9 If $A = QR$ is the full QR, what is cholesky of A^TA

$$A^TA = (QR)^T QR = R^T Q^T Q R = R^T R := LL^T$$

$$L = R^T \text{ is lower triangular}$$

10 (a) $A = \hat{Q}\hat{R}$ $\quad \underline{\hat{Q}^T\hat{Q} = I}$ $\qquad \boxed{} = \boxed{\hat{Q}}\,\boxed{\hat{R}}$

$$A^TA = \hat{R}^T\hat{Q}^T\hat{Q}\hat{R} = \hat{R}^T\hat{R}, \quad (A^TA)^{-1} = \hat{R}^{-1}\hat{R}^{-T}$$

$$P = A(A^TA)^{-1}A^T = \hat{Q}\hat{R}\,\hat{R}^{-1}\,\hat{R}^{-T}\hat{R}^T\hat{Q}^T = \hat{Q}\hat{Q}^T$$

(b) $A = \hat{U}\hat{\Sigma}V^T$, $\quad A^T = V\hat{\Sigma}^T\hat{U}^T$ $\qquad \boxed{} = \boxed{\hat{U}}\,\boxed{\hat{\Sigma}}\,\boxed{V}^T$

$$A^TA = V\hat{\Sigma}^T\hat{U}^T\hat{U}\hat{\Sigma}V^T = V\hat{\Sigma}^T\hat{\Sigma}V^T$$

$$(A^TA)^{-1} = V^{-T}\hat{\Sigma}^{-1}\hat{\Sigma}^{-T}V^T$$

$$P = A(A^TA)^{-1}A^T = \hat{U}\hat{\Sigma}\underline{V^T V}^{-T}\hat{\Sigma}^{-1}\hat{\Sigma}^{-T}\underline{V^T V}\hat{\Sigma}^T\hat{U}^T$$

$$= \hat{U}\underbrace{\hat{\Sigma}\hat{\Sigma}^{-1}}\underbrace{\hat{\Sigma}^{-T}\hat{\Sigma}^T}\hat{U}^T = \hat{U}\hat{U}^T$$

11
$$\begin{bmatrix} 1 & 1 & 4 \\ -1 & 0 & 0 \\ 1 & 1 & 2 \\ -1 & 0 & -2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ & 1 & 2 \\ & & 2 \end{bmatrix}, \text{ solve LS of } Ax = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$A \quad = \quad \hat{Q} \qquad \hat{R}$$

Compute $y = \hat{Q}^T b = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2+1-1 \\ 2 \\ 2+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}$

Solve $\begin{bmatrix} 2 & 1 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}$ $\qquad \begin{matrix} x_3 = 0 \\ x_2 + 0 = 1 \Rightarrow x_2 = 1 \\ 2x_1 + 1 = 1 \Rightarrow x_1 = 0 \end{matrix}$ $\qquad x = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

12
$$B = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} \\ 1 & 0 & -1 \\ \frac{1}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 2 & & 0 \\ & 0.02 & \\ 0.01 & & 0 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}^T$$

(a) $B = U\Sigma V^T$

$B^T = V\Sigma^T U^T = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix}\underset{\color{red}\text{reduced}}{\begin{bmatrix} 2 & & \\ & 0.02 & \\ & & 0.01 \end{bmatrix}}\frac{1}{\sqrt{2}}\begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} \\ 1 & 0 & -1 \\ \frac{1}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} \end{bmatrix}^T$

$\underset{U^T}{}$

$\underset{\color{red}\text{reduced}}{}$

cb)

$$B^T = \frac{1}{2}\begin{bmatrix}1&1&1\\1&1&-1\\1&-1&1\\1&-1&-1\end{bmatrix}\begin{bmatrix}2&&\\&0.02&\\&&0.01\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{\sqrt{2}}&1&\frac{1}{\sqrt{2}}\\1&0&-1\\\frac{1}{\sqrt{2}}&-1&\frac{1}{\sqrt{2}}\end{bmatrix}^T := ADC^T$$

compute $z = A^T b = \frac{1}{2}\begin{bmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&1&-1\end{bmatrix}\begin{bmatrix}2\\-1\\0\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\\0\\2\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix}$

Solve $\begin{bmatrix}2&&\\&0.02&\\&&0.01\end{bmatrix} y = \begin{bmatrix}1\\0\\1\end{bmatrix} \Rightarrow$
$y_1 = 0.5$
$y_2 = 0$
$y_3 = 100$

$x = Cy = \frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{\sqrt{2}}&1&\frac{1}{\sqrt{2}}\\1&0&-1\\\frac{1}{\sqrt{2}}&-1&\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}0.5\\0\\100\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{2\sqrt{2}}+\frac{100}{\sqrt{2}}\\\frac{1}{2}-100\\\frac{1}{2\sqrt{2}}+\frac{100}{\sqrt{2}}\end{bmatrix}$