Fall 2017 Math 395: Numerical Analysis

Quiz 5
12/5 Tuesday

Name: Key

1. (6) Given $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}^{-1}$

   (a) State the eigenvalues and corresponding eigenvectors of A without any computation.

   (b) Is A symmetric?

(a) $\lambda_1 = 1 \qquad \lambda_2 = -5 \qquad \lambda_3 = -1$

$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

(b) A is not symmetric.
If A were symmetric, then eigenvectors will be orthogonal to each other, but v_1, v_2, v_3 are not.

2. (5) Let $P = QDQ^{-1}$, where Q is orthonormal and D is a diagonal matrix whose diagonals are all positive. Justify why P is positive definite (also need to show symmetry).

① $Q^{-1} = Q^T \Rightarrow P = QDQ^T$

$P^T = (QDQ^T)^T = (Q^T)^T D^T Q^T = QDQ^T = P \Rightarrow P$ is symmetric

② P and D are similar $\Rightarrow$ eigenvalues of P = eigenvalues of D, all positive

3. (5) What does it mean for A and B to be similar? What does it mean for A to be diagonalizable?

A and B are similar if there exists an invertible U such that

$A = UBU^{-1}$

A is diagonalizable iff A is similar to a diagonal matrix.

4. (9) Suppose A has eigenvalues 1, 2, 3, 4.

(a) Which eigenvalue will the power method (on A) find? What is the convergence rate?

(b) Which eigenvalue will the power method on $A - 2I$ find? What is the convergence rate?

(c) Which eigenvalue will the power method on $(A - 1.8I)^{-1}$ find? What is the convergence rate?

(a) 4. rate $= \frac{3}{4}$

(b) $-1, 0, 1, 2$

Find 2. rate $= \frac{1}{2}$

(c) $\frac{1}{1-1.8}, \frac{1}{2-1.8}, \frac{1}{3-1.8}, \frac{1}{4-1.8}$

$= \underline{\frac{1}{-0.8}}, \frac{1}{0.2}, \frac{1}{1.2}, \frac{1}{2.2}$

2nd biggest

will find $\frac{1}{0.2}$

rate $= \dfrac{\frac{1}{0.8}}{\frac{1}{0.2}} = \frac{1}{4}$