Mini-Teste 2 2A 2022

① $u(x,y) = y\,a(x,y)$ e $v(x,y) = x\,b(x,y)$

$f'(z_0) = u_x(x_0,y_0) + i\,v_x(x_0,y_0) =$

$= y_0 a_x(x_0,y_0) + i\left(b_x(x_0,y_0) + x_0 b_x(x_0,y_0)\right)$

CA

$u_x(x_0,y_0) = 0 + y\,a_x(x_0,y_0)$

$v_x(x_0,y_0) = b_x(x_0,y_0) + x_0 b(x_0,y_0)$

D

② $\displaystyle\sum_{n=1}^{+\infty} \frac{(z-2022)^n}{n^{2023}}$

$\lambda = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(z-2022)^n}{n^{2023}}\right|} = \lim_n \frac{|z-2023|}{n^{\left(\frac{2023}{n}\right)}} = |z-2023|$

A série é absolutamente convergente se $|z-2023|<1$, logo, $R=1$

A

③ $f(z) = \displaystyle\sum_{n=2}^{+\infty} (-1)^n \frac{2^{3n-3}}{(3n-4)!} z^{3n-3}$, $z \in \mathbb{C}$

CA

$2022 = 3n-3 \Leftrightarrow$

$\Leftrightarrow \frac{2025}{3} = n \Leftrightarrow$

$\Leftrightarrow n = 675$

$2025 \div 3$: 22, 15, 0 → 675

$\frac{f^{(2022)}(0)}{2022!} = \frac{2^{3(675)-3}}{(3(675)-4)!}(-1)^{675} \Leftrightarrow$

$\Leftrightarrow f^{(2022)}(0) = -\frac{2^{2022}\cdot 2022!}{2021!} = -2^{2022}\cdot 2022$

C

④ $f(z) = \frac{-1}{z^5(z+1)}$, $z \in D = \{z \in \mathbb{C} : 0<|z|<1\}$

$= -\frac{1}{z^5}\cdot\frac{1}{1-(-z)} = \frac{-1}{z^5}\displaystyle\sum_{n=0}^{+\infty}\underline{(-z)^n} = -1\sum_{n=0}^{+\infty}(-1)^n (z)^{n-5} = \sum_{n=0}^{+\infty}(-z)^{n-5}$

Negativo quando n mod 2 = 0

B

⑤ $f_t(z) = \frac{e^{tz}}{(z-1)^2(2-z)}$, $z \in \mathbb{C}\setminus\{1,2\}$ ($e^{tz} \to p(z)$, $(z-1)^2(2-z) \to q(z)$)

CA $p(z) = e^{tz}$, $p(1) = e^t$

$q(z) = (z-1)^2$, $q(1) = 0$

$q'(z) = 2(z-1)$, $q'(1) = 0$

$q''(z) = 2$, $q(1) = 2 \neq 0$

Polo de ordem $2-0=2$

$\frac{d}{dz}\left(\frac{e^{tz}}{2-z}\right) = \frac{te^{tz}(2-z)+e^{tz}}{(2-z)^2}$

$\operatorname{Res}_{z=1} f = \frac{1}{(2-1)!}\lim_{z\to1}\frac{d^1}{dz^1}\left((z-1)^2\frac{e^{tz}}{(z-1)^2(2-z)}\right) =$

$= \cancel{\lim_{z\to1}\frac{e^{tz}}{2-z} = e^{tz}}$

$= \lim_{z\to1}\frac{te^{tz}(2-z)+e^{tz}}{(2-z)^2} = te^t + e^t$

A