Fall 2017 Math 395: Numerical Analysis

Quiz 2
9/14 Thursday

Name: key Total = 25

4 1. Explain why 4 vectors in $\mathbb{R}^3$ are always linearly dependent.

Explanation 1: $3 = \dim(\mathbb{R}^3)$ = the max number of vectors that can be independent

So $4 > 3$, 4 vectors have to be dependent.

Explanation 2: $\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix} \xrightarrow{\text{row elimination}} \begin{bmatrix} \boxed{a_1'} & & & \\ 0 & \boxed{b_2'} & & * \\ 0 & 0 & \boxed{c_3'} & \end{bmatrix}$

rank $= 3 < 4 = \#$ of vectors

dependent!

2. Given $a_1 = (1,0,1)$, $a_2 = (2,1,1)$, $a_3 = (3,1,2)$, find the dimension and a basis of span$\{a_1, a_2, a_3\}$

4 $\begin{array}{c} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \\ a_1 \;\; a_2 \;\; a_3 \end{array} \longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} \boxed{1} & 2 & 3 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$

~~Column~~ a_1 and a_2 are pivot columns

2 So a basis $= \{a_1, a_2\}$

1 $\dim = 2$

—More on the back—

3 3. Find the value of a, b such that the system $\left[\begin{array}{ccc:c} 2 & 3 & 8 & 9 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & a & a \\ 0 & 0 & a & b \end{array}\right]$ has infinitely many solutions.

$a = 0$

$b = 0$

4. Given $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & -1 \\ 1 & 2 & -3 \end{bmatrix}$

(a) Find rank of A.

(b) Are the columns of A linearly independent? If no, write one of them as the linear combination of the other two.

(c) Find the null space of A.

3 (a) $\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & -1 \\ 1 & 2 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 2 & -2 \end{bmatrix} \to \begin{bmatrix} \boxed{1} & 0 & -1 \\ 0 & \boxed{-1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$

rank = 2

2 (b) No, because rank < # of columns

3 $\to \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ column 3 = (-1) Colum 1 + (-1) column 2

$\begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix} = -\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$

3 (c) $x_1 - x_3 = 0$

$x_2 - x_3 = 0$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$N(A) = \left\{ x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, x_3 \text{ can be any value} \right\} = \text{span} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$