Neural Network

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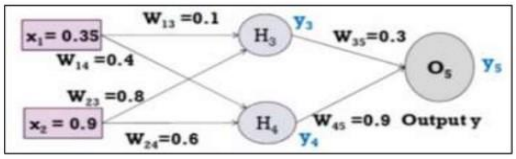
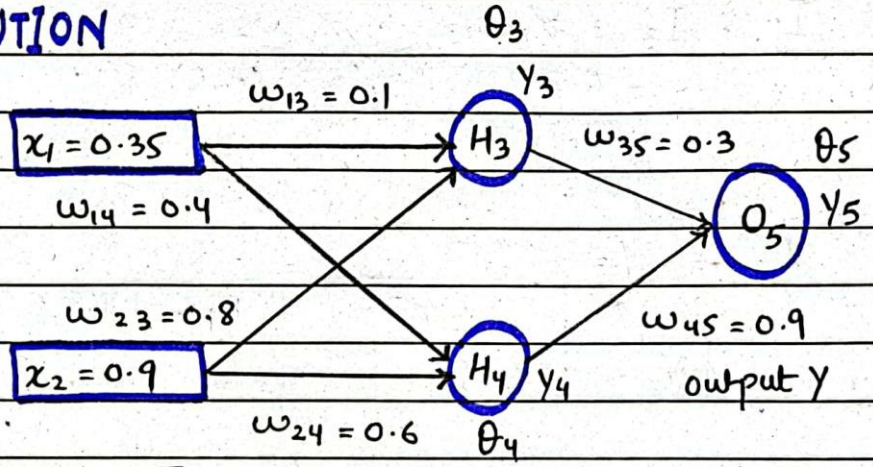
Q:
Consider the Neural Network architecture in the given figure. Assume that the neurons have sigmoid activation function.



Take the learning rate value 0.9 and the class label as 1.

Simulate the working of the given NN by calculating the hypothesis function output by forward propagation then use backpropagation algorithm to find the error of all activation nodes and hence update the weights and bias values. Using updated values of weights and bias run two more forward and backward passes each time finding errors and updating weights and bias values.
Make a table and note all values of weights, bias and network error.

SOLUTION



The NN Architecture

Learning rate = 0.9

Class label = 1

Set intial biases = 1

x_1	x_2	w_{13}	w_{14}	w_{23}	w_{24}	w_{35}	w_{45}	θ_3	θ_4	θ_5
0.35	0.9	0.1	0.4	0.8	0.6	0.3	0.9	1	1	1

Table 1

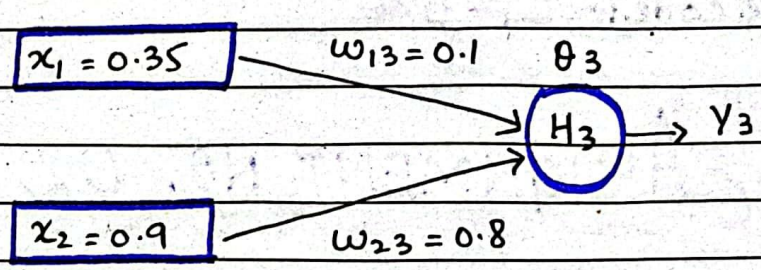
Teacher's Sign. _____

Date _____

Iteration: 01

NET Input and Output Calculations:

For Unit H_3

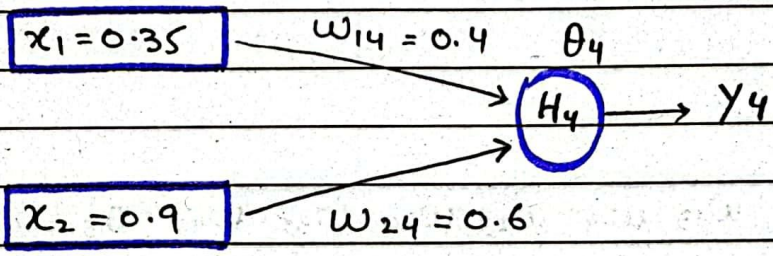


$$I_j = I_3 = x_1 w_{13} + x_2 w_{23} + \theta_3$$
$$= 0.35(0.1) + (0.9)(0.8) + 1$$
$$= 1.755$$

$$O_j = Y_3 = \frac{1}{1 + e^{-I_3}} = \frac{1}{1 + e^{-1.755}}$$

$$\boxed{Y_3 = 0.852}$$

For Unit H_4



$$I_j = I_4 = x_1 w_{14} + x_2 w_{24} + \theta_4$$
$$= 0.35(0.4) + 0.9(0.6) + 1$$
$$= 1.68$$

$$O_j = Y_4 = \frac{1}{1 + e^{-I_4}} = \frac{1}{1 + e^{-1.68}}$$

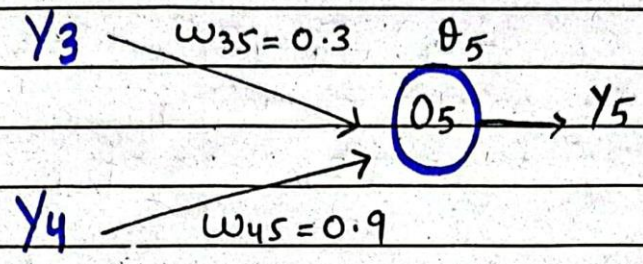
$$\boxed{Y_4 = 0.842}$$

Rainbow

Teacher's Sign. _____

Date _____

For Unit O_5 :



$$I_j = I_5 = Y_3 w_{35} + Y_4 w_{45} + \theta_5$$
$$= 0.852(0.3) + 0.842(0.9) + 1$$
$$= 2.0053$$

$$O_j = O_5 = Y_5 = \frac{1}{1 + e^{-I_5}} = \frac{1}{1 + e^{-2.0053}}$$

$$\boxed{Y_5 = 0.881}$$

In Our Network,

$$h(x) = Y_5 = 0.881$$

Target value / actual value is given in training set as 1

$$y_{target} = 1$$

$$\text{Error} = y_{target} - Y_5$$
$$= 1 - 0.881$$

$$\boxed{\text{Error} = 0.119}$$

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Error Computation:

$Err_j = O_j(1-O_j)(T_j - O_j)$ where,
O_j = is the output of unit j
T_j = actual and target value of j
$O_j = Y_j$
$O_j(1-O_j)$ is the derivative of the logistic function

Error at Unit Y5

$$Err_5 = Y_5(1-Y_5)(Y_{target} - Y_5)$$
$$= 0.881(1-0.881)(1-0.881)$$
$$= 0.0125$$

The error is propagated backward for each unit j in the hidden layers

$$Err_j = O_j(1-O_j)\sum_K Err_K W_{jk}$$

in our case $O_j = Y_j$

Error at unit 4

$$Err_4 = Y_4(1-Y_4)\,Err_5\,W_{45}$$
$$= 0.842(1-0.842)(0.0125)(0.9)$$
$$= 0.00149$$

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Error at unit 3

$$Err_3 = O_3(1-O_3)\, Err_5\, w_{35}$$
$$= Y_3(1-Y_3)\, Err_5\, w_{35}$$
$$= 0.852(1-0.852)(0.0125)(0.3)$$
$$= 0.000472$$

The weight and biases are updated to reflect the propagated errors;
Weights are updated by using the following equations

$$w_{ij} = w_{ij} + \Delta w_{ij}$$
$$\Delta w_{ij} = (l) Err_j\, O_j$$

where,
Δw_{ij} is the change in weight w_{ij} and
l is the learning rate.

Biases are updated by the following equations

$$\theta_j = \theta_j + \Delta\theta_j$$
$$\Delta\theta_j = (l)\, Err_j$$

where $\Delta\theta_j$ is the change in bias θ_j

Calculations for the Updated Weights & Biases

For Weights:

Weights	New Value Calculations
w_{35}	$w_{35} = w_{35} + \Delta w_{35}$ $\Delta w_{35} = (L) Error_5 . Y_3$ $= 0.9 \times 0.0125 \times 0.852$ $= 0.00958$

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$w_{35} = 0.3 + 0.00958$

$w_{35} = 0.309$

w_{45}

$w_{45} = w_{45} + \Delta w_{45}$

$= 0.9 + 0.9(0.0125)(0.842)$

$= 0.909$

w_{13}

$w_{13} = w_{13} + \Delta w_{13}$

$= 0.1 + 0.9(0.000472)(0.35)$

$= 0.1 + 0.000148$

$= 0.100148$

w_{14}

$w_{14} = w_{14} + \Delta w_{14}$

$= 0.4 + 0.9(0.00149)(0.35)$

$= 0.4004$

w_{23}

$w_{23} = w_{23} + \Delta w_{23}$

$= 0.8 + 0.9(0.000472)(0.9)$

$= 0.80038$

w_{24}

$w_{24} = w_{24} + \Delta w_{24}$

$= 0.6 + 0.9(0.00149)(0.9)$

$= 0.60120$

For Biases

Bias — **New value Calculations**

θ_5

$\theta_5 = \theta_5 + \Delta\theta_5$

$= 1 + 0.9(0.0125)$

$= 1.01125$

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θ_4	$\theta_4 = \theta_4 + \Delta\theta_4$ $= 1 + 0.9(0.00149)$ $= 1.00134$
θ_3	$\theta_3 = \theta_3 + \Delta\theta_3$ $= 1 + 0.9(0.000472)$ $= 1.00042$

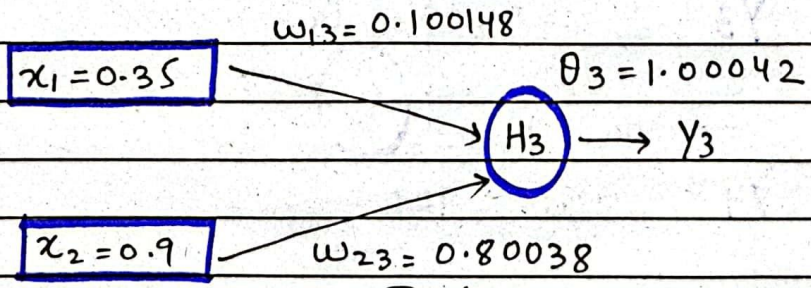
x_1	0.35
x_2	0.9
w_{13}	0.100148
w_{14}	0.4004
w_{23}	0.80038
w_{24}	0.60120
w_{35}	0.309
w_{45}	0.909
θ_3	1.00042
θ_4	1.00134
θ_5	1.01125

← Table 2

Iteration: 02

NET Input And Output Calculations:

For Unit H_3



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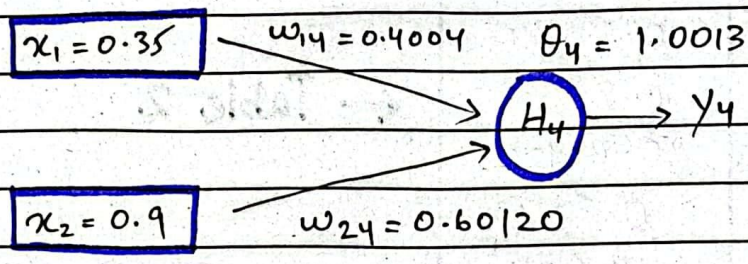
$$I_j = I_3 = x_1 w_{13} + x_2 w_{23} + \theta_3$$
$$= 0.35(0.100148) + 0.9(0.80038) + 1.00042$$
$$= 1.756$$

$$O_j = Y_3 = \frac{1}{1 + e^{-I_3}} = \frac{1}{1 + e^{-1.756}}$$

$$\boxed{Y_3 = 0.853}$$

For unit H_4

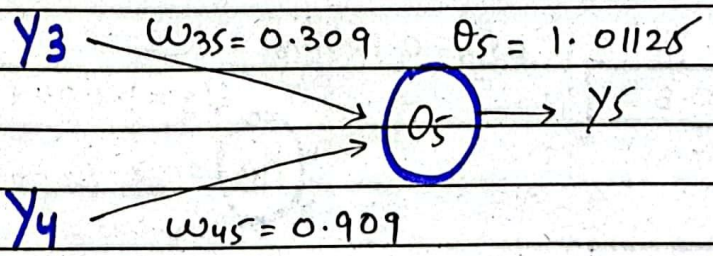


$$I_j = I_4 = x_1 w_{14} + x_2 w_{24} + \theta_4$$
$$= 0.35(0.4004) + 0.9(0.60120) + 1.00134$$
$$= 1.683$$

$$O_j = O_4 = Y_4 = \frac{1}{1 + e^{-I_4}} = \frac{1}{1 + e^{-1.683}}$$

$$\boxed{Y_4 = 0.843}$$

For unit O_5



Date

$I_j = I_5 = Y_3 w_{35} + Y_4 w_{45} + \theta_5$

$= 0.853(0.309) + 0.843(0.909) + 1.01125$

$= 2.0411$

$O_j = Y_5 = \frac{1}{1+e^{-I_5}} = \frac{1}{1+e^{-2.0411}}$

$\boxed{Y_5 = 0.885}$

Error = $Y_{target} - Y_5$

$= 1 - 0.885$

$\boxed{\text{Error} = 0.115}$

Error Computation:

Error at unit 5

$Err_5 = Y_5(1-Y_5)(Y_{target} - Y_5)$

$= 0.885(1-0.885)(1-0.885)$

$= 0.0117$

Error at unit 4

$Err_4 = Y_4(1-Y_4)(Err_5)\, w_{45}$

$= 0.843(1-0.843)(0.0117)(0.909)$

$= 0.00147$

Error at unit 3

$Err_3 = Y_3(1-Y_3)\, Err_5\, w_{35}$

$= 0.853(1-0.853)(0.0117)(0.309)$

$= 0.000453$

Date _____

Calculations for updated Weights and Biases:

For Weights:-

Weight	New Value Calculations
w_{35}	$w_{35} = w_{35} + \Delta w_{35}$ $= 0.309 + 0.9(0.0117)(0.853)$ $= 0.318$
w_{45}	$w_{45} = w_{45} + \Delta w_{45}$ $= 0.909 + 0.9(0.0117)(0.843)$ $= 0.918$
w_{13}	$w_{13} = w_{13} + \Delta w_{13}$ $= 0.100148 + 0.9(0.000453)(0.35)$ $= 0.1003$
w_{14}	$w_{14} = w_{14} + \Delta w_{14}$ $= 0.4004 + 0.9(0.00147)(0.35)$ $= 0.4009$
w_{23}	$w_{23} = w_{23} + \Delta w_{23}$ $= 0.80038 + 0.9(0.000453)(0.9)$ $= 0.8007$
w_{24}	$w_{24} = w_{24} + \Delta w_{24}$ $= 0.60120 + 0.9(0.00147)(0.9)$ $= 0.6024$

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Biases	New Value Calculations
θ_5	$\theta_5 = \theta_5 + \Delta\theta_5$ $= 1.01125 + 0.9(0.0117)$ $= 1.0218$
θ_4	$\theta_4 = \theta_4 + \Delta\theta_4$ $= 1.00134 + 0.9(0.00147)$ $= 1.0027$
θ_3	$\theta_3 = \theta_3 + \Delta\theta_3$ $= 1.00042 + 0.9(0.000453)$ $= 1.0008$

x_1	0.35
x_2	0.9
w_{13}	0.1003
w_{14}	0.4009
w_{23}	0.8007
w_{24}	0.6024
w_{35}	0.318
w_{45}	0.918
θ_3	1.0008
θ_4	1.0027
θ_5	1.0218

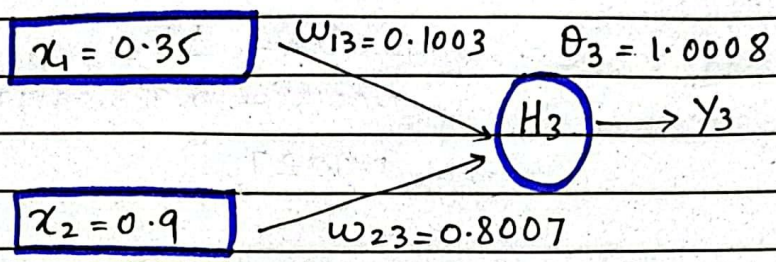
← Table 3

Date _____

Iteration: 03

NET Input And Output Calculations:

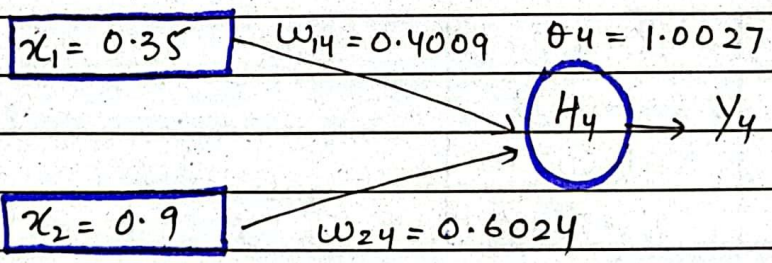
For unit H_3



$$I_j = I_3 = x_1 w_{13} + x_2 w_{23} + \theta_3$$
$$= 0.35(0.1003) + 0.9(0.8007) + 1.0008$$
$$= 1.757$$

$$O_j = O_3 = Y_3 = \frac{1}{1+e^{-I_3}} = \frac{1}{1+e^{-1.757}}$$
$$= 0.853$$

For unit H_4



$$I_j = I_4 = x_1 w_{14} + x_2 w_{24} + \theta_4$$
$$= 0.35(0.4009) + 0.9(0.6024) + 1.0027$$
$$\boxed{Y_3 = 1.69}$$

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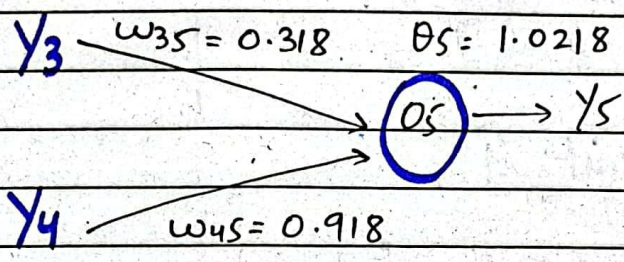
$$O_j = Y_4 = \frac{1}{1+e^{-I_4}} = \frac{1}{1+e^{-1.69}}$$

$$\boxed{Y_4 = 0.844}$$

For unit O_5



$$I_j = I_5 = Y_3 w_{35} + Y_4 w_{45} + \theta_5$$
$$= 0.853(0.318) + (0.844)(0.918) + 1.0218$$
$$= 2.07$$

$$O_j = Y_5 = \frac{1}{1+e^{-I_5}} = \frac{1}{1+e^{-2.07}}$$

$$\boxed{Y_5 = 0.9}$$

$$\text{Error} = Y_{target} - Y_5$$
$$= 1 - 0.9$$

$$\boxed{\text{Error} = 0.1}$$

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Error Computation:

Error at Unit 5

$$Err_5 = Y_5(1 - Y_5)(Y_{target} - Y_5)$$
$$= 0.9(1 - 0.9)(1 - 0.9)$$
$$= 0.009$$

Error at unit 4

$$Err_4 = Y_4(1 - Y_4)\, Err_5\, W_{45}$$
$$= 0.844(1 - 0.844)(0.009)(0.918)$$
$$= 0.00108$$

Error at unit 3

$$Err_3 = Y_3(1 - Y_3)\, Err_5\, W_{35}$$
$$= 0.853(1 - 0.853)(0.009)(0.318)$$
$$= 0.000358$$

Calculation for Updated Weights and Biases:

For Weights

Weights	New Value Calculations
W_{35}	$W_{35} = W_{35} + \Delta W_{35}$ $= 0.318 + 0.9(0.009)(0.853)$ $= 0.325$

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w_{45}	$w_{45} = w_{45} + \Delta w_{45}$ $= 0.918 + 0.9(0.009)(0.844)$ $= 0.925$
w_{13}	$w_{13} = w_{13} + \Delta w_{13}$ $= 0.1003 + 0.9(0.000358)(0.35)$ $= 0.1004$
w_{14}	$w_{14} = w_{14} + \Delta w_{14}$ $= 0.4009 + 0.9(0.00108)(0.35)$ $= 0.4012$
w_{23}	$w_{23} = w_{23} + \Delta w_{23}$ $= 0.8007 + 0.9(0.000358)(0.9)$ $= 0.8009$
w_{24}	$w_{24} = w_{24} + \Delta w_{24}$ $= 0.6024 + 0.9(0.00108)(0.9)$ $= 0.6032$

For Biases

Biases	New Value Calculations
θ_5	$\theta_5 = \theta_5 + \Delta\theta_5$ $= 1.0218 + 0.9(0.009)$ $= 1.0299$

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θ_4	$\theta_4 = \theta_4 + \Delta\theta_4$ $= 1.0027 + 0.9(0.00108)$ $= 1.0036.$
θ_3	$\theta_3 = \theta_3 + \Delta\theta_3$ $= 1.0008 + 0.9(0.000358)$ $= 1.0011$

x_1	0.35
x_2	0.9
w_{13}	0.1004
w_{14}	0.4012
w_{23}	0.8009
w_{24}	0.6032
w_{35}	0.325
w_{45}	0.925
θ_3	1.0011
θ_4	1.0036
θ_5	1.0299

← Table 4