Fall 2017 Math 395 Written Homework 3 Key 100 total. -5 for no stapling

3.9 Compute x^TAx, where $x = (x_1, x_2, x_3)$, and $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$ (A is symmetric.)

3.10 Prove that if B is invertible, then B^TB is positive definite.

3.11 Check that $C = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$ is positive definite using both criteria I and III.

3.12 Check that $D = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ is not positive definite using criteria II.

4.1 How many **exact** flops are needed for the following computations?

(a) $\langle x, y\rangle$, where x, y are vectors in $\mathbb{R}^9$.

(b) ABC, where A, B, C are 5×5 matrices.

(c) Ux, where U is $n \times n$ upper triangular matrix and x is a vector in $\mathbb{R}^n$.

4.2 Given $n \times n$ matrices A, B, C, how many flops are needed for the following computations? Answer in terms of the order $n, n^2, n^3, \cdots$.

(a) ABC

(b) $A^{-1}B$

(c) $A + B$

(d) LU, where L is $n \times n$ lower triangular, U is $n \times n$ upper triangular

4.3 Consider the linear system $\left[\begin{array}{cc:c} 2 & 2 \times 10^{20} & 2 \times 10^{20} \\ 1 & 1 & 2 \end{array}\right]$. Find its 'solution' using Gaussian elimination, in a machine where numbers are in standard IEEE double-precision format.

4.4 Solve $\begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix} x = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ ($Ax = b$) by

(a) First find the LU factorization of A.

(b) Second solve $Ly = b$.

(c) Third solve $Ux = y$.

4.5 Given $\begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ -4 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, solve the system

$$\begin{array}{rrrl} 3x_1 & -6x_2 & -3x_3 & = -3 \\ 2x_1 & & +6x_3 & = -22 \\ -4x_1 & +7x_2 & +4x_3 & = 3 \end{array} .$$

4.6 Solve the system in Problem 5 using scaled partial pivoting strategy.