Fall 2017 Math 395 Final Review

Final Written Exam:	12/12 Tuesday 12:30 - 2:30	at LS 209
Extra Office hours:	12/11 Monday 9:30-11 12/12 Tuesday 9:30-11	at Harney 121B

The final exam has two parts: final project (10%) + final written exam (25%). The final project is due on 12/14 midnight. The final written exam is comprehensive. It covers all 10 chapters. Naturally, there is a bit more focus on material after the midterm.

You are allowed to have one cheat sheet, double sided. You can put anything on the sheet, but it has to be hand-written by you. **NO CALCULATOR.**

You should read the notes thoroughly and focus on homework and quiz problems.

Concepts:

0 Be familiar with the basic linear algebra knowledges written on page 1.

1. Understand the Bisection algorithm, Newton's algorithm, and the fixed point algorithm. What are they and why do they work? What is a fixed point. Be able to apply Theorem 1.5.

2. Binary representation. Rounding down, rounding up, and rounding to the nearest. How to break the tie when rounding to the nearest in binary? How to add up two numbers in floating number arithmetic? Machine precision.

3. Compute ℓ_p norm of a vector. Block matrix multiplication. Positive definite matrices and its 3 criteria. Positive semidefinite matrix. Advantages of an ONB (Theorem 3.10). Properties of matrix norm. Compute operator norm of a matrix.

4. Compute LU factorization. FLOP counts. Partial pivoting and scaled partial pivoting. Cholesky factorization. Least square solution, projection matrix, normal equation. Solving least square via normal equation, SVD and QR.

5. Definition of SVD and reduced SVD. Write SVD as sum of rank one matrices (formula (5.3)). Compute singular values of any matrix. Compute singular values of a symmetric matrix. Rank-s approximation. How to do PCA/Data compression through SVD.

6. Difference between stability of a problem and stability of an algorithm. Stability of a linear system. Compute condition number. Gram-Schmidt algorithm. Orthogonal complement. Hyperplane. Theorem 6.7. Formula for Householder reflector. Use Householder reflector to transform x to $\pm\|x\|_2 e_1$ (need to know formula (6.7)). Compute QR or reduced QR using Gram-Schmidt or Householder.

7. Polynomial interpolation with Vandermonde, Lagrange or Newton: formula and how they are derived. What are Chebyshev nodes and why are they for? Basic concepts of spline and what does it mean for piecewise functions to be continuous/differentiable/twice differentiable. What is the L_p norm of a function on certain interval?

8. What is a convex function? What is a convex optimization problem? How to write a problem as a convex problem (if possible)? What is a convex set? What is the gradient and Hessian? Relationship between Hessian and convex function. The gradient descent method to solve unconstrained convex problem.

9. What does it mean for two matrices to be similar? What is similarity transformation? What is the main purpose for similarity transformation? What is Schur decomposition? How to find a matrix's

Hessenberg form? Why do we turn a matrix to its Hessenberg form before applying eigenvalue algorithms (shifted QR)? Power method and its convergence rate on $A, A-\mu I, (A-\mu I)^{-1}$. The (shifted) QR algorithm.

10. General iteration, Jacobi and Gauss-Seidel iteration. Spectral radius. A-norm and A-orthogonal. How are ellipses related to a PD matrix? What is a quadratic form? The gradient and Hessian of a quadratic form. What is residual and error vector? Steepest gradient descent: how stepsize was derived, what is the algorithm? How condition numbers affect convergence rate of steepest gradient descent and CGD?