Miniteste 2 R - 2022

1.

$$f(z) = y\,a(x,y) + i\,x\,b(x,y)$$

$$f'(z_0) = y_0\,a_x(x_0,y_0) + i\left(b(x_0,y_0) + x_0\,b_x(x_0,y_0)\right) \quad \text{(B)}$$

2.

$$\sum_{n=1}^{+\infty} \frac{(z+i)^n}{n^2 6^n} \qquad \lambda = \lim_n \sqrt[n]{\left|\frac{(z+i)^n}{n^2 6^n}\right|} = \lim_n \left|\frac{z+i}{n^{2/n}\,6}\right| = \frac{|z+i|}{6}$$

$$\lambda < 1 \Leftrightarrow |z+i| < 6 = R \quad \text{(C)}$$

3.

$$f(z) = \sum_{n=0}^{+\infty} (-1)^n \frac{2^{2n}}{(2n)!} z^{2n+1}, \quad z \in \mathbb{C}$$

$$21 = 2n+1 \Leftrightarrow n = 10$$

$$\frac{f^{(21)}(0)}{21!} = (-1)^{10}\frac{2^{20}}{20!} \Leftrightarrow 2^{20}\cdot 21 \quad \text{(E)}$$

4.

$$f(z) = \frac{1}{z^7(z+1)}, \quad z \in D = \{z \in \mathbb{C} : 0<|z|<1\}$$

$$= \frac{1}{z^7}\,\frac{1}{1-(-z)} = \frac{1}{z^7}\sum_{n=0}^{+\infty}(-z)^n = \sum_{n=0}^{+\infty}(-1)^n z^{n-7} = \sum_{n=0}^{+\infty}(-1)^{n-6} z^{n-7} \quad \text{(B)}$$

5.

$$f_t(z) = \frac{e^{-tz} \rightarrow p(z)}{(1-z)^2(z+1) \rightarrow q(z)}$$

$$p(z) = e^{-tz}, \quad p(1) = e^{-t}$$

$$\left.\begin{aligned} q(z) &= (1-z)^2, \quad q(1) = 0 \\ q'(z) &= -2(1-z), \quad q'(1) = 0 \\ q''(z) &= 2 \neq 0 \end{aligned}\right\} \text{Polo ordem } 2-0=2$$

$$\operatorname*{Res}_{z=1} f(z) = \lim_{z\to 1}\left[\frac{d}{dz}\left(\frac{e^{-tz}}{z+1}\right)\right] =$$

$$= \lim_{z\to 1}\frac{-te^{-tz}(z+1) - e^{-tz}}{(z+1)^2} = \frac{-te^{-t}2 - e^{-t}}{4} = -\frac{1}{4}e^{-t}(2t+1) \quad \text{(B)}$$