# 1 Ramsey model with housing

**Household:** Households supply labor exogenously, $N_t = 1$, and earns a wage $w_t$. The return on saving is $r_{t+1}$. Utility is derived from housing and consumption

$$U = \max_{\{C_t,H_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{H_t^{1-\sigma}}{1-\sigma} \right), \beta \in (0,1), \sigma > 0$$

s.t.

$$A_t + C_t + q_t H_t = (1+r_t)A_{t-1} + w_t N_t + q_t H_{t-1} 3344$$

where $H_t$ is housing consumption and $q_t$ is housing prices in terms of consumption.

**Firms:** Firms rent capital $K_{t-1}$ at the rental rate $r_t^K$ and hires labor $L_t$ at the wage rate $w_t$.¡br¿ Firms have access to the production function

$$\begin{aligned} Y_t &= F(K_{t-1}, L_t) \\ &= \Gamma_t(\alpha K_{t-1}^{-\theta} + (1-\alpha)L_t^{-\theta})^{\frac{1}{-\theta}}, \;\; \theta > -1, \alpha \in (0,1), \Gamma_t > 0 \end{aligned}$$

Profits are

$$\Pi_t = Y_t - w_t L_t - r_t^K K_{t-1}$$

**Equilibrium:**

1. Households maximize utility
2. Firms maximize profits
3. Labor market clear: $L_t = N_t = 1$
4. Goods market clear: $Y_t = C_t + I_t$
5. Housing market clear: $\bar{H} = H_t$
6. Asset market clear: $A_t = K_t$ and $r_t = r_t^k - \delta$
7. Capital follows its law of motion: $K_t = (1-\delta)K_{t-1} + I_t$

**Implication of profit maximization:** From FOCs

$$\begin{aligned} r_t^k &= F_K(K_{t-1}, L_t) = A_t \alpha K_{t-1}^{-\theta-1} Y_t^{-1} \\ w_t &= F_L(K_{t-1}, L_t) = A_t(1-\alpha)L_t^{-\theta-1} Y_t^{-1} \end{aligned}$$

**Implication of utility maximization:** From FOC for consumption/saving:

$$\begin{aligned} C_t^{-\sigma} &= \beta(1+r_{t+1})C_{t+1}^{-\sigma} \\ &= \beta(1+F_K(K_t,1)-\delta)C_{t+1}^{-\sigma} \end{aligned}$$

For housing:

$$C_t^{-\sigma} q_t = H_t^{-\sigma} + \beta C_{t+1}^{-\sigma} q_{t+1}$$

**Simpler capital accumulation equation:**

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + I_t \\ &= (1-\delta)K_{t-1} + Y_t - C_t \\ &= (1-\delta)K_{t-1} + F(K_{t-1}, 1) - C_t \end{aligned}$$

**System to solve:**

$$\boldsymbol{H}(\boldsymbol{K}, \boldsymbol{C}, \boldsymbol{q}, K_{-1}) = \begin{bmatrix} C_t^{-\sigma} - \beta(1+F_K(K_t,1))C_{t+1}^{-\sigma} \\ K_t - [(1-\delta)K_{t-1} + F(K_{t-1},1) - C_t] \\ \bar{H}^{-\sigma} + \beta C_{t+1}^{-\sigma} q_{t+1} - C_t^{-\sigma} q_t \end{bmatrix}$$