Fall 2017 Math 395: Numerical Analysis **Quiz 4**
11/16 Thursday

Name: Key

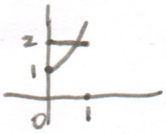
1. (6) Let $f(x) = e^x, g(x) = 2$, on the interval [0,1].
   (a) Compute $\|f-g\|_2$, the L_2 norm on the interval [0,1].
   (b) Extra credit for computing $\|f-g\|_1$.

(a) $\|f-g\|_2^2 = \int_0^1 (e^x-2)^2\,dx = \int_0^1 e^{2x}+4-4e^x\,dx$

$= \frac{1}{2}e^{2x}\Big|_0^1 + 4 - 4e^x\Big|_0^1 = \frac{1}{2}(e^2-1)+4-4(e-1) = \frac{1}{2}e^2-4e+\frac{15}{2}$

$\|f-g\|_2 = \sqrt{\frac{1}{2}e^2-4e+\frac{15}{2}} \approx 0.567$

(b) $\|f-g\|_1 = \int_0^1 |e^x-2|\,dx = \int_0^{\ln 2} 2-e^x\,dx + \int_{\ln 2}^1 e^x-2\,dx$



$f=g \Leftrightarrow x=\ln 2$

$= 2\ln 2 - e^x\Big|_0^{\ln 2} + e^x\Big|_{\ln 2}^1 - 2(1-\ln 2)$

$= 2\ln 2 - (2-1) + (e-2) - 2 + 2\ln 2$

$= 4\ln 2 + e - 5 \approx 0.491$

2. (4+4) Given three points $(-1,0), (1,1), (2,1)$
   (a) Write down the linear system to solve using the Vandermonde method. Do not solve it.
   (b) Write down the degree 2 polynomial $\varphi_1(x)$ that equals 0 at $x = -1, 2$, and equals 1 at $x = 1$.

(a) $$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

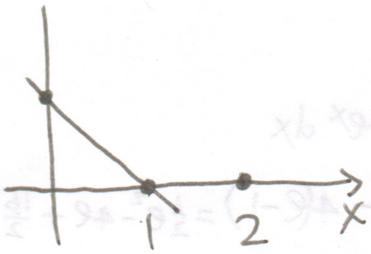
(b) $$\varphi_1(x) = \frac{(x+1)(x-2)}{(1+1)(1-2)}$$

—More on the back—

3. (6) Determine an interpolant of (0,1), (1,0), (2,0) P such that

$$P(x) = \begin{cases} P_1(x)(\text{degree } 1), & 0 \le x \le 1 \\ P_2(x)(\text{degree } 2), & 1 \le x \le 2 \end{cases}$$

and P is differentiable on [0,2].



$P_1(x) = -x+1$

since it's a line going through (0,1) (1,0)

$P_1'(1) = -1$

let $P_2(x) = a(x-1)(x-2)$

$P_2'(x) = a(x-1+x-2) = a(2x-3)$

$P_2'(1) = -1 \Rightarrow -1 = a(-1) \Rightarrow a = 1$

$P_2(x) = (x-1)(x-2)$

$$P(x) = \begin{cases} -x+1, & 0 \le x \le 1 \\ (x-1)(x-2), & 1 \le x \le 2 \end{cases}$$