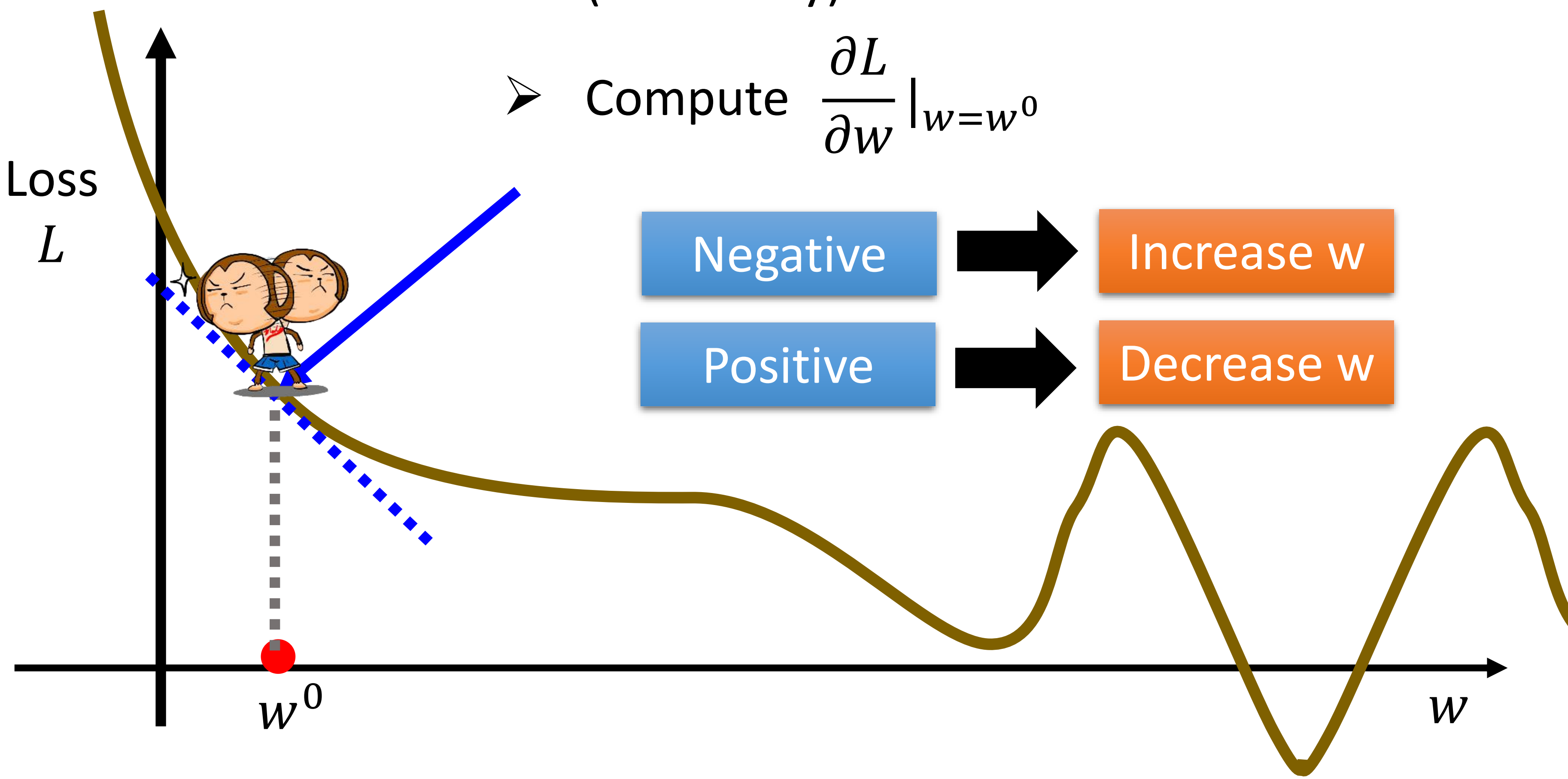


3. Optimization

w^* [redacted] $= arg \min_{w}$ [redacted] L

Gradient Descent

- (Randomly) Pick an initial value w^0
- Compute $\frac{\partial L}{\partial w}|_{w=w^0}$



Negative → Increase w

Positive → Decrease w

3. Optimization

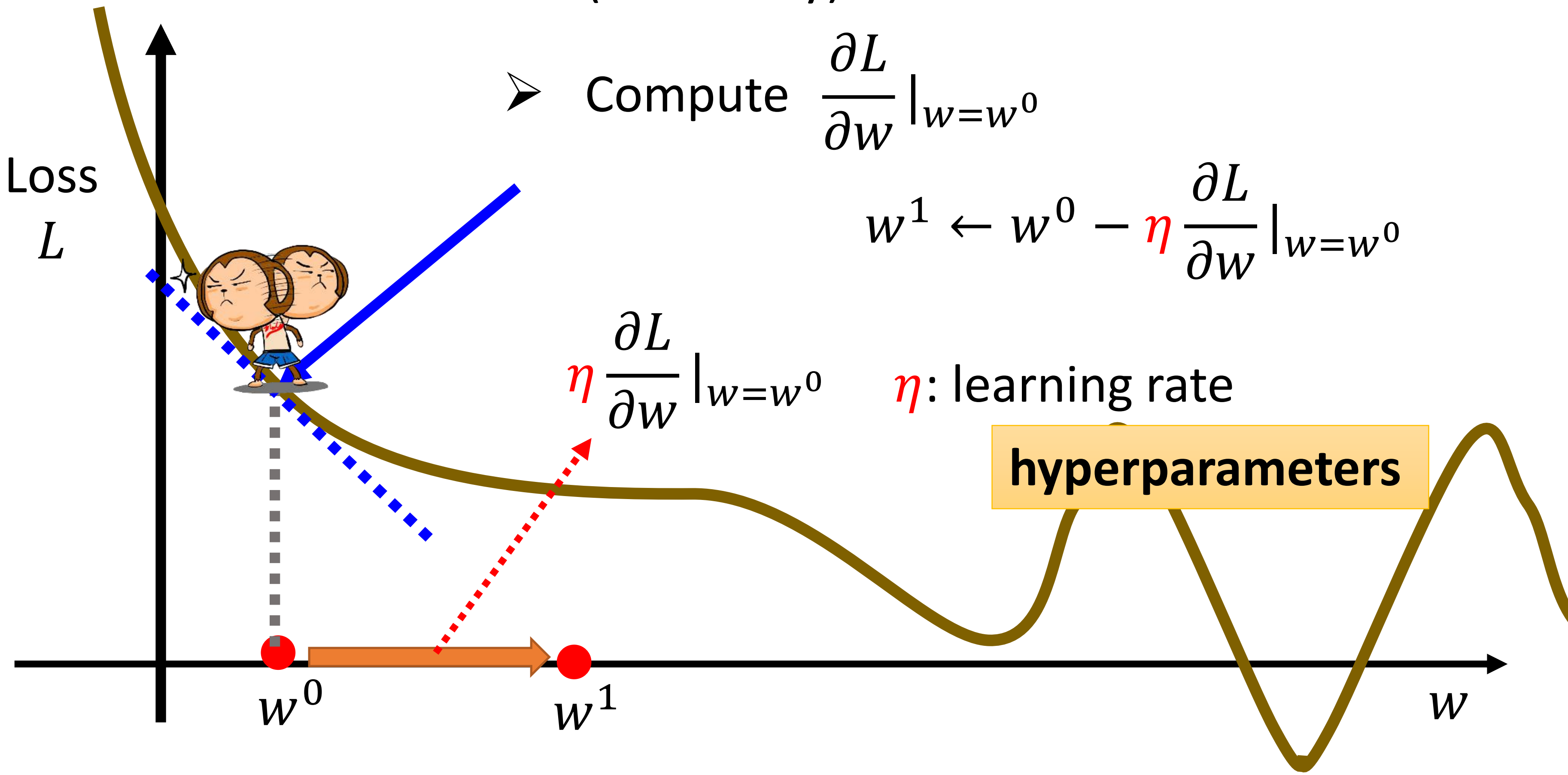
w^* [redacted] $= arg \min_{w} L$ [redacted]

Gradient Descent

- (Randomly) Pick an initial value w^0
- Compute $\frac{\partial L}{\partial w}|_{w=w^0}$

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w}|_{w=w^0}$$

η: learning rate



3. Optimization

w^* [redacted] $= arg \min_{w}$ [redacted] L

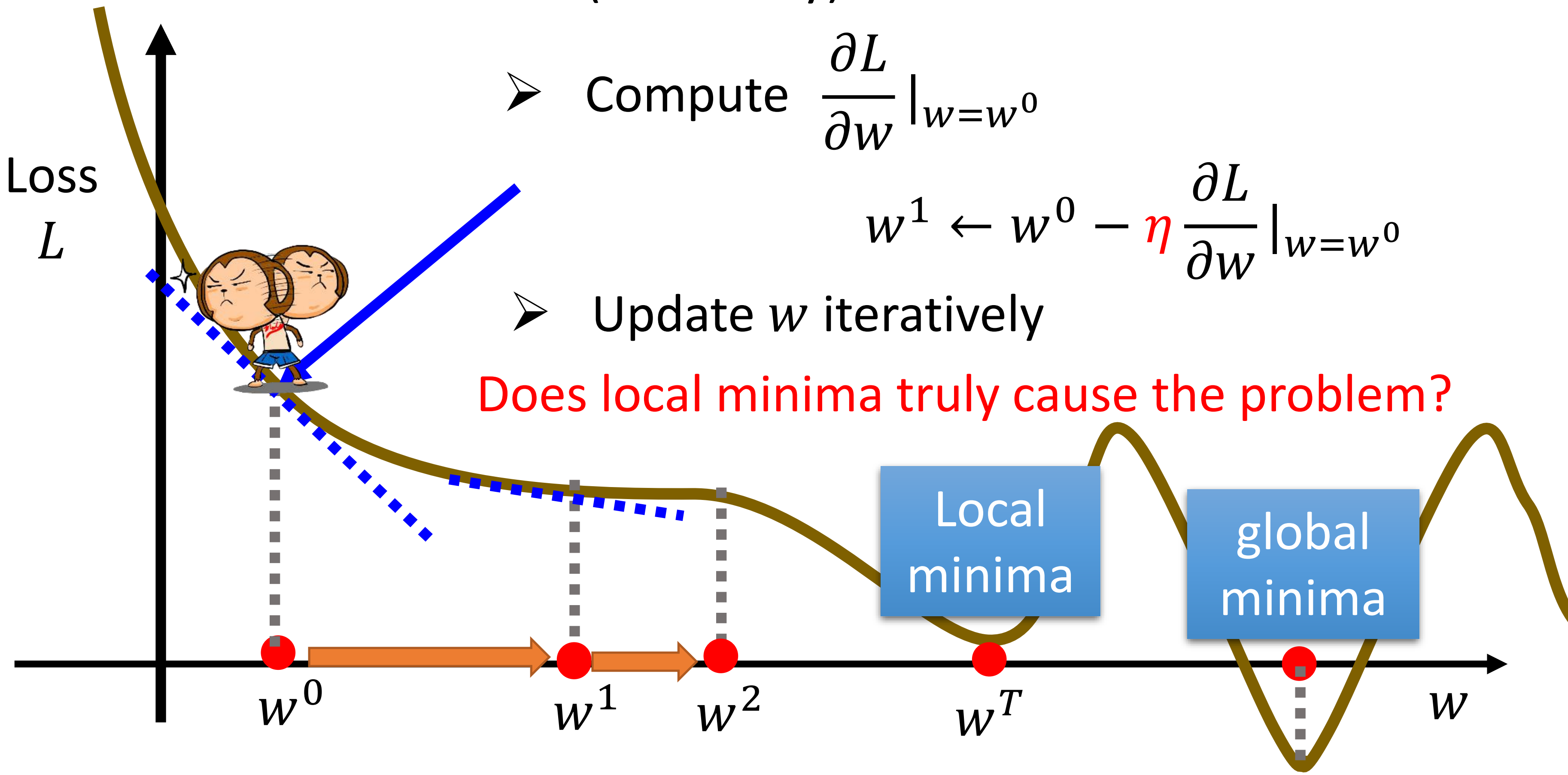
Gradient Descent

- (Randomly) Pick an initial value w^0
- Compute $\frac{\partial L}{\partial w}|_{w=w^0}$

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w}|_{w=w^0}$$

- Update w iteratively

Does local minima truly cause the problem?



3. Optimization

$$w^*, b^* = arg \min_{w,b} L$$

- (Randomly) Pick initial values w^0, b^0
- Compute

$$\boxed{\begin{array}{l} \frac{\partial L}{\partial w}|_{w=w^0,b=b^0} \\ \\ \frac{\partial L}{\partial b}|_{w=w^0,b=b^0} \end{array}}$$

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w}|_{w=w^0,b=b^0}$$

$$b^1 \leftarrow b^0 - \eta \frac{\partial L}{\partial b}|_{w=w^0,b=b^0}$$

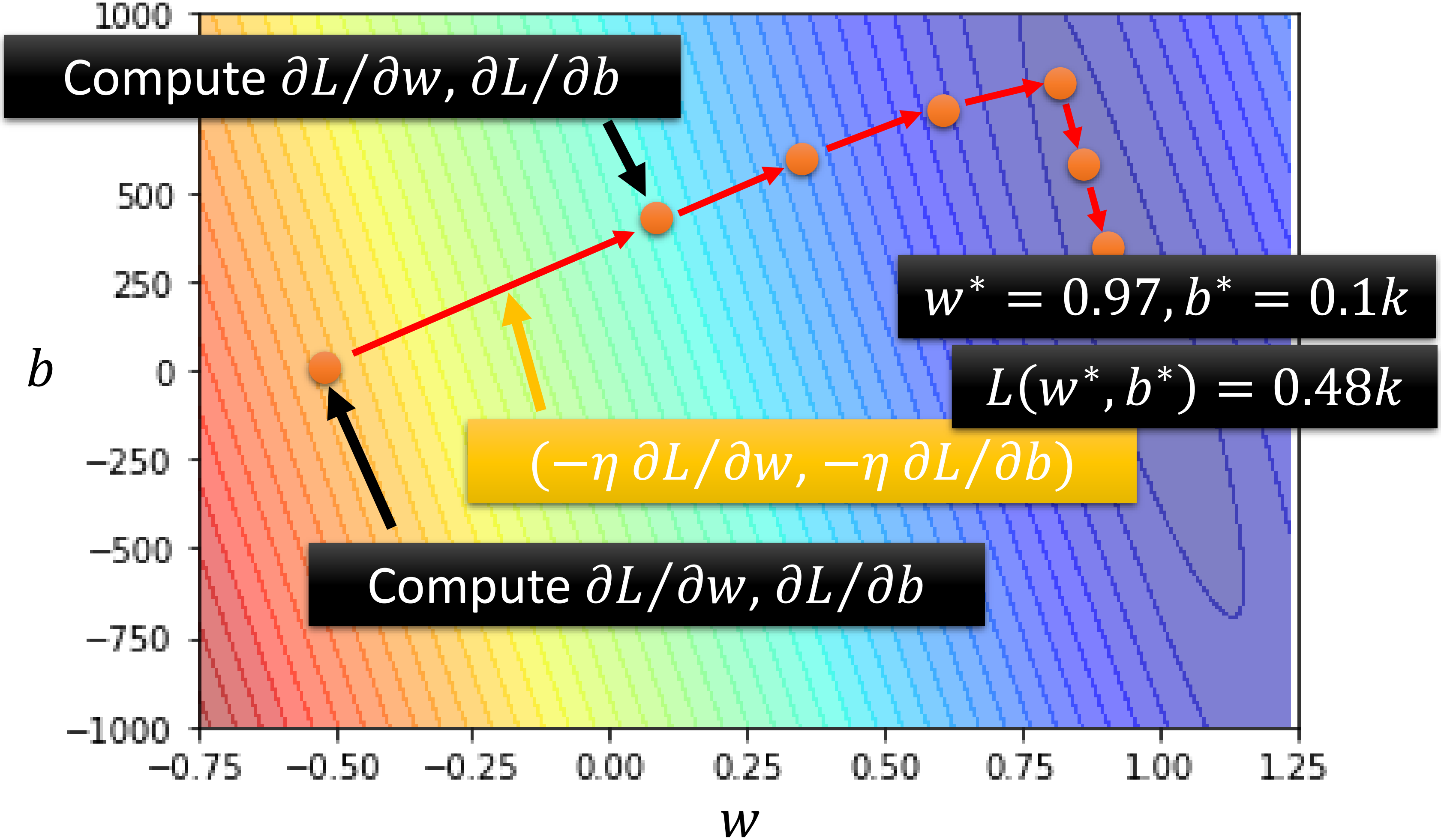
Can be done in one line in most deep learning frameworks

- Update w and b interatively

3. Optimization

Model $y = b + wx_1$

$$w^*, b^* = arg \min_{w,b} L$$



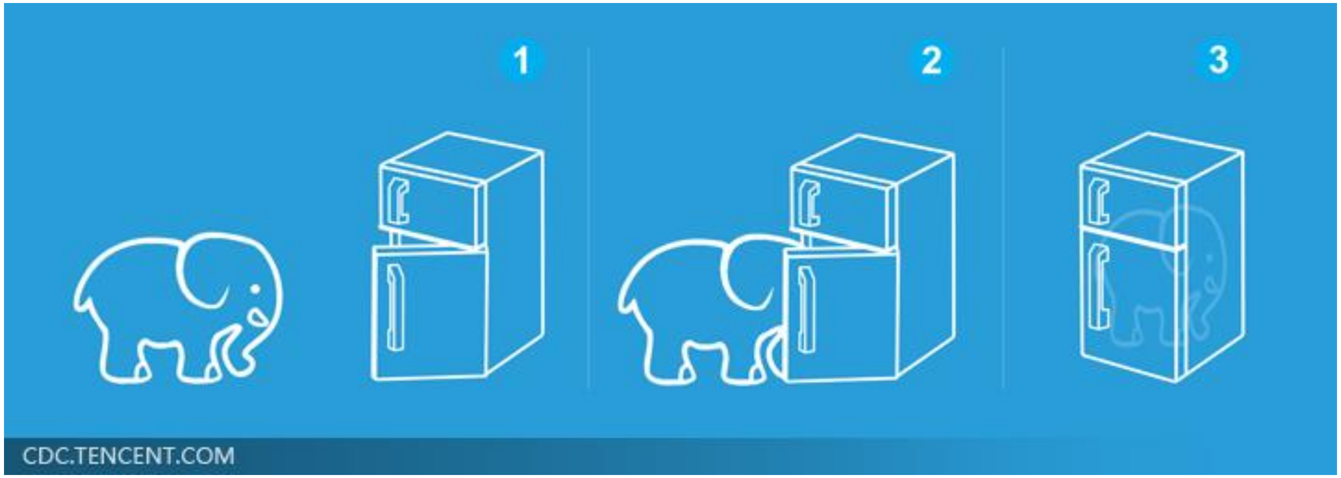
Machine Learning is so simple ……

$$w^* = 0.97, b^* = 0.1k$$

$$L(w^*, b^*) = 0.48k$$

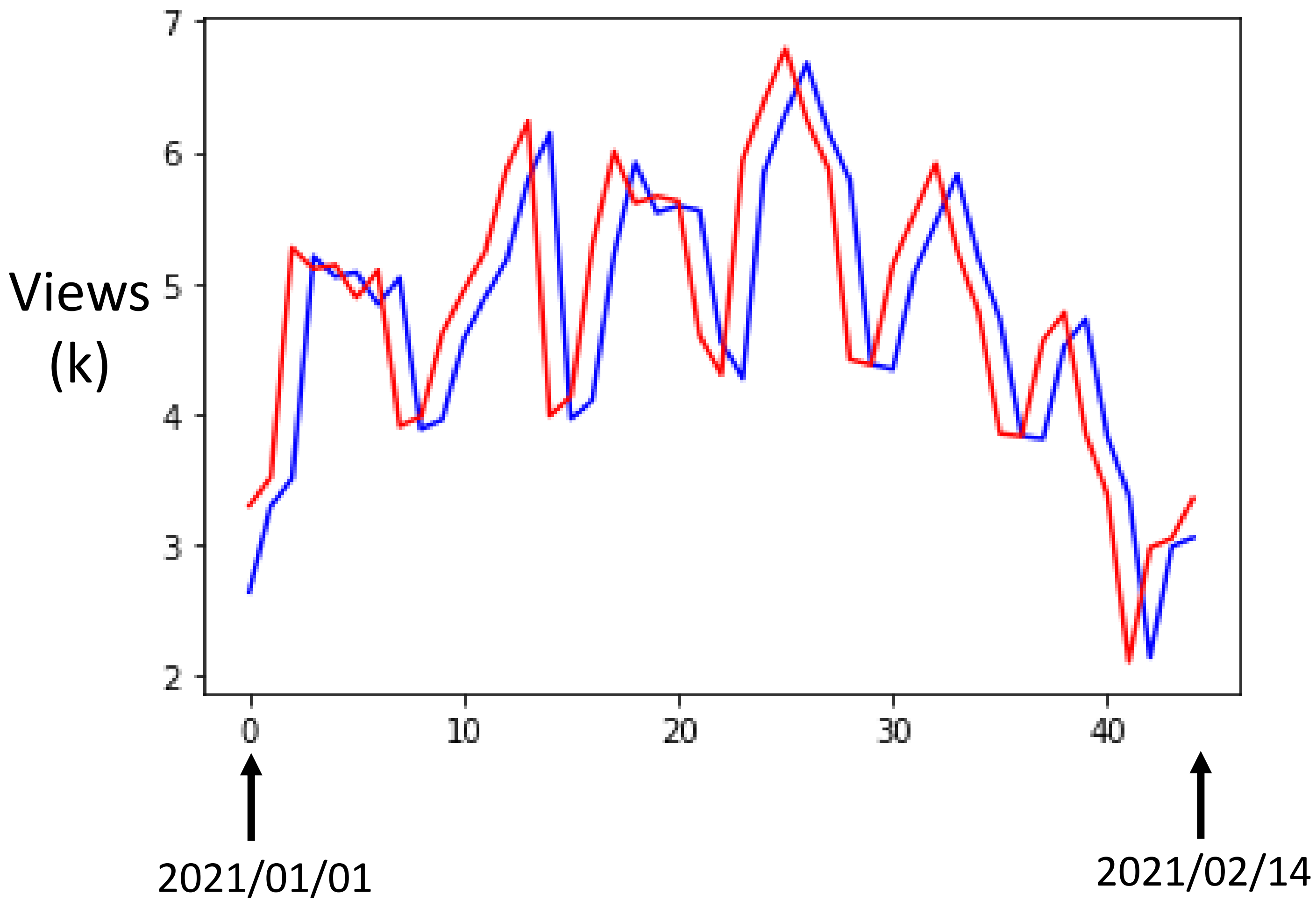
$$y = b + wx_1$$

Step 1: function with unknown → Step 2: define loss from training data → Step 3: optimization



$$y = 0.1k + 0.97x_1$$

Red: real no. of views
blue: estimated no. of views



$$y = b + wx_1$$

2017 - 2020 $L = 0.48k$

2021 $L' = 0.58k$

$$y = b + \sum_{j=1}^{7} w_j x_j$$

2017 - 2020 $L = 0.38k$

2021 $L' = 0.49k$

b	w_1^*	w_2^*	w_3^*	w_4^*	w_5^*	w_6^*	w_7^*
0.05k	0.79	-0.31	0.12	-0.01	-0.10	0.30	0.18

$$y = b + \sum_{j=1}^{28} w_j x_j$$

2017 - 2020 $L = 0.33k$

2021 $L' = 0.46k$

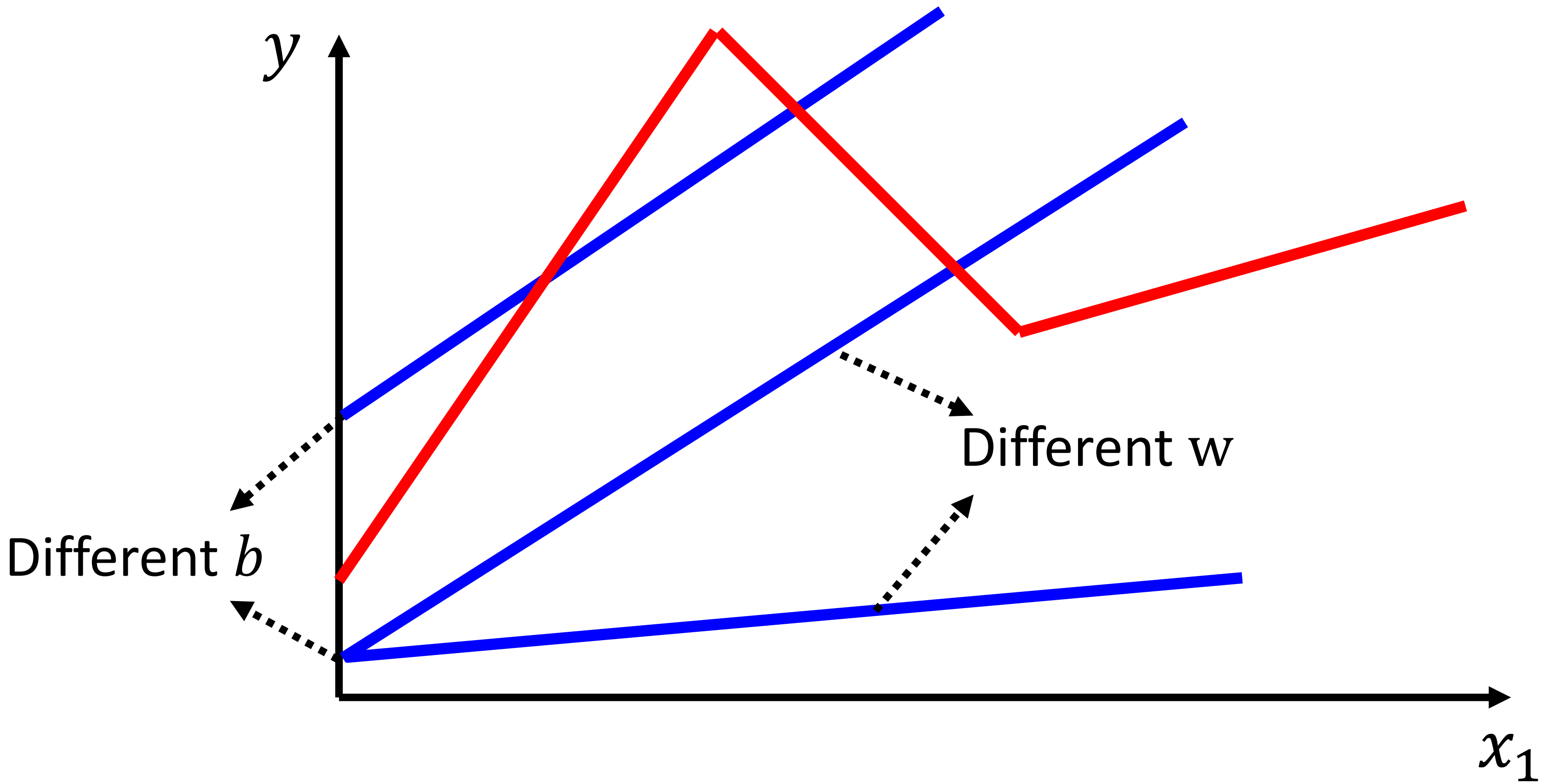
$$y = b + \sum_{j=1}^{56} w_j x_j$$

2017 - 2020 $L = 0.32k$

2021 $L' = 0.46k$

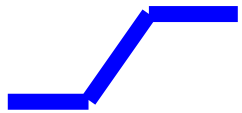
Linear models

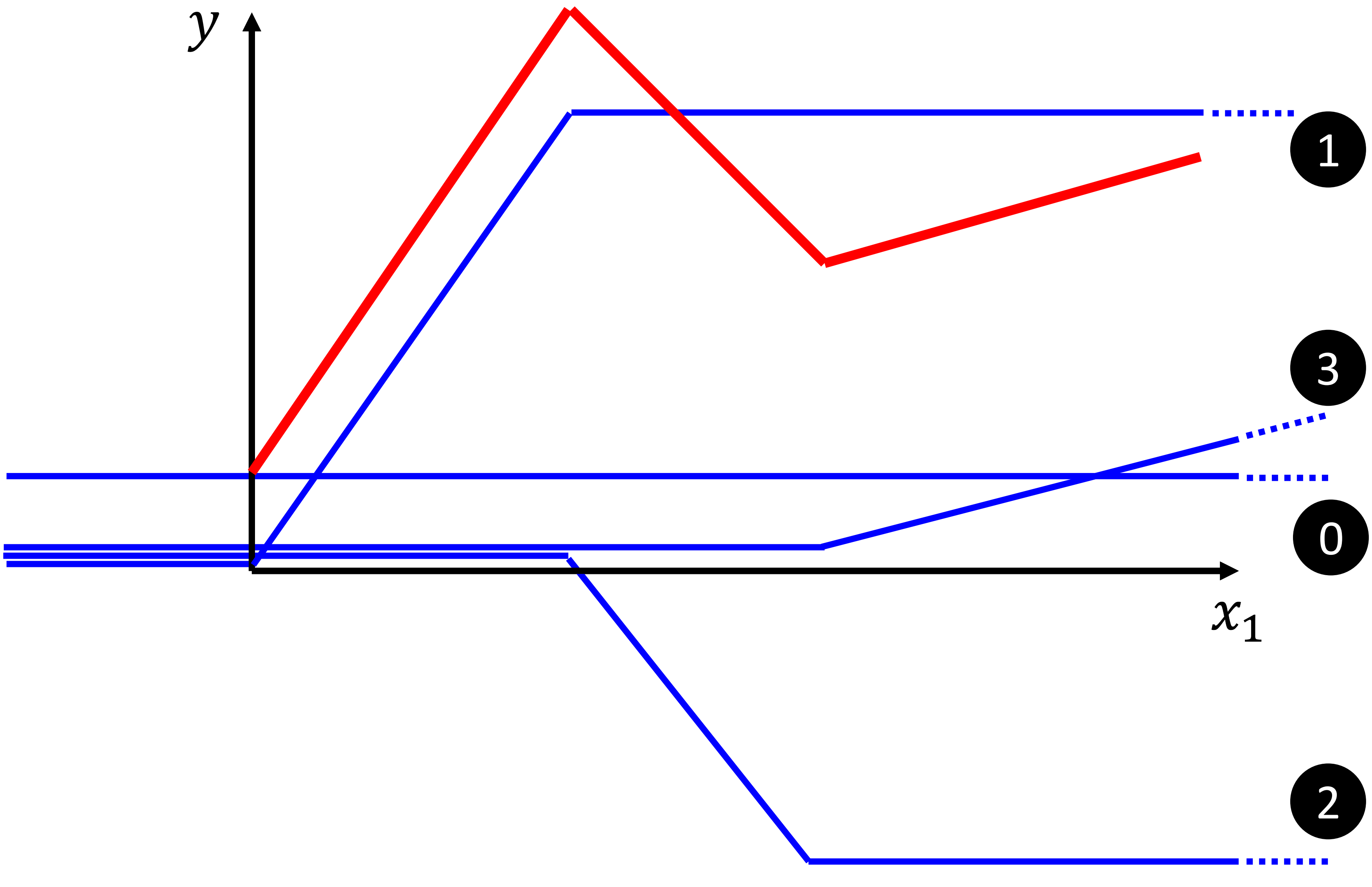
Linear models are too simple … we need more sophisticated modes.



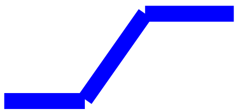
Linear models have severe limitation. ***Model Bias***

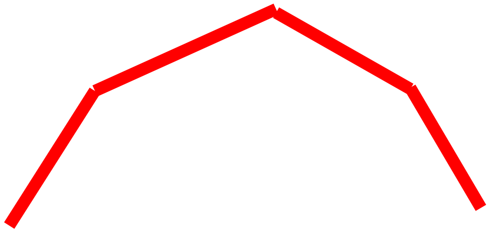
We need a more flexible model!

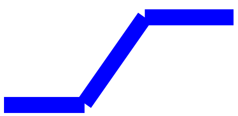
red curve = constant + sum of a set of 



All Piecewise Linear Curves

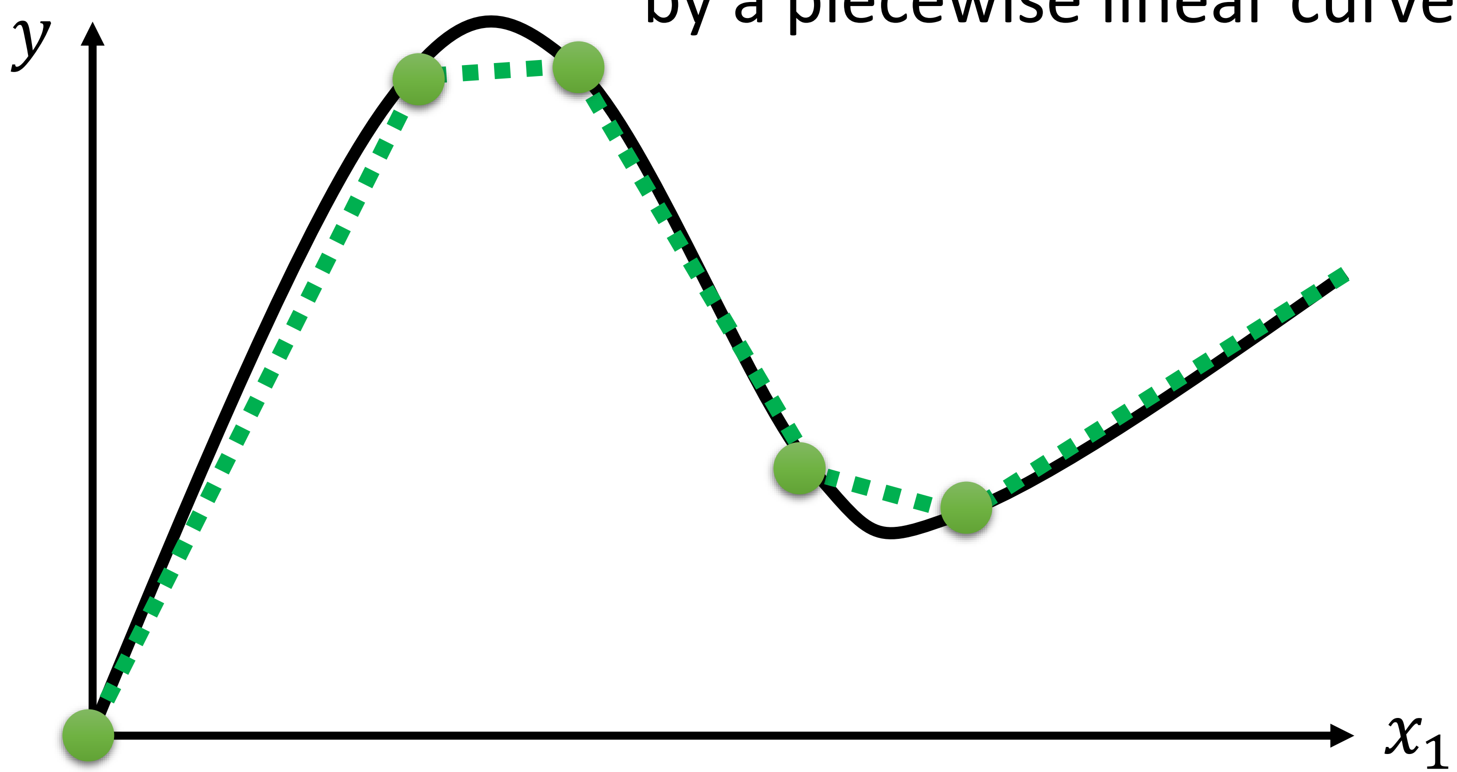
= constant + sum of a set of 



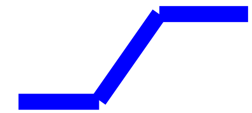
More pieces require more 

Beyond Piecewise Linear?

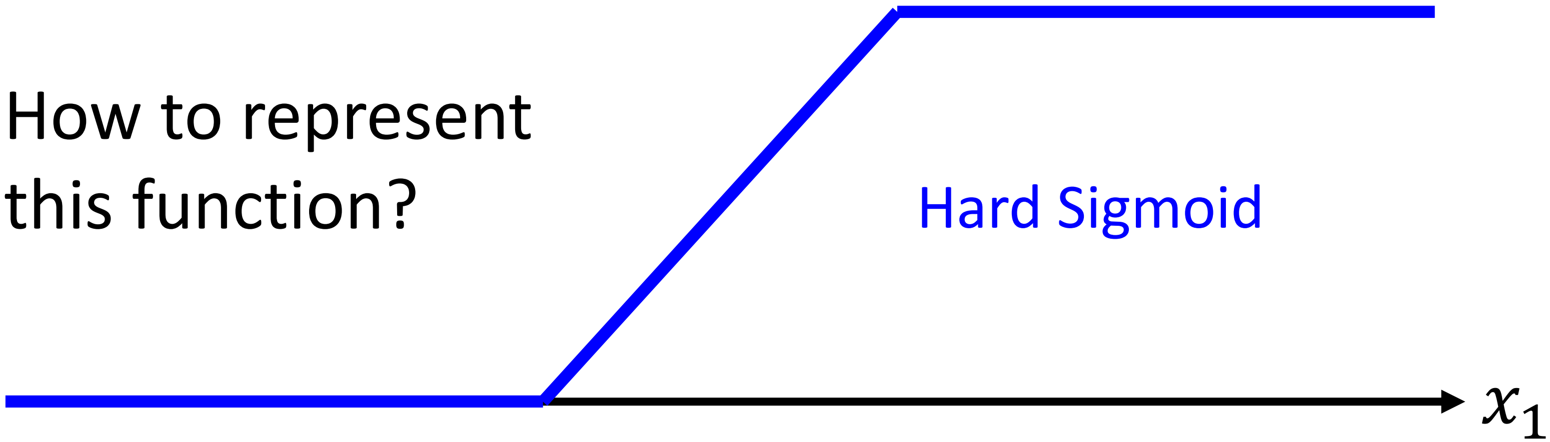
Approximate continuous curve by a piecewise linear curve.



To have good approximation, we need sufficient pieces.

red curve = constant + sum of a set of 

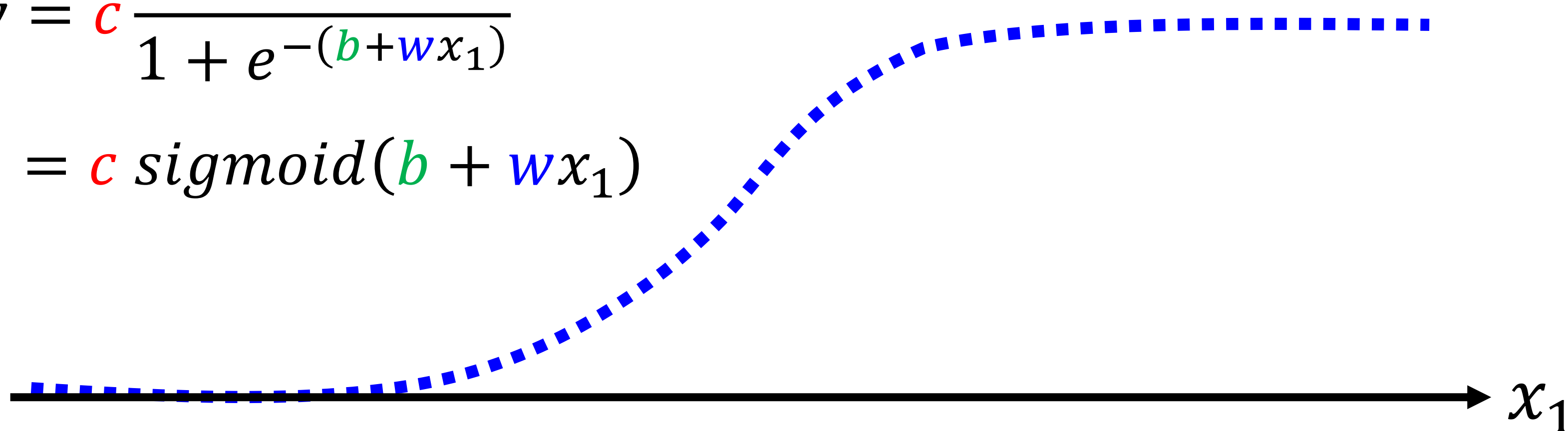
How to represent this function?

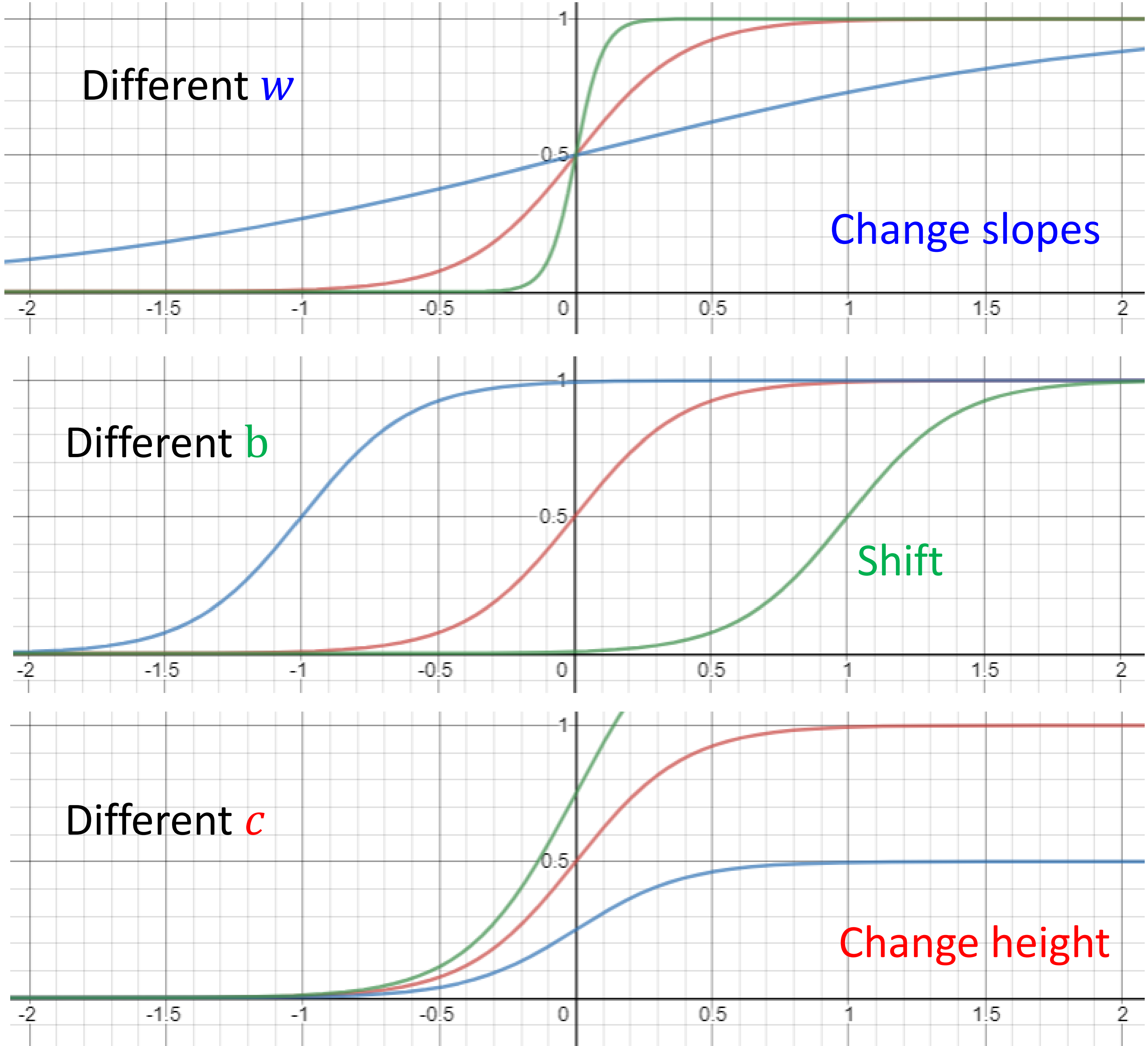


Sigmoid Function

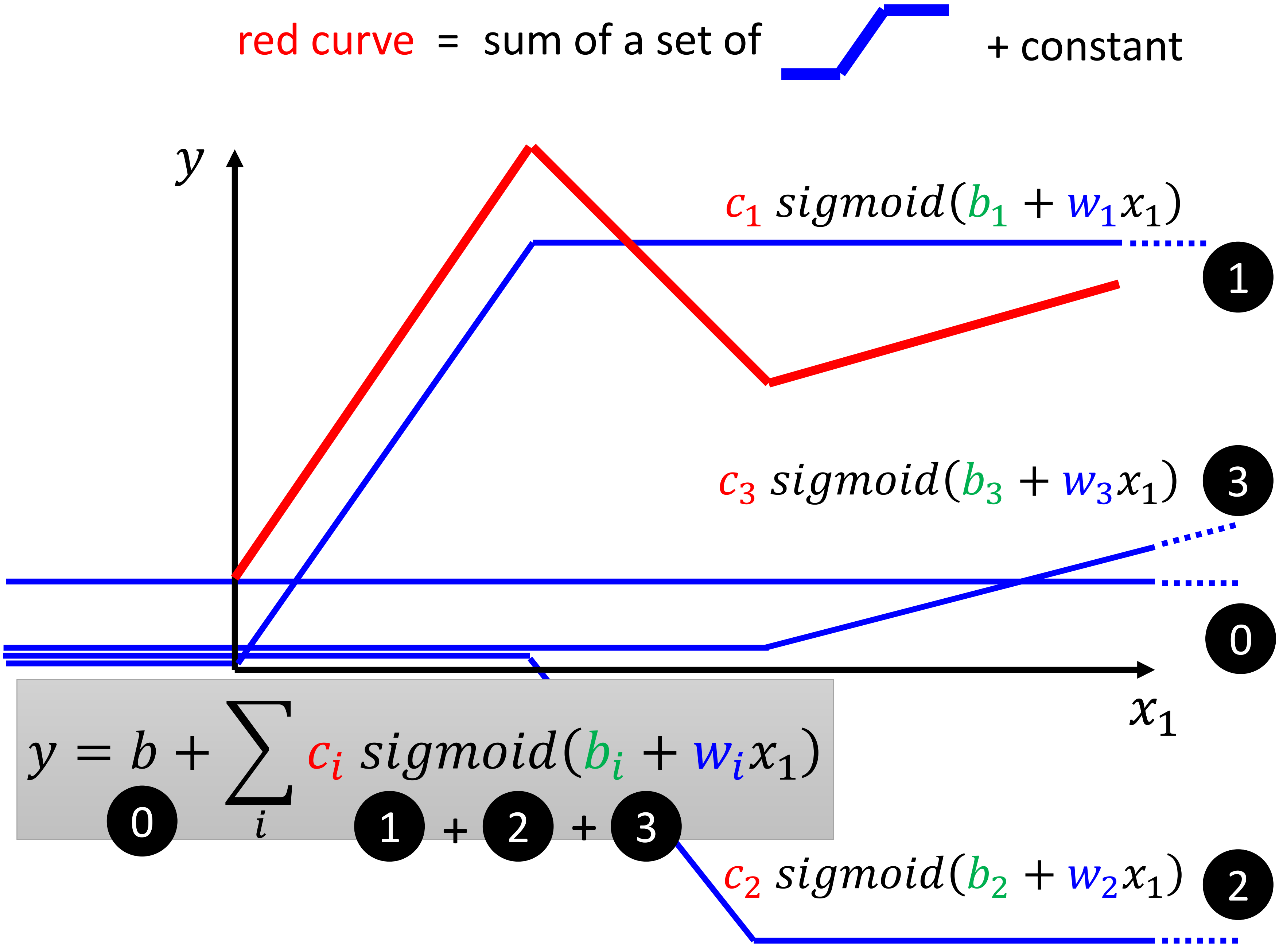
$$y = c \frac{1}{1 + e^{-(b + w x_1)}}$$

$$= c \; sigmoid(b + w x_1)$$





Different w
Change slopes
Different b
Shift
Different c
Change height


red curve = sum of a set of + constant
y
$c_1\ sigmoid(b_1 + w_1 x_1)$
1
$c_3\ sigmoid(b_3 + w_3 x_1)$
3
0
x_1
$y = b + \sum_i c_i\ sigmoid(b_i + w_i x_1)$
0
1 + 2 + 3
$c_2\ sigmoid(b_2 + w_2 x_1)$
2

New Model: More Features

$$y = b + wx_1$$

$$y = b + \sum_i c_i \; sigmoid(b_i + w_i x_1)$$

$$y = b + \sum_j w_j x_j$$

$$y = b + \sum_i c_i \; sigmoid\left(b_i + \sum_j w_{ij} x_j\right)$$

$$y = b + \sum_{i} c_i \; sigmoid\left(b_i + \sum_{j} w_{ij} x_j\right)$$

j: 1,2,3
no. of features

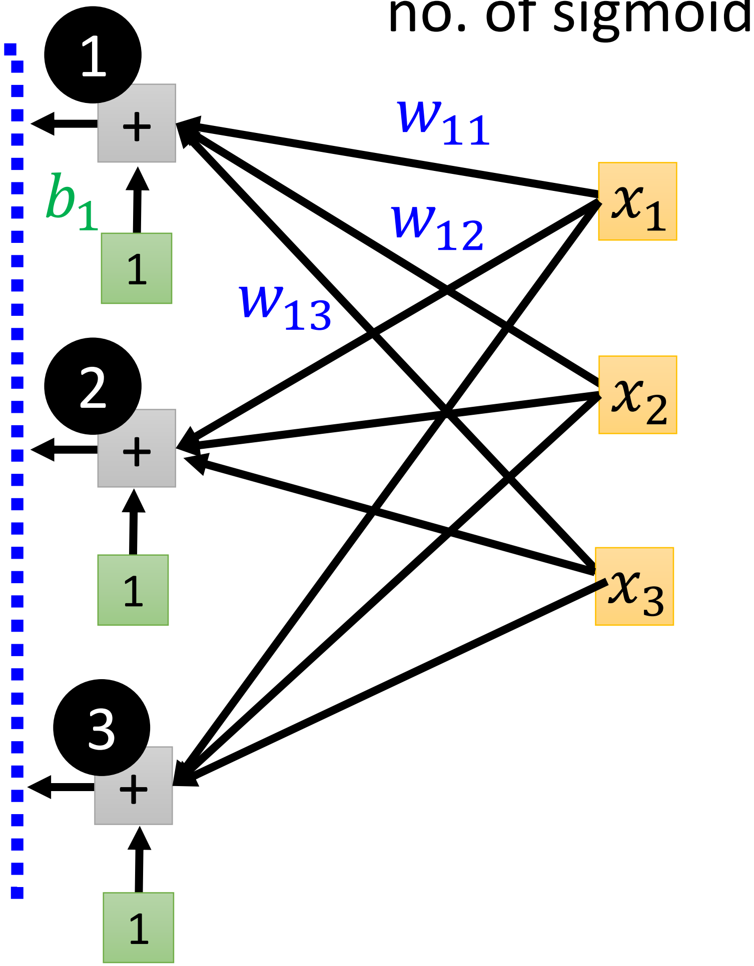
i: 1,2,3
no. of sigmoid

$$r_1 = b_1 + w_{11}x_1 + w_{12}x_2 + w_{13}x_3$$

w_{ij}: weight for x_j for i-th sigmoid

$$r_2 = b_2 + w_{21}x_1 + w_{22}x_2 + w_{23}x_3$$

$$r_3 = b_3 + w_{31}x_1 + w_{32}x_2 + w_{33}x_3$$



$$y = b + \sum_i c_i \, sigmoid\left(b_i + \sum_j w_{ij} x_j \right) \qquad \begin{matrix} i: 1,2,3 \\ j: 1,2,3 \end{matrix}$$

$$\boxed{\begin{aligned} r_1 &= b_1 + w_{11}x_1 + w_{12}x_2 + w_{13}x_3 \\ r_2 &= b_2 + w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \\ r_3 &= b_3 + w_{31}x_1 + w_{32}x_2 + w_{33}x_3 \end{aligned}}$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

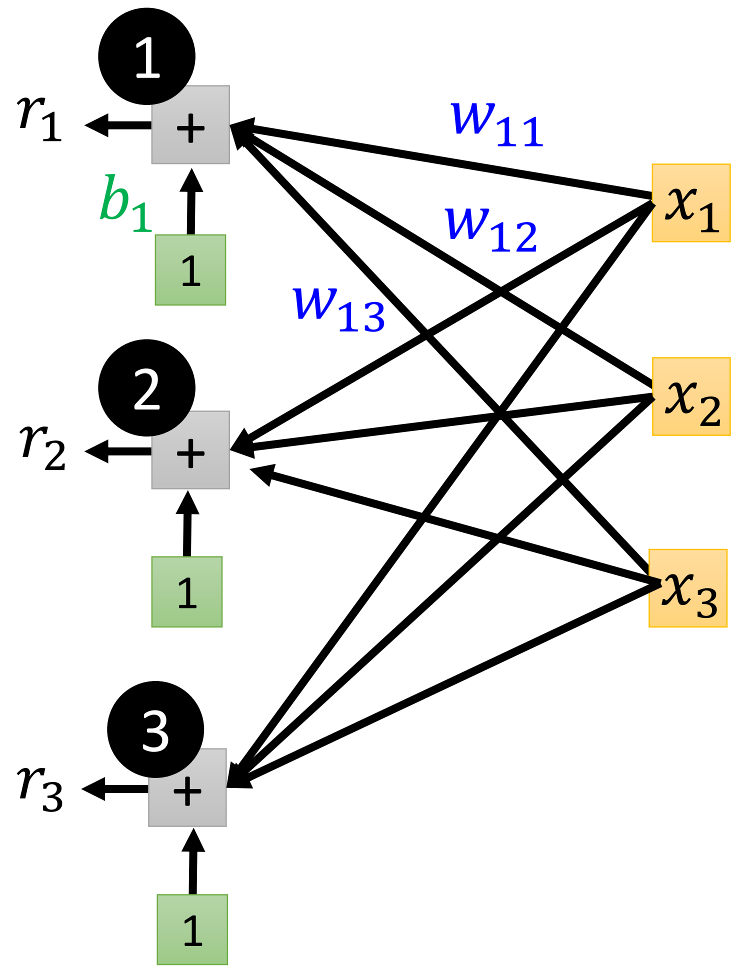
$$\boldsymbol{r} = \boldsymbol{b} + W \boldsymbol{x}$$

$$y = b + \sum_i c_i \; sigmoid\left(\boxed{b_i + \sum_j w_{ij} x_j}\right)$$

i: 1,2,3

j: 1,2,3

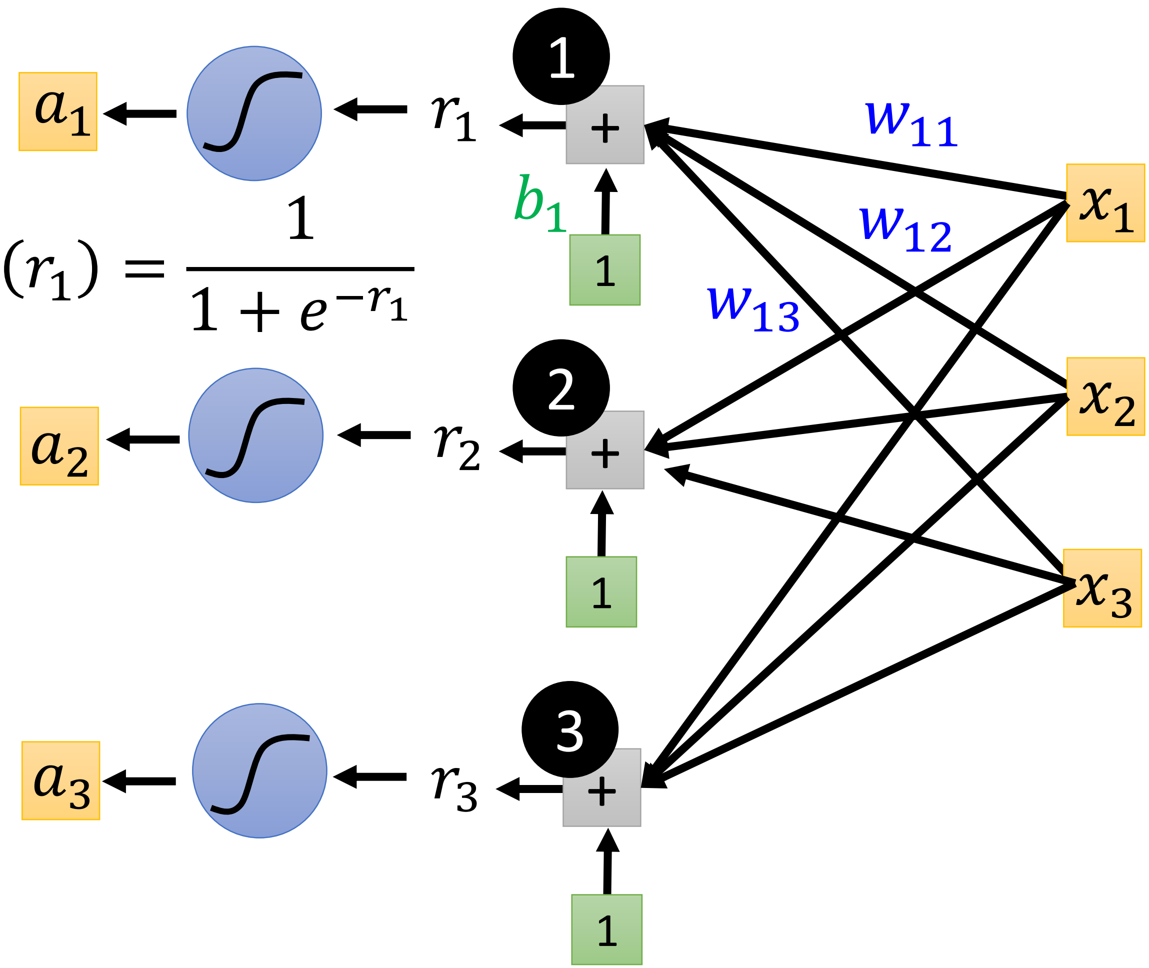
$$\boldsymbol{r} = \boldsymbol{b} + W \; \boldsymbol{x}$$



$$y = b + \sum_i c_i \boxed{sigmoid\left(b_i + \sum_j w_{ij} x_j\right)}$$

i: 1,2,3

j: 1,2,3



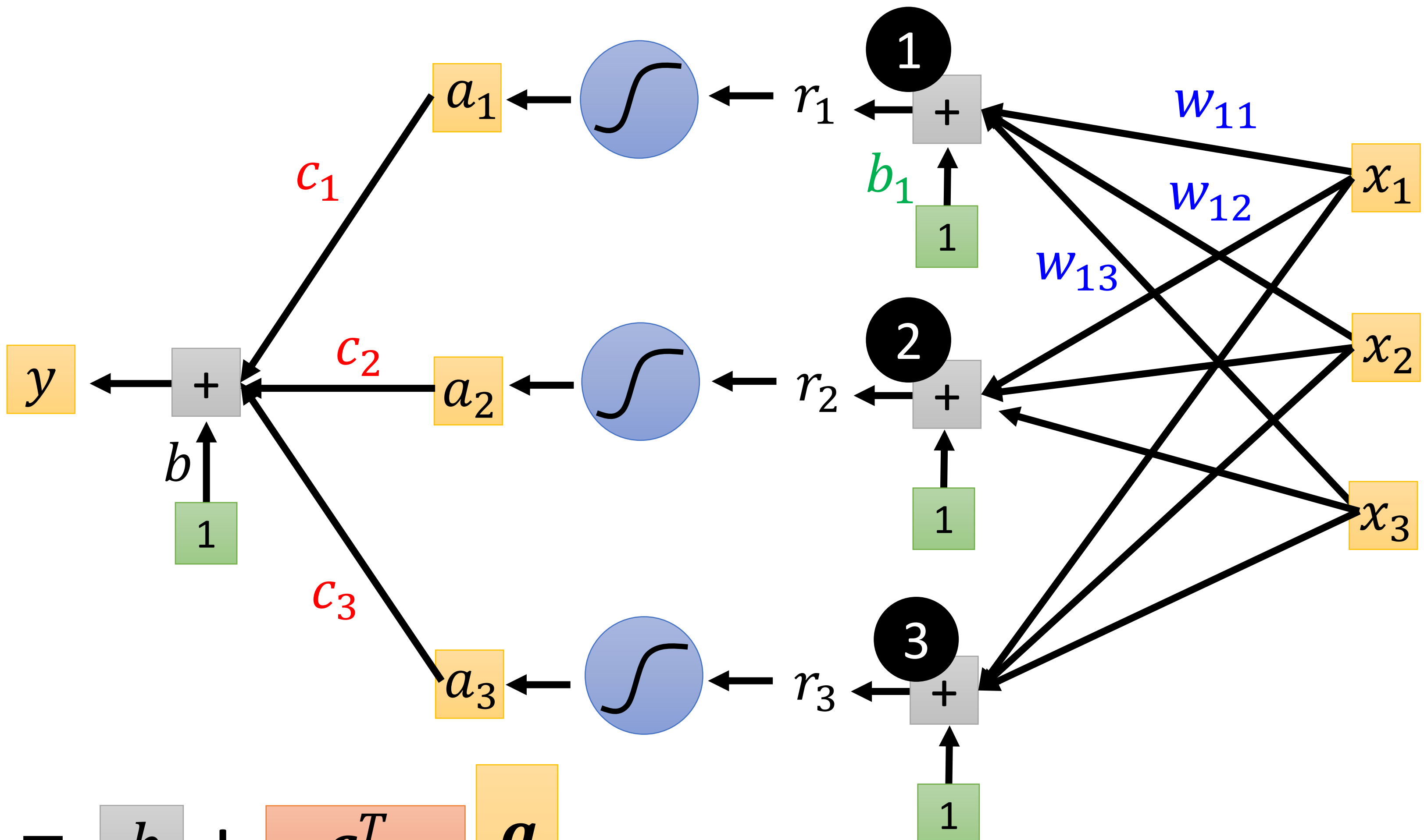
$$a_1 = sigmoid(r_1) = \frac{1}{1 + e^{-r_1}}$$

$$\boldsymbol{a} = \sigma(\boldsymbol{r})$$

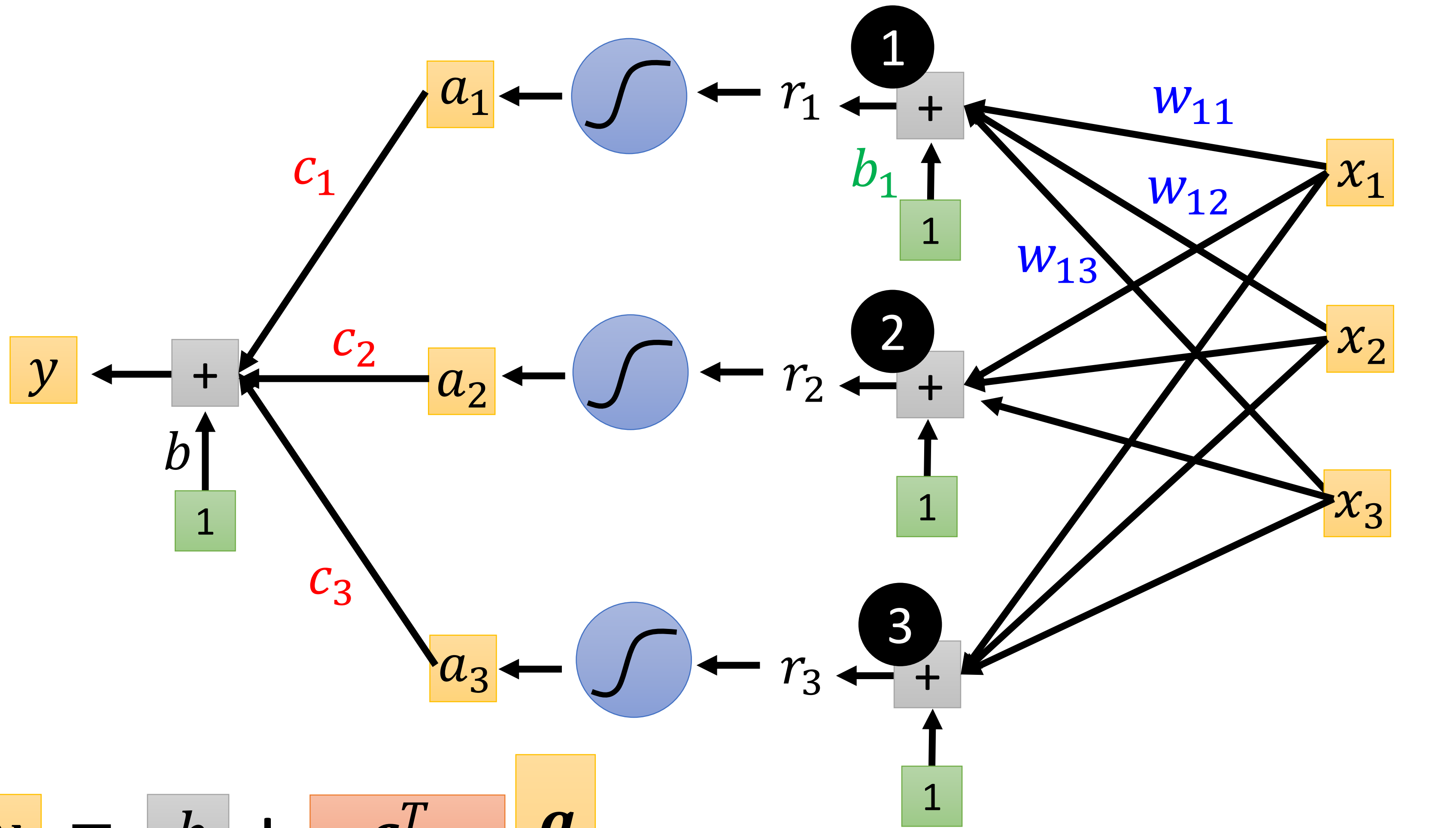
$$y = \boxed{b + \sum_i c_i \; sigmoid\left(b_i + \sum_j w_{ij} x_j\right)}$$

i: 1,2,3

j: 1,2,3

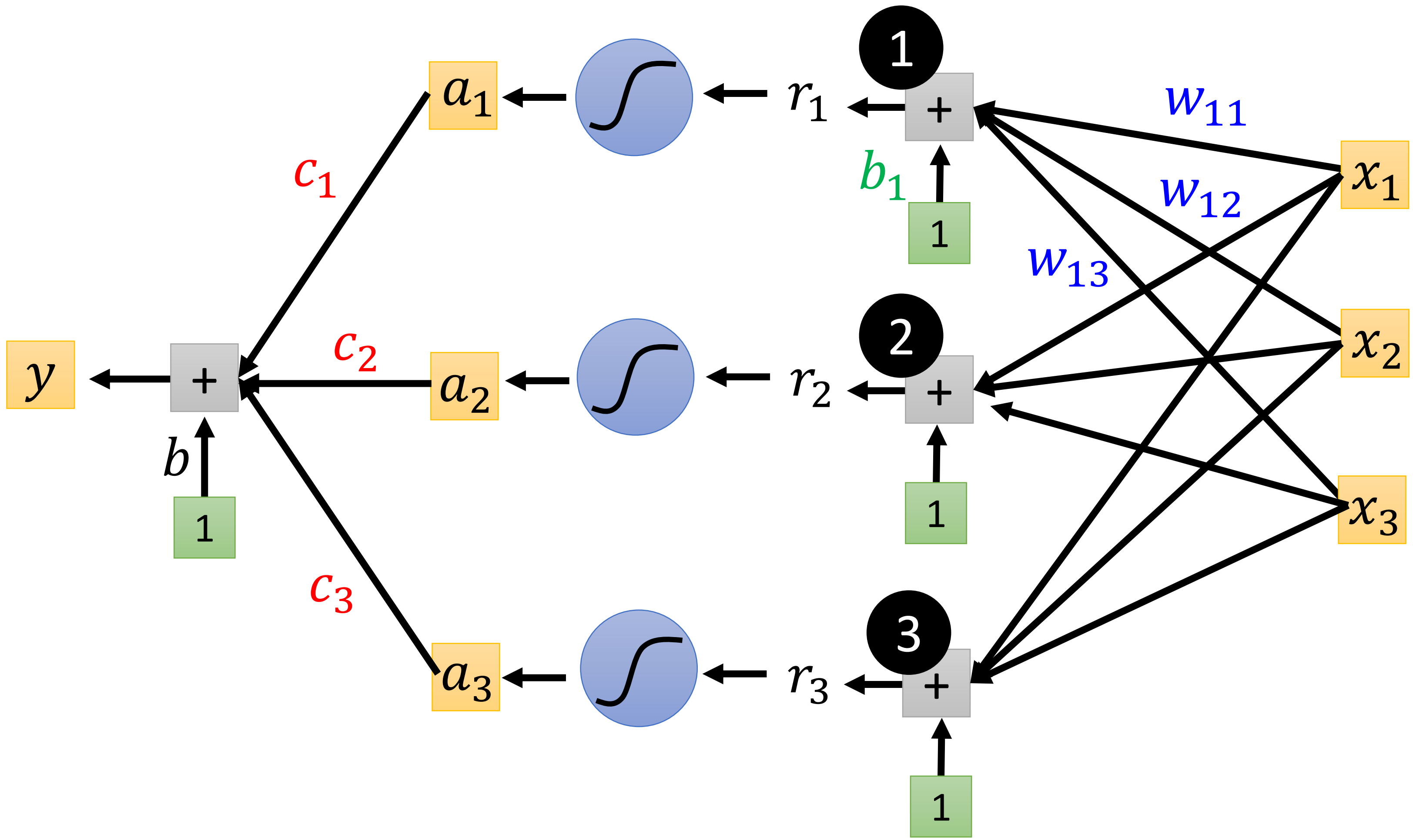


$$y = b + \mathbf{c}^T \mathbf{a}$$



$$y = b + \boldsymbol{c}^T \boldsymbol{a}$$

$$\boldsymbol{a} = \sigma(\boldsymbol{r}) \qquad \boldsymbol{r} = \boldsymbol{b} + W \boldsymbol{x}$$



$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W \boldsymbol{x})$$

Function with unknown parameters

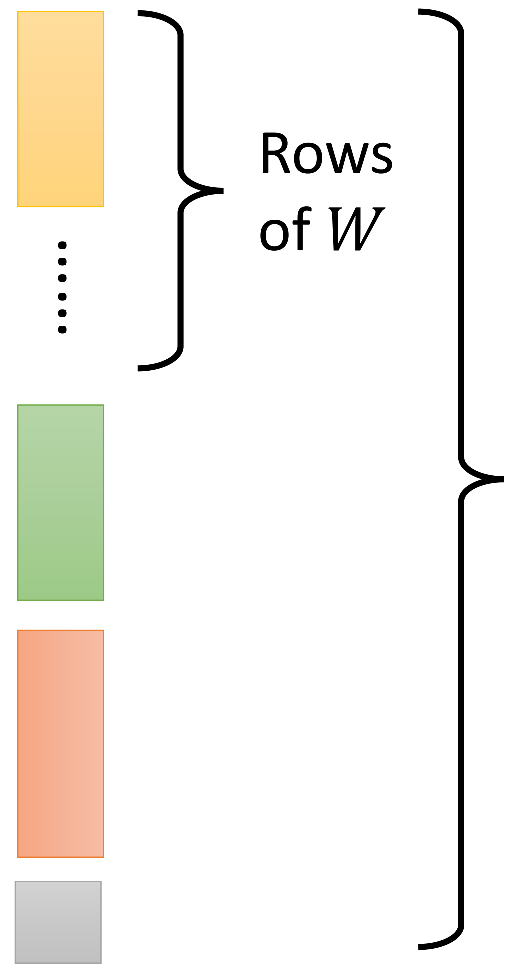
$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W\boldsymbol{x})$$

$\boldsymbol{x}$ **feature**

Unknown parameters

W $\boldsymbol{b}$

$\boldsymbol{c}^T$ b



$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \end{bmatrix}$$

Back to ML Framework

Step 1: function with unknown

Step 2: define loss from training data

Step 3: optimization

$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W\boldsymbol{x})$$

Loss

- Loss is a function of parameters $L(\theta)$
- Loss means how good a set of values is.

feature

$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W \boldsymbol{x})$$

e

label $\hat{y}$

Given a set of values

Loss: $$L = \frac{1}{N}\sum_n e_n$$

Back to ML Framework

Step 1: function with unknown

Step 2: define loss from training data

Step 3: optimization

$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W \boldsymbol{x})$$

Optimization of New Model

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \end{bmatrix}$$

$$\boldsymbol{\theta}^* = arg \min_{\boldsymbol{\theta}} L$$

➢ (Randomly) Pick initial values $\boldsymbol{\theta}^0$

$$\boldsymbol{g} = \begin{bmatrix} \frac{\partial L}{\partial \theta_1}|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \frac{\partial L}{\partial \theta_2}|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \vdots \end{bmatrix}$$

gradient

$$\begin{bmatrix} \theta_1^1 \\ \theta_2^1 \\ \vdots \end{bmatrix} \leftarrow \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \\ \vdots \end{bmatrix} - \begin{bmatrix} \eta \frac{\partial L}{\partial \theta_1}|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \eta \frac{\partial L}{\partial \theta_2}|_{\boldsymbol{\theta}=\boldsymbol{\theta}^0} \\ \vdots \end{bmatrix}$$

$$\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^0)$$

$$\boldsymbol{\theta}^1 \leftarrow \boldsymbol{\theta}^0 - \eta \boldsymbol{g}$$

Optimization of New Model

$$\boldsymbol{\theta}^* = arg \min_{\boldsymbol{\theta}} L$$

- (Randomly) Pick initial values $\boldsymbol{\theta}^0$
- Compute gradient $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^0)$

$$\boldsymbol{\theta}^1 \leftarrow \boldsymbol{\theta}^0 - \eta \boldsymbol{g}$$

- Compute gradient $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^1)$

$$\boldsymbol{\theta}^2 \leftarrow \boldsymbol{\theta}^1 - \eta \boldsymbol{g}$$

- Compute gradient $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^2)$

$$\boldsymbol{\theta}^3 \leftarrow \boldsymbol{\theta}^2 - \eta \boldsymbol{g}$$

Optimization of New Model

$$\boldsymbol{\theta}^* = arg\min_{\boldsymbol{\theta}} L$$

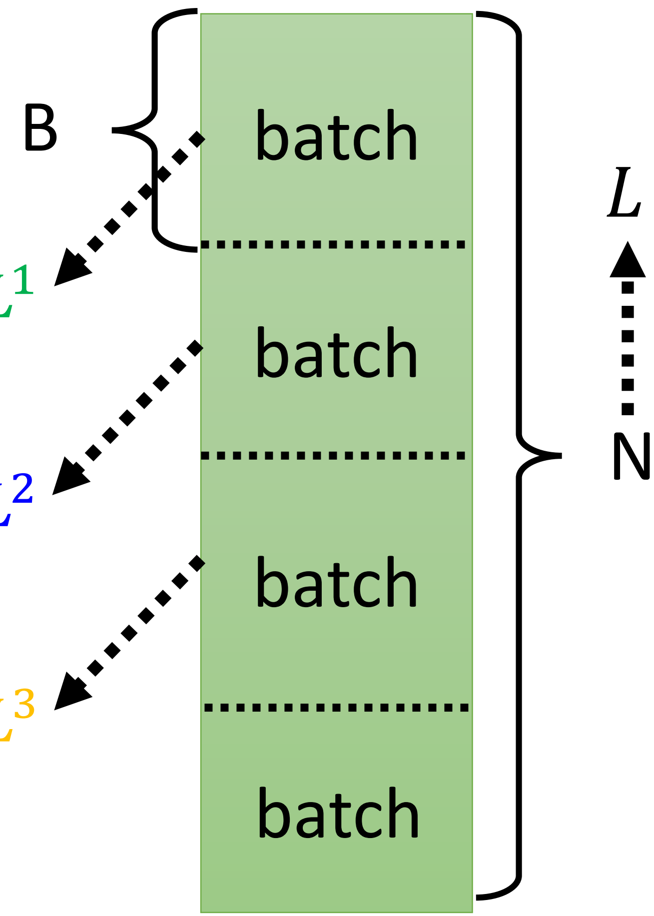
- (Randomly) Pick initial values $\boldsymbol{\theta}^0$
- Compute gradient $\boldsymbol{g} = \nabla L^1(\boldsymbol{\theta}^0)$ $\quad L^1$

  update $\boldsymbol{\theta}^1 \leftarrow \boldsymbol{\theta}^0 - \eta \boldsymbol{g}$
- Compute gradient $\boldsymbol{g} = \nabla L^2(\boldsymbol{\theta}^1)$ $\quad L^2$

  update $\boldsymbol{\theta}^2 \leftarrow \boldsymbol{\theta}^1 - \eta \boldsymbol{g}$
- Compute gradient $\boldsymbol{g} = \nabla L^3(\boldsymbol{\theta}^2)$ $\quad L^3$

  update $\boldsymbol{\theta}^3 \leftarrow \boldsymbol{\theta}^2 - \eta \boldsymbol{g}$

1 **epoch** = see all the batches once



Optimization of New Model

Example 1

- 10,000 examples (N = 10,000)
- Batch size is 10 (B = 10)

How many update in **1 epoch**?

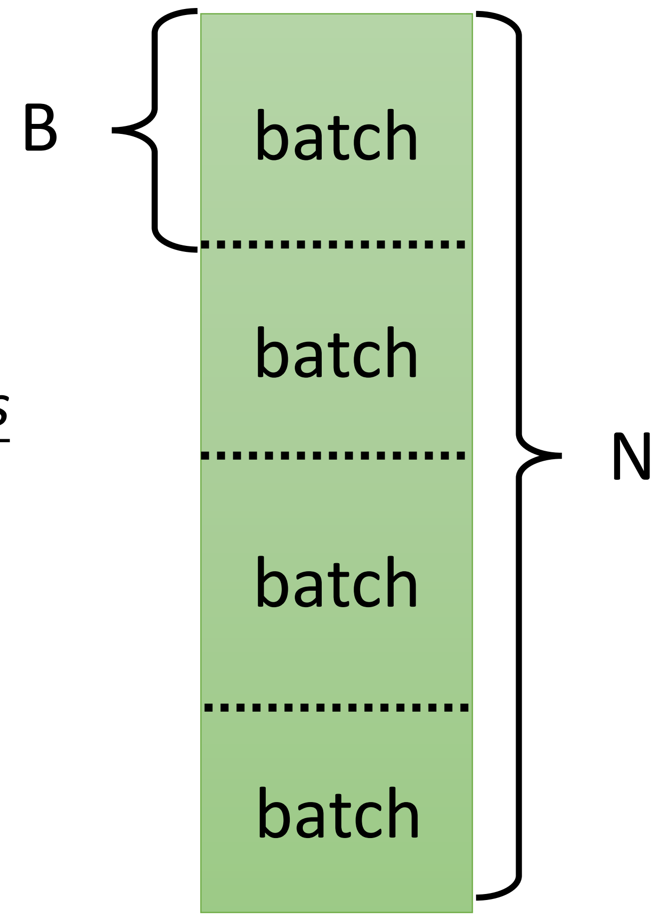
1,000 updates

Example 2

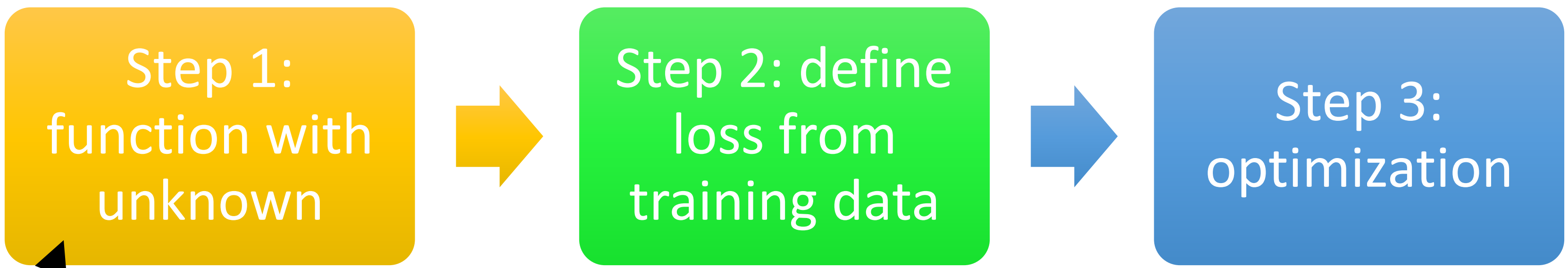
- 1,000 examples (N = 1,000)
- Batch size is 100 (B = 100)

How many update in **1 epoch**?

10 updates



Back to ML Framework

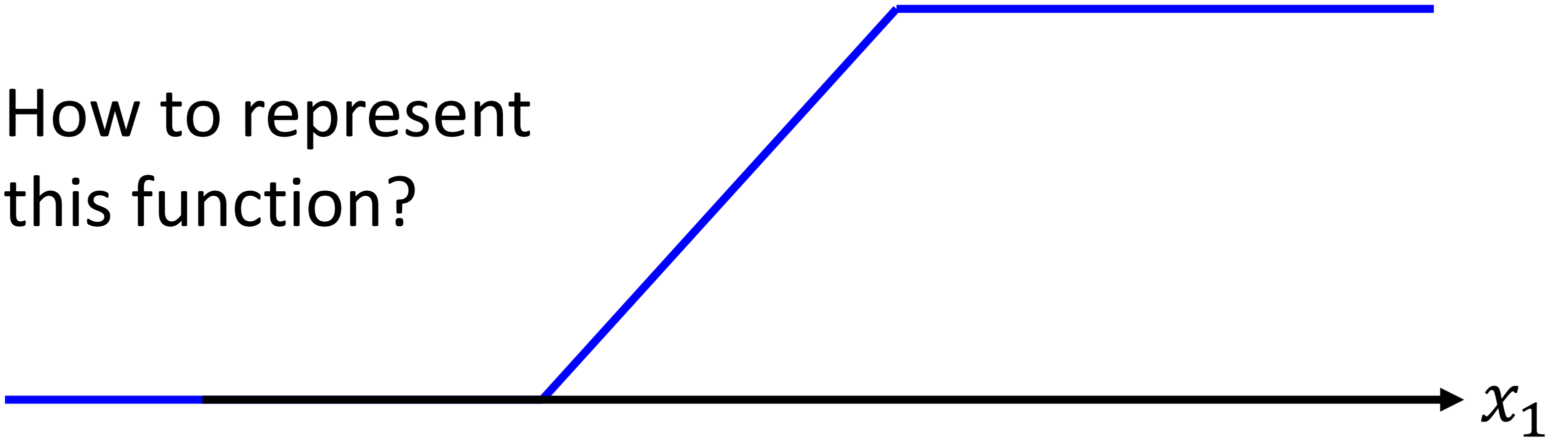


$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W \boldsymbol{x})$$

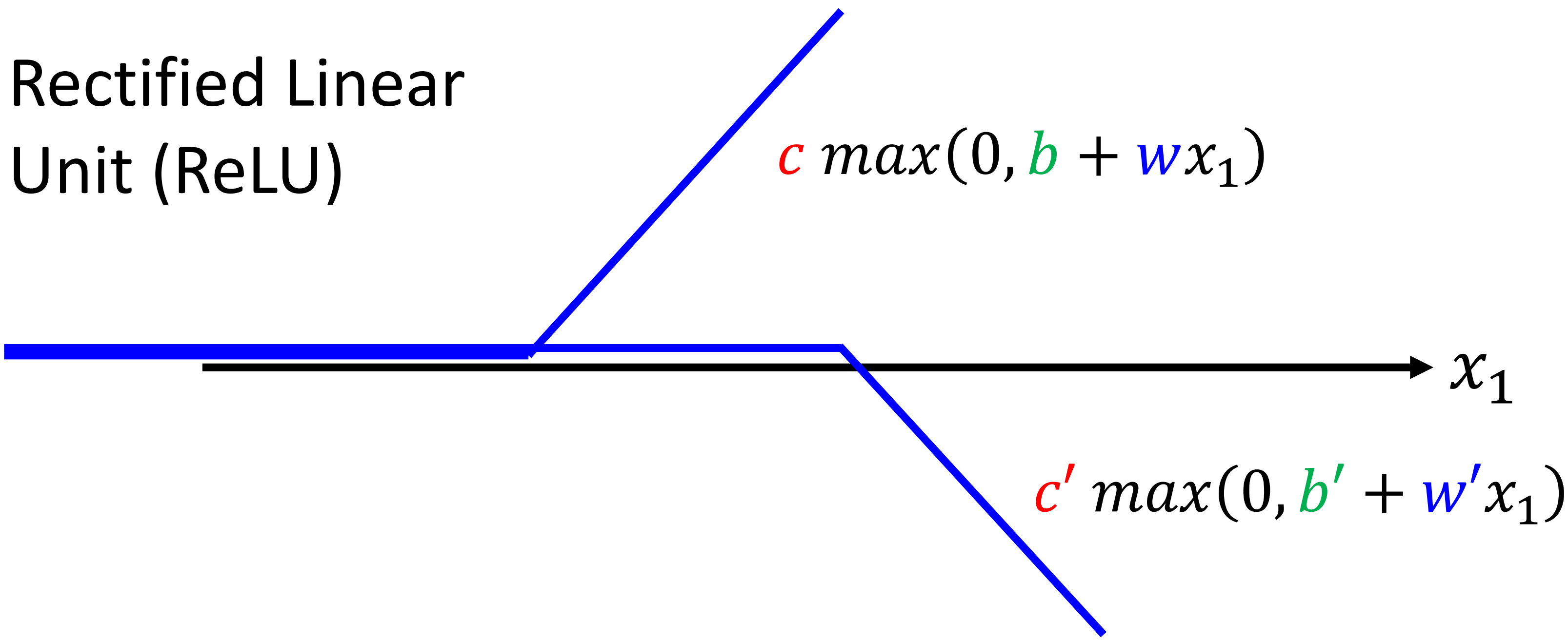
More variety of models …

Sigmoid → ReLU

How to represent this function?



Rectified Linear Unit (ReLU)



Sigmoid → ReLU

$$y = b + \sum_{\boxed{i}} c_i \, \underline{sigmoid}\left(b_i + \sum_j w_{ij} x_j \right)$$

Activation function

$$y = b + \sum_{\boxed{i}} c_i \, \underline{max}\left(0, b_i + \sum_j w_{ij} x_j \right)$$

Which one is better?

Experimental Results

$$y = b + \sum_i c_i \, max\left(0, b_i + \sum_j w_{ij} x_j\right)$$

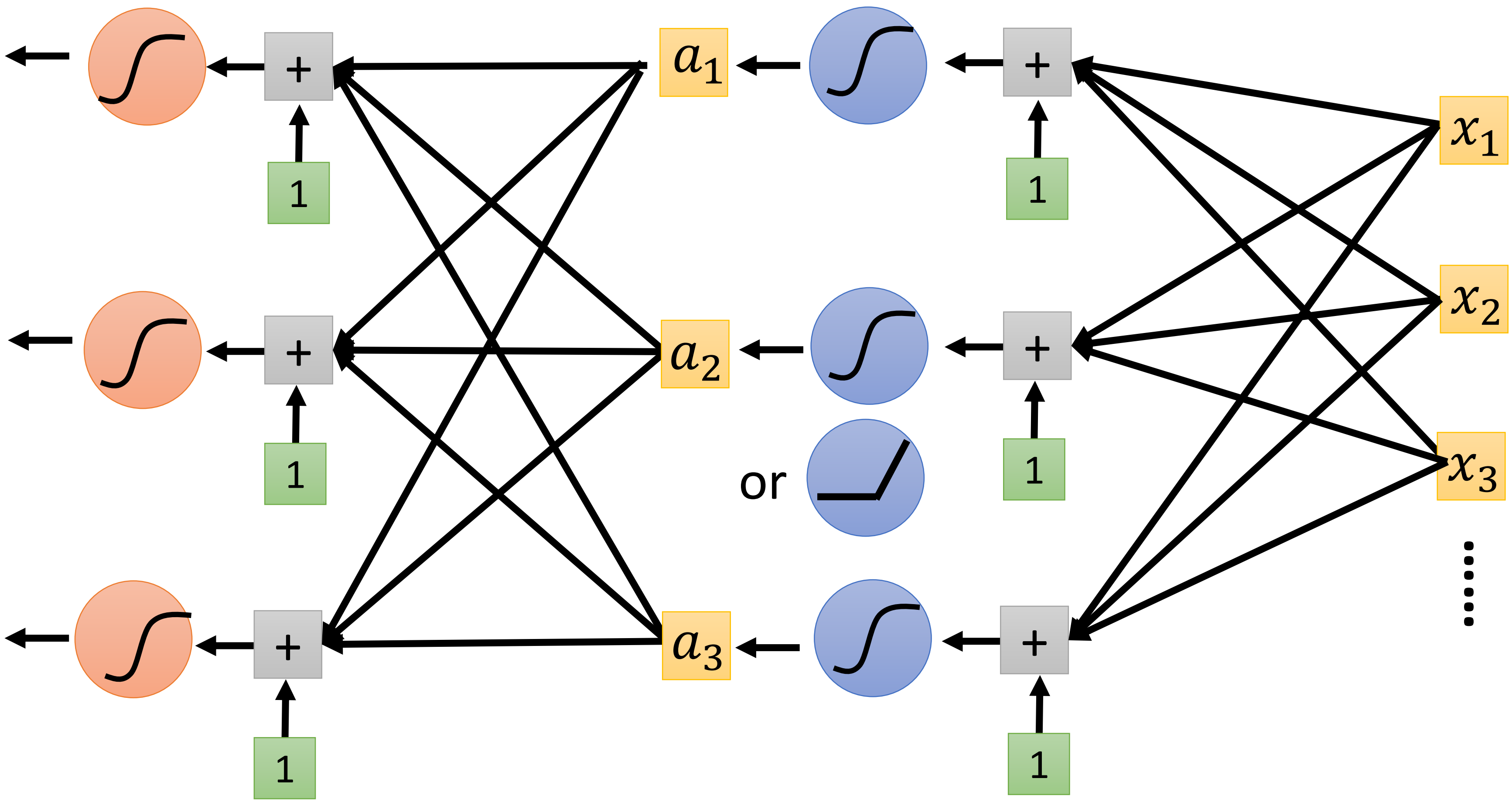
	linear	10 ReLU	100 ReLU	1000 ReLU
2017 – 2020	0.32k	0.32k	0.28k	0.27k
2021	0.46k	0.45k	0.43k	0.43k

Back to ML Framework

Step 1: function with unknown → Step 2: define loss from training data → Step 3: optimization

$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W\boldsymbol{x})$$

Even more variety of models …



$$\boldsymbol{a}' = \sigma(\boldsymbol{b}' + W'\boldsymbol{a}) \qquad \boldsymbol{a} = \sigma(\boldsymbol{b} + W\boldsymbol{x})$$

Experimental Results

- Loss for multiple hidden layers
    - 100 ReLU for each layer
    - input features are the no. of views in the past 56 days

	1 layer	2 layer	3 layer	4 layer
2017 – 2020	0.28k	0.18k	0.14k	0.10k
2021	0.43k	0.39k	0.38k	0.44k

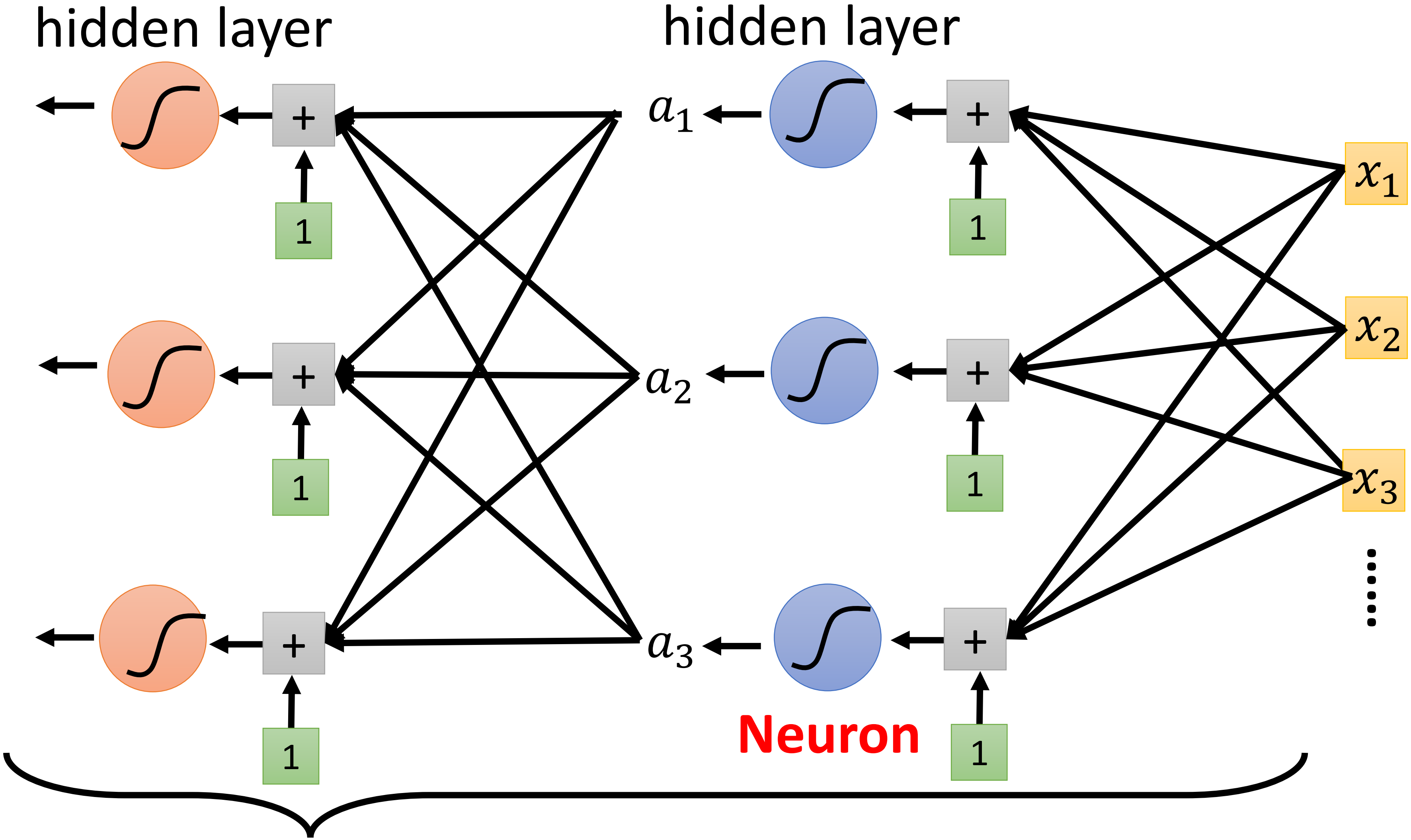
Back to ML Framework

Step 1: function with unknown → Step 2: define loss from training data → Step 3: optimization

$$y = b + \boldsymbol{c}^T \sigma(\boldsymbol{b} + W \boldsymbol{x})$$

It is not ***fancy*** enough.

Let's give it a ***fancy*** name!



Neural Network This mimics human brains … (???)

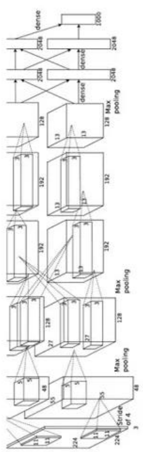
Many layers means **Deep** → Deep Learning

Deep = Many hidden layers

http://cs231n.stanford.edu/slides/winter1516_lecture8.pdf

8 layers

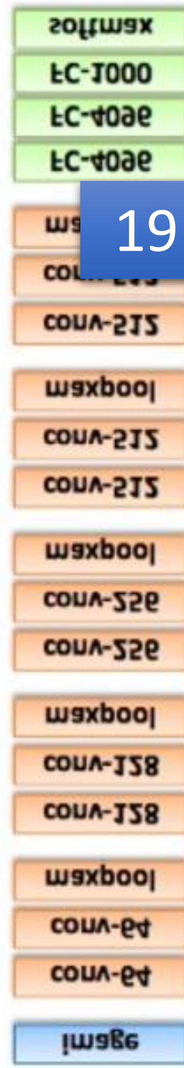
16.4%



AlexNet (2012)

19 layers

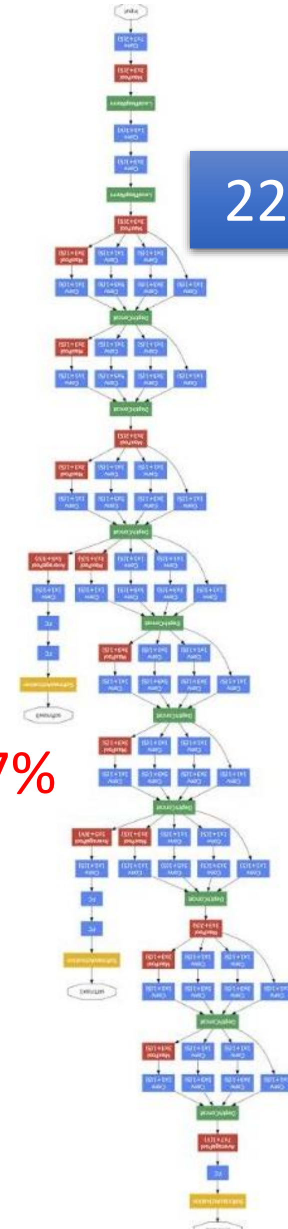
7.3%



VGG (2014)

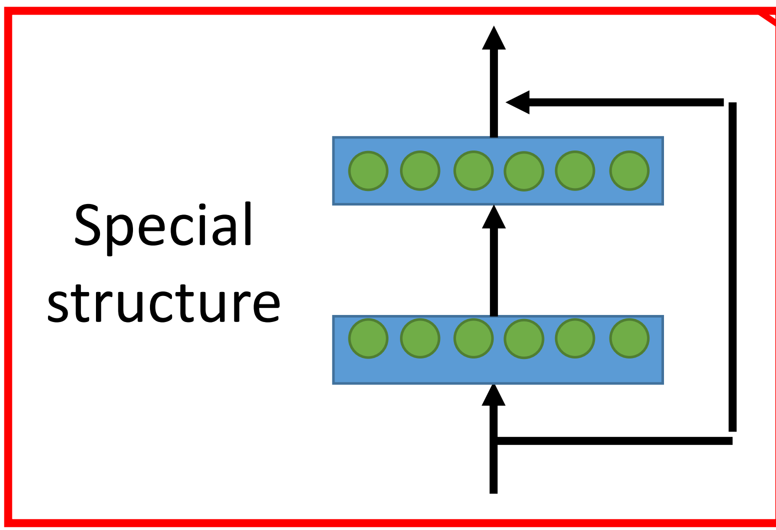
22 layers

6.7%

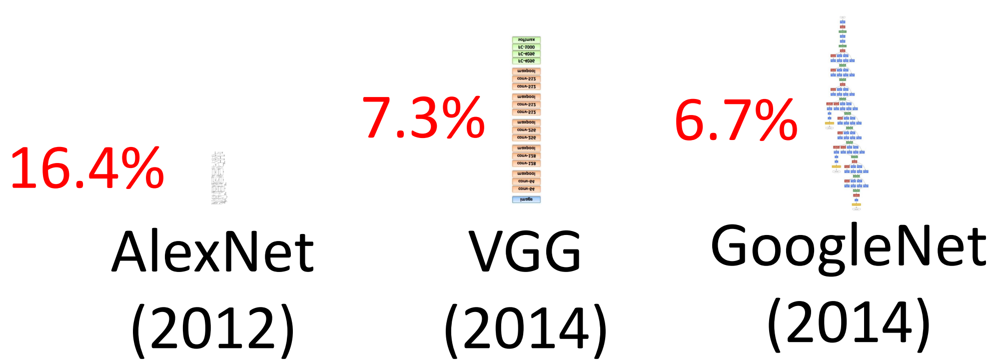


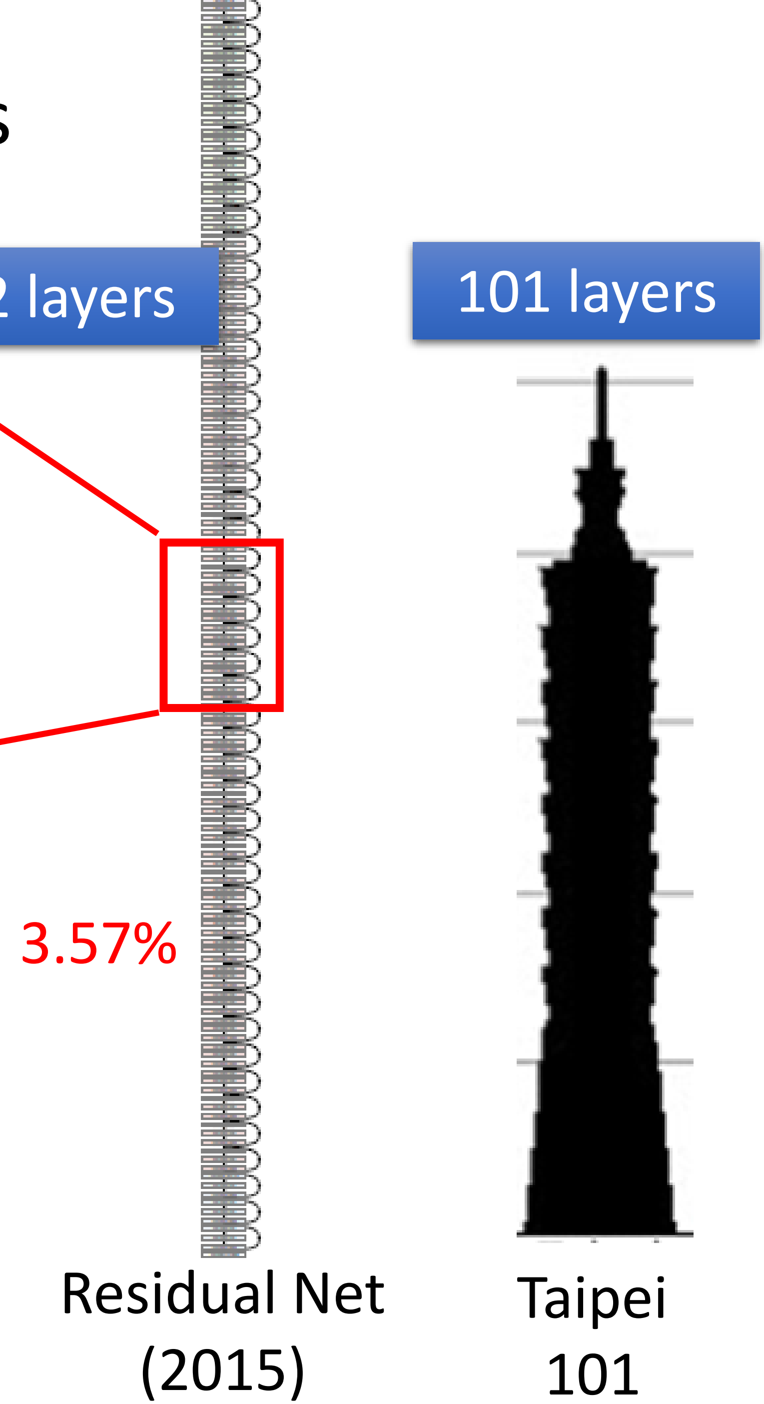
GoogleNet (2014)

Deep = Many hidden layers



Why we want "***Deep***" network,
not "***Fat***" network?





Why don’t we go deeper?

- Loss for multiple hidden layers
  - 100 ReLU for each layer
  - input features are the no. of views in the past 56 days

	1 layer	2 layer	3 layer
2017 – 2020	0.28k	0.18k	0.14k
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Why don't we go deeper?

- Loss for multiple hidden layers
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	1 layer	2 layer	3 layer	4 layer
2017 – 2020	0.28k	0.18k	0.14k	0.10k
2021	0.43k	0.39k	0.38k	0.44k

Better on training data, worse on unseen data

➡ **Overfitting**