Numerical Methods

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Contents

Preliminaries

Before this course, you need to be familiar with

- Calculus I
- Calculus II: especially Sequences and Series
- Linear Algebra
  1. Vectors, norm, dot product
  2. Linear Independence and Dependence
  3. Span, subspace, bases, dimension of a subspace/vector space
  4. Properties of orthogonal or orthonormal sets/bases; Gram-Schmidt algorithm
  5. Matrix multiplication; Determinant; Inverse
  6. Solving linear equations: algorithm, existence and uniqueness.
  7. Projections; Least square
  8. Eigenvalues and Eigenvectors
  9. SVD

Notations

- $\{x : \text{ descriptions of } x\}$ is the set of all x that meets the description after the ":" sign. For example, $\{x : x^2 - 1 < 0\}$ is the set of all number x such that $x^2 - 1 < 0$. We can solve this inequality further and see that $\{x : x^2 - 1 < 0\} = (-1, 1)$
- $\in$: belongs to. For example, $a \in \{x : x^2 - 1 < 0\}$ means that a is a number satisfying $a^2 - 1 < 0$.
- s.t.: such that

Chapter 1

Solving nonlinear equations

If you recall the equations that you are able to solve, you would realize that there are very few. You know how to solve $x^2 - x - 1 = 0$ using the famous quadratic formula, but have you ever wondered formulas for finding roots of polynomials whose degree is higher than 2? Unfortunately, even as simple as a polynomial equation $x^5 - x - 1 = 0$, a math Ph.D feels hopeless to find an exact answer by hand (so you shouldn't feel bad). In fact, there are no formula for polynomials of degree bigger than 4. Equations like $\cos x = x$? does not have a closed form solution either.

When we need to solve nonlinear equations that frequently occur in every aspect of sciences and applications, we use iterative methods.

1.1 Bisection Method

Bibliography

[1] Anne, Greenbaum, and Timothy P. Chartier. *Numerical methods: design, analysis, and computer implementation of algorithms*. Princeton University Press, 2012.