Summarizing Nested AMMs

Robert Leifke[1]

University of Michigan
rleifke@umich.edu

Abstract. Nested AMMs are a variation of Geometric Mean Market Makers (G3M), a type of Constant Function Market Maker that is able to support the exchange of multiple related on-chain assets with an aggregate trading function that classes nested token pairs. The constant curve described is a generalized invariant based on a constant elasticity of substitution (CES) utility function that clumps the related tokens together to a stable asset and can adjust from a constant product to constant sum curve based on the slippage of a nested pair expressed as the substitution parameter σ and the slippage to the stable token η.

Keywords: Constant Function Market Maker · CES Utility Function.

1 Introduction

Constant Function Market Makers (CFMMs) are a subclass of Automated Market Makers (AMM) that utilize a trading function φ to maintain a desired ratio of reserves $R \in \boldsymbol{R_+}^n$ held by the AMM. This creates a generalized pricing algorithm, represented as a curve that multiple agents pool liquidity to. Thereby, a liquidity provider is in effect holding a portfolio of tokens and exposed to the volatility risks of the underliers. More importantly, the invariant quotes a price to an agent for some scalar quantity in a two-asset trade. This proposal $(\varDelta, \varLambda)$ is accepted by the AMM if the trading function $\varphi(R)$ is unchanged, the fee γ is < 1, and the amount of reserves R is sufficient as described by

$$\varphi(R + \gamma\varDelta - \varLambda) = \varphi(R) \tag{1}$$

Due to the storage constraints of a blockchain, the high fees per transaction have propelled constant function AMMs as the go-to mechanism for on-chain decentralized exchange.

With in CFMMs, a trading function can be modified to have a constant weighted geometric mean before and after are trade. Instead of uniformly distributing across reserves, a new invariant could allow differentiated weights on multiple assets. The function

$$\varphi(R) = \prod_{i=1}^{n} R_i^{w_i} \tag{2}$$

is a geometric mean trading function by adding a weight vector $w_1(t), ..., w_n(t))$ where $w_i(t) \geq 0$ assuming it satisfies

$$\sum_{i=1}^{n} w_1(t) = 1 \tag{3}$$

This is useful for customizing LP exposure to a portfolio of reserves R via continuous rebalancing to a predefined set of weights and can even replicate a desired payoff as Evans et al (2020) showed in *Lquidity Provider Returns in Geometric Mean Markets*.

2 Nested AMMs

To expand to a broader spectrum of related tokens and multiasset trades, a nested AMM was conceived. The AMM invariant that allows for functions within a function was inspired by constant elasticity substitution (CES) utility functions and formalized by Zhang et al. (2021). The CES utility function refers to the aggregate function of more than two utility functions. The function exhibits constant elasticity of substitution when the nested functions have a ratio indicating the proportional change of price to quantities to be the same.

$$Q = F * (a * K^{\rho} + (1-a) * L^{\rho})^{\frac{\nu}{\rho}} \tag{4}$$

Applying this to a trading function φ for multi-token trades $(\Delta_a, \Delta_b, \Delta_y)$ the invariant

$$\varphi(a, b, y) = (a^{1-\sigma} + b^{1-\sigma})^{\frac{1}{1-\sigma}})^{1-\eta} + y^{1-n} \tag{5}$$

In this case, the substitution parameter ρ from the CES utility function is equal to the σ and η incdicating the slippage between the related nested pairs and the non-related token. The nest pair (a,b) are a composite product as the slippage, σ should be close to 0 increasing their substitution elasticity, meaning that if $\sigma = 0$, then a and b would be perfect substitutes.

2.1 Adding More Tokens to a Nest

To ensure deep liquidity there will be more multiple tokens pointing to the same underlier in a nest, $\varphi(x_1...x_n, y)$ which is able by generalizing the G3M to

$$((\sum_{i=1}^{n} w_i {x_i}^{1-\sigma})^{\frac{1}{1-\sigma}})^{1-\eta} + \omega_y y^{1-\eta} \tag{6}$$

Where each token x within a nest can be weighted w_i and then sum of all those weighted tokens are taken. Thereby making them more sensitive each tokens real degree of volatility as well.

3 Conclusion

There are variety of benefits of a nested AMM implementation. For one it reduces liquidity fragmentation across multiple related tokens denominated by a stable and allows for customized LP exposure to a portfolio of tokens.

3.1 Swapping

Take a $cUSD <> WETH$ pair on Celo for example and a $cUSD <> aaETH$ pair for example. Both are the same asset, but since they pass via different bridges are not fungible between each other due to the the differentiated token representations. In this scenario, instead of providing scarce cUSD liquidity to two separate pools to trade the same assets. These two assets with low slippage can be nested and be anchored by the same stable token. This pool looks like $cUSD <> WETH <> aaETH$ Thereby, allowing for $cUSD <> aaETH$ trades and $cUSD <> WETH$ trades utilizing the same liquidity pool. Offering superior prices to generic two-token constant product curves.

References

1. Anthony Zhang: Nested AMMs. University of Chicago. 2021.
2. Alex Evan: Optimal Fees for Geometric Market Makers. Placeholder VC. 2020.
3. Michael Egorov: Stableswap - efficient mechanism for Stablecoin liquidity. Curve Finance. 2019.