

GENERAL

(Total 8 modules each of 50 marks)

MODULE I : Group A : Classical Algebra (20 marks)
Group B : Analytical Geometry of two dimensions (15 marks)
Group C : Vector Algebra (15 marks)

MODULE II : Group A : Differential Calculus (25 marks)
Group B : Integral Calculus (10 marks)
Group C : Differential Equations (15 marks)

MODULE III : Group A : Modern Algebra (25 marks)
Group B : Analytical Geometry of three dimensions (25 marks)

MODULE IV : Group A : Differential Calculus (25 marks)
Group B : Integral Calculus (15 marks)
Group C : Differential Equations (10 marks)

MODULE V : Group A : Numerical Methods (20 marks)
Group B : Linear Programming (30 marks)

MODULE VI : Any **one** of the following groups :

Group A : Analytical Dynamics (50 marks)
Group B : Probability & Statistics (50 marks)

MODULE VII : Computer Science & Programming (50 marks)

MODULE V : Any **one** of the following groups :

Group A : A Course of Calculus (50 marks)
Group B : Discrete Mathematics (50 marks)

MODULE I

Group A (20 marks)

Classical Algebra

- 01. Complex Numbers :** De Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a^z , ($a \neq 0$). Inverse circular and Hyperbolic functions.
- 02. Polynomials :** Fundamental Theorem of Classical Algebra (Statement only). Polynomials with real co-efficients: The n th degree polynomial equation has exactly n roots. Nature of roots of an equation (Surd or Complex roots occur in pairs). Statement of Descarte's Rule of signs and its applications.
- Statements of :
- (i) If the polynomial $f(x)$ has opposite signs for two real values of x , e.g. a and b , the equation $f(x) = 0$ has an odd number of real roots between a and b ; if $f(a)$ and $f(b)$ are of same sign, either no real root or an even number of roots lies between a and b .
 - (ii) Rolle's Theorem and its direct applications.

Relation between roots and co-efficients. Symmetric functions of roots, Transformations of equations. Cardan's method of solution of a cubic.

- 03. Determination up to the third order :** Properties, Cofactor and Minor. Product of two determinants. Adjoint, Symmetric and Skew-symmetric determinants. Solutions of linear equations with not more than three variables by Cramer's Rule.
- 04. Matrices of Real Numbers :** Equality of matrices. Addition of matrices. Multiplication of a matrix by a scalar. Multiplication of matrices – Associative properties. Transpose of matrix – its properties. Inverse of a non-singular square matrix. Symmetric and Skew-symmetric matrices. Scalar matrix. Orthogonal matrix. Elementary operations on matrices.

Rank of a matrix : Determination of rank either by considering minors or by sweep-out process. Consistency and solution of a system of linear of equations with not more than 3 variables by matrix method.

Group B (15 marks)

Analytical Geometry of 2 Dimensions

- 01. Transformations of Rectangular axes :** Translation, Rotation and their combinations. Invariants.
- 02. General equation of second degree in x and y :** Reduction to canonical forms. Classification of conic.
- 03. Pair of straight lines :** Condition that the general equation of 2^{nd} degree in x and y may represent two straight lines. Points of intersection of two intersecting straight lines. Angle between two lines given by $ax^2 + 2hxy + by^2 = 0$. Equation of bisectors. Equation of two lines joining the origin to the points in which a line meets a conic.

- 04. Equations of pair of tangents from an external point, chord of contact, poles and polars in case of General conic :** Particular cases for Parabola, Ellipse, Circle, Hyperbola.
- 05.** Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equation of chord joining two points. Equations of tangent and normal.

Group C (15 marks)

Vector Algebra

Addition of Vectors. Multiplication of a Vector by a scalar. Collinear and Coplanar Vectors. Scalar and Vector products of two and three vectors. Simple applications to problems of Geometry. Vector equation of plane and straight line. Volume of Tetrahedron. Application to problems of Mechanics (Work done and Moment).

MODULE II

Group A (25 marks)

Differential Calculus

- 01.** Rational Numbers. Geometrical representation. Irrational number. Real number represented as point on a line – Linear Continuum. Acquaintance with basic properties of real number (No deduction or proof is included).
- 02. Sequence :** Definition of bounds of a sequence and monotone sequence. Limit of a sequence. Statements of limit theorems. Concept of convergence and divergence of monotone sequences – applications of the theorems, in particular, definition of ϵ . Statement of Cauchy's general principle of convergence and its application.
- 03. Infinite series of constant terms :** Convergence and Divergence (definitions). Cauchy's principle as applied to infinite series (application only). Series of positive terms : Statements of Comparison test, D'Alembert's Ratio test. Cauchy's n th root test and Raabe's test – Applications. Alternating series: Statement of Leibnitz test and its applications.
- 04. Real-valued functions defined on an interval :** Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval.

Acquaintance (no proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.

- 05. Derivative** – its geometrical and physical interpretation. Sign of derivative – Monotonic increasing and decreasing functions. Relation between continuity and derivability. Differential – application in finding approximation.

- 06. Successive derivative** – Leibnitz's Theorem and its application.
- 07.** Application of the principle of Maxima and Minima for a function of single variable in geometrical, physical and other problems.
- 08. Applications of Differential Calculus :** Tangents and Normals, Pedal equation and Pedal of a curve. Rectilinear Asymptotes (Cartesian only).
Definition and examples of singular points (viz. Node, Cusp, Isolated point).

Group B (10 marks)

Integral Calculus

- 01.** Integration of the form :

$\int \frac{dx}{a+b \cos x}$, $\int \frac{l \sin x + m \cos x}{n \sin x + p \cos x} dx$ and Integration of Rational functions.

- 02.** Evaluation of definite integrals.

- 03.** Integration as the limit of a sum (with equally spaced as well as unequal intervals)

Group C (15 marks)

Differential Equations

- 01.** Order, degree and solution of an ordinary differential equation (ODE) in presence

of arbitrary constants. Formation of ODE.

First order equations :

- (i) Variables separable.
- (ii) Homogeneous equations and equations reducible to homogeneous forms.
- (iii) Exact equations and those reducible to such equation.
- (iv) Euler's and Bernoulli's equations (Linear).
- (v) Clairaut's Equations : General and Singular solutions.

- 02.** Simple applications : Orthogonal Trajectories.

MODULE III

Group A (25 marks)

Modern Algebra

- 01. Basic concept :** Sets, Sub-sets, Equality of sets, Operations on sets : Union, intersection and complement. Verification of the laws of Algebra of sets and De Morgan's Laws. Cartesian product of two sets.

Mappings, One-One and onto mappings. Composition of Mappings

—
concept only, Identity and Inverse mappings. Binary Operations in a set.
Identity element. Inverse element.

- 02. Introduction of Group Theory :** Definition and examples taken from various branches (examples from number system, roots of unity, 2×2 real matrices, non-singular real matrices of a fixed order). Elementary properties using definition of Group. Definition and examples of sub-group – Statement of necessary and sufficient condition – its applications.
- 03.** Definitions and examples of (i) Ring, (ii) Field, (iii) Sub-ring, (iv) Sub-field.
- 04. Concept of Vector space over a Field :** Examples, Concepts of Linear combinations, Linear dependence and independence of a finite set of vectors, Sup-space. Concepts of generators and basis of a finite-dimensional vector space. Problems on formation of basis of a vector space (No proof required).
- 05.** Real Quadratic Form involving not more than three variables – Problems only.
- 06.** Characteristic equation of a square matrix of order not more than three – determination of Eigen Values and Eigen Vectors – Problems only. Statement and illustration of Cayley-Hamilton Theorem.

Group B (25 marks)

Analytical Geometry of 3 dimensions

- 01. Rectangular Cartesian co-ordinates :** Distance between two points. Division of a line segment in a given ratio. Direction cosines and direction ratios of a straight line. Projection of a line segment on another line. Angle between two straight lines.
- 02. Equation of a Plane :** General form. Intercept and Normal form. Angle between two planes. Signed distance of a point from a plane. Bisectors of angles between two intersecting planes.
- 03. Equations of Straight line :** General and symmetric form. Distance of a point from a line. Coplanarity of two straight lines. Shortest distance between two skew-lines.
- 04.** Sphere and its tangent plane.
- 05.** Right circular cone.

MODULE IV

Group A (25 marks)

Differential Calculus

- 01.** Statement of Rolle's theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy. Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders. Taylor's and Maclaurin's Infinite series for functions like e^x , $\sin x$, $\cos x$, $(1+x)^n$, $\log(1+x)$ [with restrictions wherever necessary]
- 02. Indeterminate Forms :** L'Hospital's Rule : Statement and problems only.
- 03. Functions of two and three variables :** Their geometrical representations. Limit and Continuity (definitions only) for functions of two variables. Partial derivatives : Knowledge and use of Chain Rule. Exact differentials (emphasis on solving problems only). Functions of two variables – Successive partial derivatives : Statement of Schwarz's Theorem on commutative property of mixed derivatives. Euler's theorem on homogeneous function of two and three variables. Maxima and minima of functions of not more than three variables – Lagrange's Method of undetermined multiplier – Problems only. Implicit function in case of function of two variables (existence assumed) and derivative.

Group B (15 marks)

Integral Calculus

- 01.** Reduction formulae of $\int \sin^m x \cos^n x \, dx$, $\int \frac{\sin^m x}{\cos^n x} dx$, $\int \tan^n x \, dx$ and associated problems (m and n are non-negative integers).
- 02. Definition of Improper Integrals :** Statements of (i) μ -test, (ii) Comparison test (Limit form excluded) – Simple problems only. Use of Beta and Gamma functions (convergence and important relations being assumed).
- 03.** Working knowledge of Double integral.
- 04. Applications :** Rectification, Quadrature, Volume and Surface areas of solids formed by revolution of plane curve and areas – Problems only.

Group C (10 marks)

Differential Equations

- 01. Second order linear equations :** Second order linear differential equations with constant. Coefficients. Euler's Homogeneous equations.

MODULE V

Group A (20 marks)

Numerical Methods

01. Approximate numbers, Significant figures, Rounding off numbers. Error – Absolute, Relative and Percentage.
02. **Operators** - Δ , ∇ and E (Definitions and some relations among them).
03. **Interpolation** : The problem of Interpolation, Equispaced arguments – Difference Tables, Deduction of Newton's Forward Interpolation Formula. Remainder term (expression only). Newton's Backward Interpolation formula (statement only) with remainder term. Unequally – spaced arguments – Lagrange's Interpolation Formula (statement only). Numerical problems on Interpolation with both equi- and unequally-spaced arguments.
04. **Number Integration** : Trapezoidal and Simpson's $\frac{1}{3}$ rd formula (statement only). Problems on Numerical Integration.
05. **Solution of Numerical Equation** : To find a real root of an algebraic or transcendental equation. Location of root (Tabular method), Bisection method. Newton-Raphson method with geometrical significance. Numerical problems.

(Note : emphasis should be given on problems)

Group B (30 marks)

Linear Programming

01. Motivation of Linear Programming problem. Statement of L.P.P. formulation of L.P.P. Slack and Surplus variables. L.P.P. in matrix form. Convex set, Hyperplane, Extreme points, Convex Polyhedron, Basic solutions and Basic Feasible Solutions (B.F.S.) Degenerate and Non-degenerate B.F.S.

The set of all feasible solutions of an L.P.P. is a convex set. The objective function of an L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions. A B.F.S. to an L.P.P. corresponds to an extreme point of the convex set of feasible solutions.

Fundamental Theorem of L.P.P. (Statement only). Reduction of a feasible solution to a B.F.S. Standard form of an L.P.P. Solution by graphical method (for two variables), by simplex method and method of penalty. Concept of duality. Duality theory. The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Dual problems with at most one unrestricted variable, one constraint of equality.

Transportation and Assignment problem and their optimal solutions.

MODULE VI

(50 marks)

(Any one of the following groups)

Group A

Analytical Dynamics

- 01.** Velocity and Acceleration of a particle. Expressions for velocity and acceleration in rectangular Cartesian and polar co-ordinates for a particle moving in a plane. Tangential and normal components of velocity and acceleration of a particle moving along a plane curve.
- 02. Concept of Force :** Statement and explanation of Newton's laws of motion. Work, power and energy. Principles of conservation of energy and momentum. Motion under impulsive forces. Equations of motion of a particle (i) moving in a straight line, (ii) moving in a plane.
- 03.** Study of motion of a particle in a straight line under (i) constant forces, (ii) variable forces (S.H.M., Inverse square law, Damped oscillation, Forced and Damped oscillation, Motion in an elastic string). Equation of Energy. Conservative forces.
- 04. Motion in two dimensions :** Projectiles in vacuo and in a medium with resistance varying linearly as velocity. Motion under forces varying as distance from a fixed point.
- 05.** Central orbit. Kepler's laws of motion. Motion under inverse square law.

OR

Group B

Probability and Statistics

- 01. Elements of Probability Theory :** Random experiment, Outcome, Event, Mutually Exclusive Events, Equality like and Exhaustive, Classical definition of Probability, theorems of Total Probability, Conditional Probability and Statistical Independence. Bayes' theorem. Problems. Shortcomings of the classical definition. Axiomatic approach – Problems. Random Variable and its Expectation. Theorems on mathematical expectation. Joint distribution of two random variables.
Theoretical Probability Distribution – Discrete and Continuous (p.m.f. pd.d.f.) Binomial, Poisson and Normal distributions and their properties.
- 02.** Elements of Statistical Methods. Variables, Attributes, Primary data and secondary data. Population and sample. Census and Sample Survey. Tabulation – Chart and Diagram, graph, Bar diagram, Pie diagram etc. Frequency Distribution – Un-grouped and grouped cumulative frequency distribution. Histogram, Frequency curve, Measure of Central Tendencies – Average : AM, GM, HM, Mean, Median and Mode (their advantages and disadvantages). Measures of Dispersions – Range, Quartile Deviation, Mean Deviation, Variance/S.D., Moments, Skewness and Kurtosis.
- 03. Sampling Theory :** Meaning and objects of sampling. Some ideas about the methods of selecting samples. Statistic and Parameter, Sampling Distribution – standard error of a statistic (e.g. sample mean, sample proportion). Four fundamental distributions derived from the normal : (i) Standard Normal Distribution, (ii) Chi-square distribution, (iii) Student's distribution, (iv) Snedecor's F-distribution.

- Estimation and Test of Significance. Statistical Inference. Theory of estimation – Point estimation and Interval estimation. Confidence Inter/Confidence Limit. Statistical Hypothesis – Null Hypothesis and Alternative Hypothesis. Level of significance. Critical Region. Type I and Type II error. Problems.
- 04.** Bivariate Frequency Distribution. Scatter Diagram, Correlation co-efficient – Definition and properties. Regression lines.
- 05. Time Series :** Definition. Why to analyze Time series data ? Components. Measurement of Trend – (i) Moving Average Method, (ii) Curve Fittings (linear and quadratic curve). (Ideas of other curves, e.g. exponential curve etc.). Ideas about the measurement of other components.
- 06. Index Number :** Meaning of Index Number. Construction of Price Index Number. Consumer Price Index Number. Calculation of Purchasing Power of Rupee.

MODULE VII

(50 marks)

Computer Science & Programming

- 01. Boolean algebra** – Basic Postulates and Definition. Two-element Boolean algebra. Boolean function. Truth table. Standard form of Boolean function – DNF and CNF. Minterms and maxterms. Principle of Duality. Some laws and theorem of Boolean algebra. Simplification of Boolean expressions – Algebraic method and Karnaugh Map method. Application of Boolean algebra – Switching Circuits, Circuit having some specified properties, Logical Gates – AND, NOT, OR, NAND, NOR etc.
- 02. Computer Science and Programming :** Historical Development, Computer Generation, Computer Anatomy – Different Components of a Computer System. Operating System, Hardware and Software.
- Positional Number System. Binary to Decimal and Decimal to Binary. Other systems. Binary Arithmetic. Octal, Hexadecimal, etc. Storing of data in a Computer – BIT, BYTE, WORD, etc. Coding of a data – ASCII, etc.
- Programming Language :** Machine Language, Assembly language and High level language. Compiler and Interpreter. Object Programme and Source Programme. Ideas about some HLL – e.g. BASIC, FORTRAN, C, C++, COBOL, PASCAL, etc.
- Algorithms and Flow Charts** – their utilities and important features, Ideas about the complexities of an algorithm. Application in simple problems.
- FORTRAN 77/99 :** Introduction, Data Type – Keywords, Constants and Variables – Integer, Real, Complex, Logical, Character, Subscripted Variables, Fortran Expressions.
- I/O Statements** – formatted and unformatted. Programme execution control – Logical if, if-then-else, etc. Arrays, dimension statement. Repetitive Computation – Do, Bested Do etc.
- Sub Programs** – (i) Function Sub Programme
(ii) Subroutine Sub Programme

Elements of BASIC Programming Language : Reading, Printing, Branch & Loop, Array, Functions.

Application to Simple Problems. An exposure to M.S. Office, e-mail, Internet (Through Demonstration only).

MODULE VIII

(50 marks)

(Any one of the following groups)

Group A

A Course of Calculus

- 01.** Concept of Point-wise and Uniform convergence of sequence of functions and series of functions with special reference of Power Series. Statement of Weierstrass M-Test for Uniform convergence of sequence of functions and of series of functions. Simple applications. Statement of important properties like boundedness, continuity, differentiability and integrability of the limit function of uniformly convergent sequence of functions and of the sum function of uniformly convergent series of functions. Determination of Radius of convergence of Power Series.

Statement of properties of continuity of sum function power series. Term by term integration and Term by term differentiation of Power Series. Statements of Abel's Theorems on Power Series. Convergence of Power Series. Expansions of elementary functions such as e^x , $\sin x$, $\log(1+x)$, $(1+x)^n$. Simple problems.

- 02.** Fourier series on $(-\pi, \pi)$: Periodic function. Determination of Fourier co-efficients. Statement of Dirichlet's conditions of convergence and statement of the theorem on convergence of Fourier Sine and Cosine series.
- 03.** Third and Fourth order ordinary differential equation with constant co-efficients. Euler's Homogeneous Equation.
- 04.** Second order differential equation : (a) Method of variation of parameters. (b) Method of undetermined co-efficients. (c) Simple eigenvalue problem.
- 05.** Simultaneous linear differential equation with constant co-efficients.
- 06.** Laplace Transform and its application to ordinary differential equation. Laplace Transform and Inverse Laplace Transform. Statement of Existence theorem. Elementary properties of Laplace Transform and its Inverse. Application to the solution of ordinary differential equation of second order with constant eo-efficients.
- 07.** Partial Differential Equation (PDE) : Introduction, Formation of PDE, Solutions of PDE, Lagrange's method of solution.

OR

Group B

Discrete Mathematics

- 01. Integers** : Principle of Mathematical Induction. Division algorithm. Representation of integer in an arbitrary base. Prime integers. Some properties of prime integers. Fundamental theorem of Arithmetic. Euclid's Theorem. Linear Diophantine Equations. (Statement of Principle of Mathematical Induction, Strong form of Mathematical induction. Applications in different problems. Proofs of division algorithm. Representation of an integer uniquely in an arbitrary base, change of an integer from one base to another base. Computer operations with integers – Divisor of an integer, g.c.d. of two positive integers, prime integer, Proof of Fundamental theorem, Proof of Euclid's Theorem. To show how to find all prime numbers less than or equal to a given positive integer. Problems related to prime number. Linear Diophantine equation – when such an equation has solution, some applications).
- 02. Congruences** : Congruence relation on integers, Basic properties of this relation. Linear Congruences, Chinese Remainder Theorem. System of Linear Congruences. (Definition of Congruence – to show it is an equivalence relation, to prove the following : $a \equiv b \pmod{m}$ implies (i) $(a+c) \equiv (b+c) \pmod{m}$ (ii) $ac \equiv bc \pmod{m}$ (iii) $a^n \equiv b^n \pmod{m}$, for any polynomial $f(x)$ with integral coefficients $f(a) \equiv f(b) \pmod{m}$ etc. Linear Congruence, to show how to solve these congruences, Chinese remainder theorem – Statement and proof and some applications. System of linear congruences, when solution exists – some applications).
- 03. Application of Congruences** : Divisibility tests. Computer file, Storage and Hashing functions. Round-Robin Tournaments. Check-digit in an ISBN, in Universal Product Code, in major Credit Cards. Error detecting capability. (Using Congruence, develop divisibility tests for integers base on their expansions with respect to different bases, if d divides $(b-1)$ then $n = (a_k a_{k-1} \dots a_1)_b$ is divisible by d if and only if the sum of the digits is divisible by d etc. Show that congruence can be used to schedule Round-Robin tournaments. A university wishes to store a file for each of its students in its computer. Systematic methods of arranging files have been developed based on Hashing functions $h(k) \equiv k \pmod{m}$. Discuss different properties of this congruence and also problems based on this congruence. Check digits for different identification numbers – International standard book number, universal product code etc. Theorem regarding error detecting capability).
- 04. Congruence Classes** : Congruence classes, addition and multiplication of congruence classes. Fermat's little theorem. Euler's Theorem. Wilson's theorem. Some simple applications. (Definition of Congruence Classes, properties of Congruence classes, addition and multiplication, existence of inverse. Fermat's little theorem. Euler's theorem. Wilson's theorem – Statement, proof and some applications).
- 05. Recurrence Relations and Generating functions** : Recurrence Relations. The method of Iteration. Linear difference equations with constant coefficients. Counting with generating functions.
- 06. Boolean Algebra** : Boolean Algebra, Boolean functions, Logic gates, Minimization of circuits.