Model Project

1 Formal definition

Overlapping Generations Model

Preferences Agents in this economic environment lives for 2 periods: young and old generations.
Therefor we account for 2 types of households: N_y young households and N_o old households.
These households value consumption according to when they are young or old: c_y and c_o.
The agents discount time with factor β.
Their preferences are given by this utility function: $u_t(c_t)$. This implies the following life-time utility as:

$$V(c_y, c_o) = u(c_y) + \beta \cdot u(c_o) \tag{1}$$

Technology Firms produce output Y using input capital K nad labour L. Therefore the production function is as follows:

$$Y = F(K, L) = AK^{\alpha}L^{(1-\alpha)} \tag{2}$$

For firms profit maximization they use:

$$\underset{K,L}{MAX}(F(K, L) - wL - rK) \tag{3}$$

Where w and r are the factor prizes for labour and capital.
Government The government collects taxes on labour, τ_L and on capital, τ_K.
Household problem Households maximize utility subject to their budget constraint in each period as following:

$$\underset{c_y,c_o}{max}(u(c_y) + \beta \cdot u(c_o)) \quad s.t. \quad c_y + s = (1 - \tau_L)w + T_y \tag{4}$$

$$c_o = Rs + T_o \tag{5}$$

Where $R = (1 + (1 - \tau_K)r)$ and is the variabel for tax' gross interest rate.

The government's budget constraint clears market if:

$$K = S = N_y \cdot s^* \tag{6}$$

$$ARC : C + S + G = Y + (1 - \delta)K \tag{7}$$

Step 1: Substitute the budget set into preferences.

$$max_s \quad ln((1-\tau_L)w + T_y - s) + \beta ln(Rs + T_0) \tag{8}$$

This is now a function containing one unknown variable, s.
We can now derive this function with respect to s:

$$\frac{\partial V}{\partial s} = 0 \Leftrightarrow \frac{1}{(1-\tau_L)w + T_y - s} = \frac{\beta R}{Rs - T_0} \tag{9}$$

Step 2: Solve for optimal household savings s^*.

$$s^* = \frac{\beta R((1-\tau_L)w + T_y) - T_o}{(1+\beta)R} \tag{10}$$

In equilibrium household savings equals the capital stock, S=K.
Therefore aggregate capital stock is: $K = S = N_y \cdot s^*$
Using the following equation system we can solve for a steady state equilibrium:
Assuming L=1 we have the following unknown variables: K, Y, R, w and q.

$$K = N_y s^* = N_y \frac{\beta R((1-\tau_L)w + T_y) - T_o}{(1+\beta)R} \tag{11}$$

$$\alpha \cdot \frac{Y}{K} = q \tag{12}$$

$$(1-\alpha) \cdot \frac{Y}{L} = w \tag{13}$$

$$F(K, L) = AK^{\alpha} L^{(1-\alpha)} \tag{14}$$

Tryin to solve this we initially assume that government is completely exogenous.
Note: we see that L = 1 since L = $N_y * l$ and we know that l and N_y are both equal to 1. -Substitute Y out and get q(K) and w(K).
Using q(K) in R gives R(K).

$$w = (1-\alpha) \cdot AK^{\alpha} R = 1 + (1-\tau_K)(\alpha \cdot AK^{\alpha-1} - \delta) \tag{15}$$

plug then w(K) and R(K) into the equation of K:

$$K = N_y \frac{\beta(1 + (1-\tau_K)(\alpha * AK^{\alpha-1} - \delta))((1-\alpha) * AK^{\alpha}) + t_y) - t_0}{(1+\beta)(1 + (1-\tau_K)(\alpha AK^{\alpha-1} - \delta))} \tag{16}$$

In order to find the root of this function we initially have to set the parameters for the

model.
$N_y = N_o = 1, \quad \alpha = 0.3,$
$A = 1, \quad \beta = 0.9, \quad \delta = 0.1$
It was assumed that government was exogenous, thus these are the government parameters: $\tau_L = 0.40, \quad \tau_K = 0.25, \quad T_y = T_o = 0$
Then solve for K^* and make sure that the market clears by aggregate resource constraint(ARC):

$$C + N_y \cdot s + G = Y + (1 - \delta)K \tag{17}$$

2 Gauss-seidl

Instead of using substitution and then solving for one equation with one unknown we will use the Gauss-Seidl algorithm

The first step is to start with a guess for the capital K_{old}

The second step is to solve for the prices w, R, q

$$w = (1 - \alpha) * K_{old}^{\alpha} * L^{-\alpha} \tag{18}$$
$$R = 1 + (1 - \tau_K) * (q - \delta) \tag{19}$$
$$q = \alpha * A * K_{old}^{\alpha - 1} * L^{1-\alpha} \tag{20}$$

The third step is to solve for the optimal household savings s^*

$$s^* = N_y \frac{\beta R((1 - \tau_L)w + t_y) - t_0}{(1 + \beta)R} \tag{21}$$

The fourth step is to aggregate over all households to get the new capital K_{new}

$$K_{new} = N_y * s^* \tag{22}$$

The fifth step is to then calculate the error term and see if the algorithm has converged.

$$Error = \frac{K_{old} - K_{new}}{K_{old}} \tag{23}$$

The sixth and last step is to update the old capital $K_{old} = K_{new} * \lambda + K_{old} * (1 - \lambda)$ and then repeat the process from the second step. λ is an updating parameter between 0 and 1.