Written Exam Economics Summer 2019

Introduction to Programming and Numerical Analysis

Date 20th – 27th May 2019

This exam question consists of 8 pages in total.

(An electronic version of the notebook with the exam questions is also available).

Answers only in English.

You should hand-in a single zip-file named with your groupname only. The zip-file should contain:

1. A general README.md for your portfolio
2. A Feedback.txt file with a list of the groups each group member have given peer feedback to with links to the GitHub issues
3. Your data analysis project (in the folder /dataproject)
4. Your model analysis project (in the folder /modelproject)
5. Your exam project (in the folder /examproject)

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

In [1]:

```
import numpy as np
```

1. Human capital accumulation

Consider a worker living in **two periods**, $t \in \{1, 2\}$.

In each period she decides whether to **work ($l_t = 1$) or not ($l_t = 0$)**.

She can *not* borrow or save and thus **consumes all of her income** in each period.

If she **works** her **consumption** becomes:

$$c_t = wh_tl_t \text{ if } l_t = 1$$

where w is **the wage rate** and h_t is her **human capital**.

If she does **not work** her consumption becomes:

$$c_t = b \text{ if } l_t = 0$$

where b is the **unemployment benefits**.

Her **utility of consumption** is:

$$\frac{c_t^{1-\rho}}{1-\rho}$$

Her **disutility of working** is:

$$\gamma l_t$$

From period 1 to period 2, she **accumulates human capital** according to:

$$h_2 = h_1 + l_1 + \begin{cases} 0 & \text{with prob. } 0.5 \\ \Delta & \text{with prob. } 0.5 \end{cases}$$

where Δ is a **stochastic experience gain**.

In the **second period** the worker thus solves:

$$\begin{aligned} v_2(h_2) &= \max_{l_2} \frac{c_2^{1-\rho}}{1-\rho} - \gamma l_2 \\ \text{s.t.} & \\ c_2 &= wh_2l_2 \\ l_2 &\in \{0, 1\} \end{aligned}$$

In the **first period** the worker thus solves:

$$
\begin{aligned}
v_1(h_1) &= \max_{l_1} \frac{c_1^{1-\rho}}{1-\rho} - \gamma l_1 + \beta \mathbb{E}_1 \left[v_2(h_2) \right] \\
& \text{s.t.} \\
c_1 &= w h_1 l_1 \\
h_2 &= h_1 + l_1 + \begin{cases} 0 & \text{with prob. } 0.5 \\ \Delta & \text{with prob. } 0.5 \end{cases} \\
l_1 &\in \{0, 1\}
\end{aligned}
$$

where β is the **discount factor** and $\mathbb{E}_1 \left[v_2(h_2) \right]$ is the **expected value of living in period two**.

The **parameters** of the model are:

In [2]:

```
rho = 2
beta = 0.96
gamma = 0.1
w = 2
b = 1
Delta = 0.1
```

The **relevant levels of human capital** are:

In [3]:

```
h_vec = np.linspace(0.1,1.5,100)
```

Question 1: Solve the model in period 2 and illustrate the solution (including labor supply as a function of human capital).

Question 2: Solve the model in period 1 and illustrate the solution (including labor supply as a function of human capital).

Question 3: Will the worker never work if her potential wage income is lower than the unemployment benefits she can get? Explain and illustrate why or why not.

2. AS-AD model

Consider the following **AS-AD model**. The **goods market equilibrium** is given by

$$
y_t = -\alpha r_t + v_t
$$

where y_t is the **output gap**, r_t is the **ex ante real interest** and v_t is a **demand disturbance**.

The central bank's **Taylor rule** is

$$i_t = \pi_{t+1}^e + h\pi_t + by_t$$

where i_t is the **nominal interest rate**, π_t is the **inflation gap**, and π_{t+1}^e is the **expected inflation gap**.

The **ex ante real interest rate** is given by

$$r_t = i_t - \pi_{t+1}^e$$

Together, the above implies that the **AD-curve** is

$$\pi_t = \frac{1}{h\alpha}[v_t - (1 + b\alpha)y_t]$$

Further, assume that the **short-run supply curve (SRAS)** is given by

$$\pi_t = \pi_t^e + \gamma y_t + s_t$$

where s_t is a **supply disturbance**.

Inflation expectations are adaptive and given by

$$\pi_t^e = \phi\pi_{t-1}^e + (1-\phi)\pi_{t-1}$$

Together, this implies that the **SRAS-curve** can also be written as

$$\pi_t = \pi_{t-1} + \gamma y_t - \phi\gamma y_{t-1} + s_t - \phi s_{t-1}$$

The **parameters** of the model are:

In [4]:

```
par = {}

par['alpha'] = 5.76
par['h'] = 0.5
par['b'] = 0.5
par['phi'] = 0
par['gamma'] = 0.075
```

Question 1: Use the `sympy` module to solve for the equilibrium values of output, y_t, and inflation, π_t, (where AD = SRAS) given the parameters (α, h, b, α, γ) and y_{t-1} , π_{t-1}, v_t, s_t, and s_{t-1}.

Question 2: Find and illustrate the equilibrium when $y_{t-1} = \pi_{t-1} = v_t = s_t = s_{t-1} = 0$. Illustrate how the equilibrium changes when instead $v_t = 0.1$.

Persistent disturbances: Now, additionaly, assume that both the demand and the supply disturbances are AR(1) processes

$$v_t = \delta v_{t-1} + x_t$$
$$s_t = \omega s_{t-1} + c_t$$

where x_t is a **demand shock**, and c_t is a **supply shock**. The **autoregressive parameters** are:

In [5]:

```
par['delta'] = 0.80
par['omega'] = 0.15
```

Question 3: Starting from $y_{-1} = \pi_{-1} = s_{-1} = 0$, how does the economy evolve for $x_0 = 0.1$, $x_t = 0, \forall t > 0$ and $c_t = 0, \forall t \geq 0$?

Stochastic shocks: Now, additionally, assume that x_t and c_t are stochastic and normally distributed

$$x_t \sim \mathcal{N}(0, \sigma_x^2)$$
$$c_t \sim \mathcal{N}(0, \sigma_c^2)$$

The **standard deviations of the shocks** are:

In [6]:

```
par['sigma_x'] = 3.492
par['sigma_c'] = 0.2
```

Question 4: Simulate the AS-AD model for 1,000 periods. Calculate the following five statistics:

1. Variance of y_t, $var(y_t)$
2. Variance of π_t, $var(\pi_t)$
3. Correlation between y_t and π_t, $corr(y_t, \pi_t)$
4. Auto-correlation between y_t and y_{t-1}, $corr(y_t, y_{t-1})$
5. Auto-correlation between π_t and π_{t-1}, $corr(\pi_t, \pi_{t-1})$

Question 5: Plot how the correlation between y_t and π_t changes with ϕ. Use a numerical optimizer or root finder to choose $\phi \in (0, 1)$ such that the simulated correlation between y_t and π_t comes close to 0.31.

Quesiton 6: Use a numerical optimizer to choose $\sigma_x > 0$, $\sigma_c > 0$ and $\phi \in (0, 1)$ to make the simulated statistics as close as possible to US business cycle data where:

1. $var(y_t) = 1.64$
2. $var(\pi_t) = 0.21$
3. $corr(y_t, \pi_t) = 0.31$
4. $corr(y_t, y_{t-1}) = 0.84$
5. $corr(\pi_t, \pi_{t-1}) = 0.48$

3. Exchange economy

Consider an **exchange economy** with

1. 3 goods, (x_1, x_2, x_3)
2. N consumers indexed by $j \in \{1, 2, \ldots, N\}$
3. Preferences are Cobb-Douglas with log-normally distributed coefficients

$$
\begin{aligned}
u^j(x_1, x_2, x_3) &= \left(x_1^{\beta_1^j} x_2^{\beta_2^j} x_3^{\beta_3^j}\right)^{\gamma} \\
\beta_i^j &= \frac{\alpha_i^j}{\alpha_1^j + \alpha_2^j + \alpha_3^j} \\
\boldsymbol{\alpha}^j &= (\alpha_1^j, \alpha_2^j, \alpha_3^j) \\
\log(\boldsymbol{\alpha}^j) &\sim \mathcal{N}(\mu, \Sigma)
\end{aligned}
$$

4. Endowments are exponentially distributed,

$$
\begin{aligned}
\boldsymbol{e}^j &= (e_1^j, e_2^j, e_3^j) \\
e_i^j &\sim f, f(z; \zeta) = 1/\zeta \exp(-z/\zeta)
\end{aligned}
$$

Let $p_3 = 1$ be the **numeraire**. The implied **demand functions** are:

$$
x_i^{\star j}(p_1, p_2, \boldsymbol{e}^j) = \beta_i^j \frac{I^j}{p_i}
$$

where consumer j's income is

$$
I^j = p_1 e_1^j + p_2 e_2^j + p_3 e_3^j
$$

The **parameters** and **random preferences and endowments** are given by:

In [7]:

```
# a. parameters
N = 50000
mu = np.array([3,2,1])
Sigma = np.array([[0.25, 0, 0], [0, 0.25, 0], [0, 0, 0.25]])
gamma = 0.8
zeta = 1

# b. random draws
seed = 1986
np.random.seed(seed)

# preferences
alphas = np.exp(np.random.multivariate_normal(mu, Sigma, size=N))
betas = alphas/np.reshape(np.sum(alphas,axis=1),(N,1))

# endowments
e1 = np.random.exponential(zeta,size=N)
e2 = np.random.exponential(zeta,size=N)
e3 = np.random.exponential(zeta,size=N)
```

Question 1: Plot the histograms of the budget shares for each good across agents.

Consider the **excess demand functions:**

$$z_i(p_1, p_2) = \sum_{j=1}^{N} x_i^{\star j}(p_1, p_2, e^j) - e_i^j$$

Question 2: Plot the excess demand functions.

Quesiton 3: Find the Walras-equilibrium prices, (p_1, p_2), where both excess demands are (approximately) zero, e.g. by using the following tâtonnement process:

1. Guess on $p_1 > 0$, $p_2 > 0$ and choose tolerance $\epsilon > 0$ and adjustment aggressivity parameter, $\kappa > 0$.
2. Calculate $z_1(p_1, p_2)$ and $z_2(p_1, p_2)$.
3. If $|z_1| < \epsilon$ and $|z_2| < \epsilon$ then stop.
4. Else set $p_1 = p_1 + \kappa \frac{z_1}{N}$ and $p_2 = p_2 + \kappa \frac{z_2}{N}$ and return to step 2.

Question 4: Plot the distribution of utility in the Walras-equilibrium and calculate its mean and variance.

Question 5: Find the Walras-equilibrium prices if instead all endowments were distributed equally. Discuss the implied changes in the distribution of utility. Does the value of γ play a role for your conclusions?