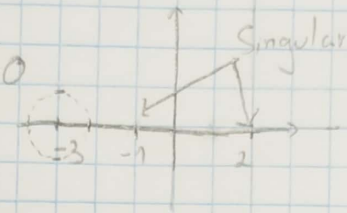
$$\text{Comp}\,\gamma = \int_\gamma |\gamma'(t)|\,dt$$

Mini-teste 4A 2013

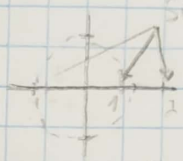
① $\oint_\gamma \frac{1}{(z-1)(z+2)^3}\,dz = 0$



(B)

O caminho não abrange as singularidades

② $\oint_\gamma \frac{1}{(z-1)(z+2)^3}\,dz = \oint_\gamma \frac{p(z)}{q(z)}\,dz =$



$\text{Sing} = \{-1, 2\}$

CA

Ordem do pólo $z = 1$

$p(z) = 1 \neq 0$

$q(z) = (z-1)\,q(1) = 0$

$q'(z) = 1 \neq 0$

pólo de ordem $1 = 0+1$

$$= 2\pi i \sum_{k \in \{\text{Sing}\}} \operatorname*{Res}_{z=k} f =$$

$$= 2\pi i \operatorname*{Res}_{z=-1} f = 2\pi i \frac{1}{(1-1)!} \lim_{z\to 1} \frac{d^0}{dz^0}\left[(z-1) f(z)\right] =$$

$$= 2\pi i \lim_{z\to -1} \frac{1}{(z+2)^3} = 2\pi i \frac{1}{27}$$

(E)

③ $\int_0^{+\infty} \frac{1}{(x^2+1)^2}\,dx = \frac{x}{2(x^2+1)} + \frac{1}{2}\int_0^{+\infty} \frac{1}{x^2+1}\,dx = \frac{x}{2(x^2+1)} + \frac{1}{2}\arctan(x)\Big|_0^{+\infty} =$

↑ Redução fórmula

$$= \frac{1}{2}\left[0 + \frac{\pi}{2} - 0 - 0\right] = \frac{\pi}{4}$$

(C)

CA

$\lim_{x\to+\infty} \frac{x}{x^2+1} = 0$

$\lim_{x\to+\infty} \arctan(x) = \frac{\pi}{2}$