

# Exercício 1 (Exemplo de Questão)

$$\textcircled{1} \quad x(t) = 2 + 3 \sin(t) \cos(2t) + 5 \sin(4t) - \cos(3t)$$

$\uparrow$   
0

$\uparrow$   
1

$\uparrow$   
4

$\uparrow$   
3

R:  $\omega \in \{0, 1, 3, 4\}$

$$\textcircled{2} \quad \omega \in \{8\pi, 16\pi, 20\pi\}$$

$$\omega_0 = \text{mdc}(\omega) = 4\pi$$

$$\omega_0 = \frac{2\pi}{T_0} \Rightarrow T_0 = 2\pi/\omega_0 = 2\pi/4\pi = 1/2$$

$$\textcircled{3} \quad x(t) = 4 \cos(2t)^2 = 4 \left[ \frac{1 + \cos(4t)}{2} \right] = 2 + 2 \cos(4t)$$

$$y(t) = 2 + 2 \cos(4t)$$

$$\textcircled{4} \quad x(t) = 2 \cos(4\pi t)$$

$$E = \int_{-2}^2 |x(t)|^2 dt = \int_{-2}^2 |4 \cos(4\pi t)|^2 dt \quad \begin{matrix} \leftarrow \text{Energia finita} \\ \text{Potência nula} \end{matrix}$$

$$\textcircled{5} \quad x(t) = 2 + 4 \sin(3t) \quad x(-t) = 2 + 4 \sin(-3t) = 2 - 4 \sin(3t)$$

$\uparrow$  Nem par nem ímpar

$$x(t) = 4 \sin(3t)^2 \quad x(-t) = 4 \sin(-3t)^2 = 4 \sin(3t)^2$$

$\uparrow$  Par

$$x(t) = 2 + 4 \cos(2t) \quad x(-t) = 2 + 4 \cos(-2t) = 2 + 4 \cos(2t)$$

$\uparrow$  Par

$$x(t) = 4 \sin(3t) \cos(2t) \quad x(-t) = -4 \sin(3t) \cos(2t)$$

$\uparrow$  Ímpar

$$\textcircled{6} \quad x[n] = 2n (\underbrace{\mu[n-1] - \mu[n-3]}_{\text{Valor 1 quando } n=1, n=2} + \underbrace{\delta[n-4]}_{\text{Valor 1 quando } n=4})$$

$$E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \sum_{n=1}^4 |x[n]|^2 = 2^2 + 4^2 + 8^2 = 84 \text{ J}$$

$$\textcircled{7} \quad x(t) = t^2 - 4 \quad [-4, -2, 0, 2, 4]$$

$$E = \int_{-4}^4 |t^2 - 4|^2 dt \approx \frac{2}{3} \left[ |x(-4)|^2 + |x(4)|^2 + 2(|x(0)|^2) + 4(|x(-2)|^2 + |x(2)|^2) \right] =$$

$$= \frac{2}{3} [144 + 144 + 32] = \frac{640}{3} \text{ J}$$

$$E \approx \frac{640}{3} \text{ J}$$

$$\textcircled{8} \quad x[n] = 2n (\mu[n+1] - \mu[n-6])$$

$$y[n] = 2[an-b] = x[3n+2] = 2(3n+2) [\mu[3n+3] - \mu[3n-4]] \\ = (6n+4) [\mu[3n+3] - \mu[3n-4]]$$

⑨

$$y[n] = (n+2)x[n-1] + 2x[n-3] \rightarrow \text{Linear}$$

$$T\{x_1[n]\} = (n+2)x_1[n-1] + 2x_1[n-3]$$

$$+ \{x_2[n]\} = (n+2)x_2[n-1] + 2x_2[n-3]$$

$$y_c[n] = (n+2)[ax_1[n-1] + bx_2[n-1]] + 2[ax_1[n-3] + bx_2[n-3]]$$

(=, logo linear)

$$y_c[n] = a(n+2)x_1[n-1] + 2ax_1[n-3] + b(n+2)x_2[n-1] + 2bx_2[n-3]$$

$$= (n+2)[ax_1[n-1] + bx_2[n-1]] + 2[ax_1[n-3] + bx_2[n-3]]$$

(...)

$$y[n] = 2x[n-1] - 3x[n+4] \rightarrow \text{Linear}$$

$$T\{x_1[n]\} = 2x_1[n-1] - 3x_1[n+4]$$

$$+ \{x_2[n]\} = 2x_2[n-1] - 3x_2[n+4]$$

$$y_c[n] = 2[ax_1[n-1] + bx_2[n-1]] - 3[ax_1[n+4] + bx_2[n+4]]$$

$$y_c[n] = a[2x_1[n-1] - 3x_1[n+4]] + b[2x_2[n-1] - 3x_2[n+4]]$$

$$= 2[ax_1[n-1] - bx_2[n-1]] - 3[x_1[n+4] + x_2[n+4]]$$

$$y[n] = 2(n+1)x[n-1]x[n-4] \rightarrow \text{N\~ao Linear}$$

$$y_1[n] = 2(n+1)x_1[n-1]x_1[n-4]$$

$$y_2[n] = 2(n+1)x_2[n-1]x_2[n-4]$$

$$y_c[n] = 2(n+1)[ax_1[n-1] + bx_2[n-1]][ax_1[n-4] + bx_2[n-4]]$$

$$y_{c_1}[n] = 2a(n+1)x_1[n-1]x_1[n-4] + 2b(n+1)x_2[n-1]x_2[n-4]$$

Resposta: a) Linear, Variante no tempo, causal

b) Linear, N\~ao variante no tempo, n\~ao causal

c) N\~ao linear, Variante no tempo, causal

$$(10) y[n] = 3\delta[n-1] - \delta[n-2] + 2\delta[n-3]$$

$$y[3] = 2$$

$$\begin{aligned} (11) \sum_{k=0}^2 x[k]h[2-k] &= x[0]h[2] + x[1]h[1] + \cancel{x[2]h[0]} \\ &= 1 \times (-2) + 3 \times 3 \\ &= 9 - 2 = 7 \end{aligned}$$

(12)

$$G(z) = \frac{-0,3z^{-3} + 1,9z^{-4}}{(1-0,5z^{-1})(1+0,6z^{-1})} =$$

$$= \frac{-0,3z^{-3} + 1,9z^{-4}}{1+0,6z^{-1}-0,5z^{-1}-0,3z^{-2}} = \frac{-0,3z^{-3} + 1,9z^{-4}}{1+0,1z^{-1}-0,3z^{-2}} \cdot \frac{z^4}{z^4}$$

$$= \frac{-0,3z + 1,9}{z^4 + 0,1z^3 - 0,3z^2} \quad \left| \begin{array}{l} \text{zeros:} \\ z = 19/3 \end{array} \right| \quad \left| \begin{array}{l} \text{poles} \\ z^2(z^2 + 0,1z - 0,3) = 0 \end{array} \right|$$

Estável //

1 zero 4 pólos //

$$\hookrightarrow z=0 \text{ duplo } z = \frac{-0,1 \pm \sqrt{0,01 + 1,2}}{2}$$

$$z = \frac{-0,1 + 1,1}{2} \quad z = \frac{-0,1 - 1,1}{2}$$

$$= 0,5 \vee -0,6$$

0,35 //

$$G(1) = \frac{-0,3 + 1,9}{(1-0,5)(1+0,6)} = \frac{1,6}{0,8} = 2 //$$

$$(13) \quad G(z) = \frac{0,4z^{-3}}{1-0,8z^{-1}} = \frac{0,4}{1-0,8z^{-1}} \times z^{-3} = H(z) \quad \left| \begin{array}{l} \text{zeros} \\ \text{poles} \end{array} \right|$$

$$z^{-1} \left( \begin{array}{l} g[n] = 0,4 \times 0,8^{n-3} u[n-3] = h[n] \end{array} \right)$$



$$\textcircled{14} \quad \begin{cases} y[n] = 0,5 x[n-1] + 0,3 x[n-3] + 1,1 y[n-1] - 0,3 y[n-2] \\ x[n] = 5 u[n-2] - 2 \delta[n-5] \end{cases}$$

$$Y(z) = 0,5 z^{-1} X(z) + 0,3 z^{-3} X(z) + 1,1 z^{-1} Y(z) - 0,3 z^{-2} Y(z)$$

$$\Leftrightarrow Y(z) [1 - 1,1 z^{-1} + 0,3 z^{-2}] = X(z) [0,5 z^{-1} + 0,3 z^{-3}]$$

$$\Leftrightarrow \frac{Y(z)}{X(z)} = \frac{0,5 z^{-1} + 0,3 z^{-3}}{1 - 1,1 z^{-1} + 0,3 z^{-2}} = G(z) = H(z) \Big|_{\text{circuitos}}$$

$$X(z) = \frac{5 z^{-2}}{1 - z^{-1}} - 2 z^{-5}$$

$$Y(z) = H(z) X(z) = \left[ \frac{0,5 z^{-1} + 0,3 z^{-3}}{1 - 1,1 z^{-1} + 0,3 z^{-2}} \right] \left[ \frac{5 z^{-2}}{1 - z^{-1}} - 2 z^{-5} \right]$$

$$\lim_{n \rightarrow \infty} y[n] = \lim_{z \rightarrow 1} (1 - z^{-1}) Y(z) =$$

$$= \lim_{z \rightarrow 1} \left[ \frac{5 z^{-2}}{\cancel{(1 - z^{-1})}} \times \cancel{(1 - z^{-1})} \times \frac{0,5 z^{-1} + 0,3 z^{-3}}{1 - 1,1 z^{-1} + 0,3 z^{-2}} \right]$$

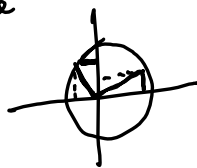
$$= 5 \times 4 = 20$$

(9) -  $\Omega = 3 \text{ rad}$   $H(3) = 3j = 3 e^{j\pi/2}$

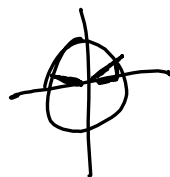
$x[n] = 2 \sin[3n]$   $|H(3)| = 3$

$x[n] = e^{i\Omega n}$

$x[n] = \underset{\Omega_i}{2} \cos[\underset{\theta_i}{3n + \pi/2}]$



$y[n] = 2 |H(3)| \cos(3n - \pi/2 + \angle H(3))$   
 $= 6 \cos(3n)$



(16)  $u(\sin(3t+1))^2 = 4 \left[ \frac{1 - \cos(6t+2)}{2} \right] =$

$= 2 - 2 \cos(6t+2)$   $\omega_0 = 6$   $m=0$  &  $m=1$

$x(t) = 2 \cos(5t) + \sin(5t-1) =$   
 $= 2 \cos(5t) + \cos(5t-1-\pi/2)$

$m=1$

$x(t) = 4 \sin(6t) \cos(9t-6) =$   
 $= 4 \cdot \frac{1}{2} [\cos(6t-9t+6) - \cos(6t+9t-6)]$

$= 2 [\cos(6-3t) - \cos(15t-6)]$

$= 2 \cos(6-3t) - 2 \cos(15t-6)$

$m=1 \wedge m=5$   $\omega_0 = 3$

(H)

$x(t) = 1 + \cos(5t-1)$   $\omega_0 = 5$   
 $m=0$  &  $m=1$

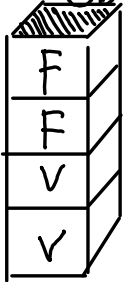
R: C  
A  
H  
C

(17)

$$C_3 = 2|C_3| \quad \theta_3 = -\angle C_3$$

$$C_4 = 2|C_4|$$

$$C_0 = |C_0|$$



(18)

$$5\omega_0 = 100\pi \Rightarrow \omega_0 = \frac{100\pi}{5} = 20\pi \text{ Rad/s}$$

$$\omega_2 = 2\omega_0 = 40\pi \text{ Rad/s} \quad \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = 20 \text{ Hz}$$

$$\omega_5 = 100\pi \quad \omega_5 = 2\pi f_5 \Rightarrow f_5 = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

(19) Verdadero