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Calculating the area and centroid of a polygon

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Sample source code

[This C function](#) returns the area of a polygon.

[JAVA code](#) submitted by Ramón Talavera.

[PolygonUtilities.java](#) contributed by

Christopher Fuhrman

[Pascal/Dephi](#) example by Rodrigo Alves Pons.

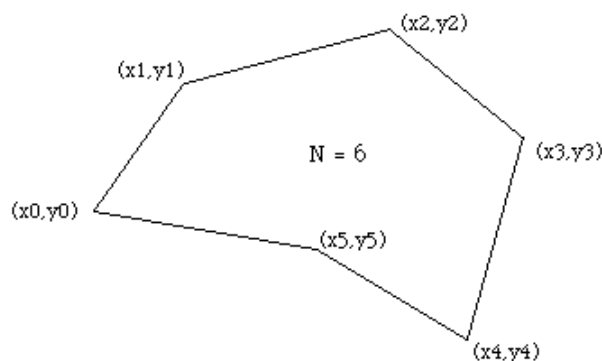
[Basic version](#) also by Rodrigo Alves Pons.

[JavaScript](#) by Raymond Hill.

[Python](#) and [example](#) by Jorg Rødscø.

Area

The problem of determining the area of a polygon seems at best messy but the final formula is particularly simple. The result and sample source code (C) will be presented here. Consider a polygon made up of line segments between N vertices (x_i, y_i) , $i=0$ to $N-1$. The last vertex (x_N, y_N) is assumed to be the same as the first, ie: the polygon is closed.



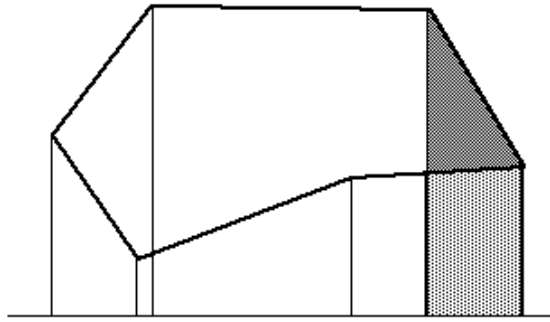
The area is given by

$$A = \frac{1}{2} \sum_{i=0}^{N-1} (x_i y_{i+1} - x_{i+1} y_i)$$

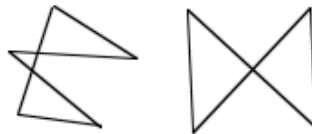
Note for polygons with holes. The holes are usually defined by ordering the vertices of the enclosing polygon in the opposite direction to those of the holes. This algorithm still works except that the absolute value should be taken after adding the polygon area to the area of all the holes. That is, the holes areas will be of opposite sign to the bounding polygon area.

The sign of the area expression above (without the absolute value) can be used to determine the ordering of the vertices of the polygon. If the sign is positive then the polygon vertices are ordered counter clockwise about the normal, otherwise clockwise.

To derive this solution, project lines from each vertex to some horizontal line below the lowest part of the polygon. The enclosed region from each line segment is made up of a triangle and rectangle. Sum these areas together noting that the areas outside the polygon eventually cancel as the polygon loops around to the beginning.



The only restriction that will be placed on the polygon for this technique to work is that the polygon must not be self intersecting, for example the solution will fail in the following cases.



Centroid

The centroid is also known as the "centre of gravity" or the "center of mass". The position of the centroid assuming the polygon to be made of a material of uniform density is given below. As in the calculation of the area above, x_N is assumed to be x_0 , in other words the polygon is closed.

$$c_x = \frac{1}{6A} \sum_{i=0}^{N-1} (x_i + x_{i+1}) (x_i y_{i+1} - x_{i+1} y_i)$$

$$c_y = \frac{1}{6A} \sum_{i=0}^{N-1} (y_i + y_{i+1}) (x_i y_{i+1} - x_{i+1} y_i)$$

Centroid of a 3D shell described by 3 vertex facets

The centroid C of a 3D object made up of a collection of N triangular faces with vertices (a_i, b_i, c_i) is given below. R_i is the average of the vertices of the i 'th face and A_i is twice the area of the i 'th face. Note the faces are assumed to be thin sheets of uniform mass, they need not be connected or form a solid object. This reduces to the equations above for a 2D 3 vertex polygon.

$$C = \frac{\sum_{i=0}^{N-1} A_i R_i}{\sum_{i=0}^{N-1} A_i}$$


$$R_i = (a_i + b_i + c_i) / 3$$

$$A_i = \|(b_i - a_i) \otimes (c_i - a_i)\|$$

Second moment of a polygon

The following assume anticlockwise orientated polygon vertices, use the negative value for clockwise polygons.

$$I_x = \frac{1}{12} \sum_{i=0}^{N-1} [y_i^2 + y_i y_{i+1} + y_{i+1}^2] [x_i y_{i+1} - x_{i+1} y_i]$$

$$I_y = \frac{1}{12} \sum_{i=0}^{N-1} [x_i^2 + x_i x_{i+1} + x_{i+1}^2] [x_i y_{i+1} - x_{i+1} y_i]$$

$$2 I_{xy} = \frac{1}{12} \sum_{i=0}^{N-1} [x_i y_{i+1} + 2 x_i y_i + x_{i+1} y_{i+1} + x_{i+1} y_i] [x_i y_{i+1} - x_{i+1} y_i]$$