① $|z|=2$

$$||1|-|z^2|| \le |1-z^2| \le |1|+|z^2| \Leftrightarrow$$

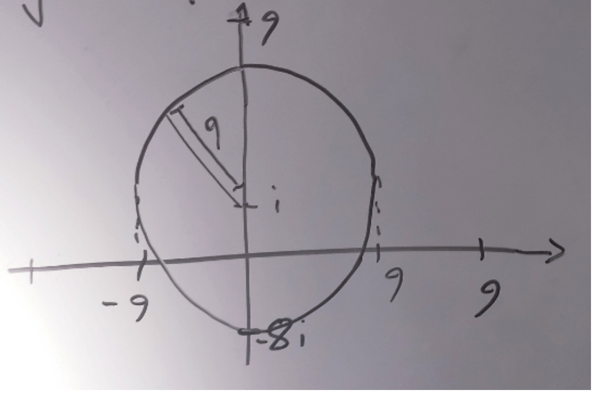
$$\Leftrightarrow |1-2^2| \le |1-z^2| \le 1+2^2 \Leftrightarrow$$

$$\Leftrightarrow 3 \le |1-z^2| \le 5$$

② $|\bar{z}+i| \le 9$

se $z = x+iy$, $x,y \in \mathbb{R}$

$$\Rightarrow \sqrt{x^2+(y+i)^2} \le 9$$



③ $z^3+27=0 \Leftrightarrow$

$\Leftrightarrow z^3=-27$

$z^3=\rho^3 e^{i\theta 3}$

$-27=27e^{i\pi}$

$$z^3=-27 \Leftrightarrow \begin{cases} \rho^3 = 27 \\ \theta 3 = \pi + 2\pi K \end{cases} \Leftrightarrow \begin{cases} \rho = 3 \\ \theta = \dfrac{\pi+2\pi K}{3} \end{cases}$$

$$\sqrt[3]{27} = 3e^{i\left(\frac{\pi+2\pi K}{3}\right)} \quad ; K \in \{0,1,2\}$$

$$K=0 \longrightarrow 3e^{i\frac{\pi}{3}} = \frac{3}{2} + i\frac{3\sqrt{3}}{2} = \frac{3}{2}(1+\sqrt{3}i)$$

$$K=1 \longrightarrow 3e^{i\pi} = -3$$

$$K=2 \longrightarrow 3e^{i\frac{5\pi}{3}} = 3e^{i\left(-\frac{\pi}{3}\right)} = \frac{3}{2}(1-i\sqrt{3})$$

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(4) $w = \frac{(1+i)^4}{1-i}$; $z = x + iy$

$e^z = w \Leftrightarrow e^x e^{iy} = e^{i\frac{5\pi}{4}} \Leftrightarrow$

CA

$w = \frac{\left(\sqrt{2}\, e^{i\frac{\pi}{4}}\right)^4}{\sqrt{2}\, e^{-i\frac{\pi}{4}}} = \frac{4\sqrt{2}}{2} e^{i\left(\pi + \frac{\pi}{4}\right)} = 2\sqrt{2} e^{i\left(\frac{5\pi}{4}\right)}$

$z = \ln|w| + i\arg(w) = \ln(2\sqrt{2}) + i\arg(z) = \ln(2) + \ln(\sqrt{2}) + \left(\frac{-3\pi}{4} - 2\pi K\right)i,\ K \in Z$

$= \ln(2) + \frac{1}{2}\ln(2) + i\left(\frac{-3}{4} - 2K\right)\pi,\ K \in Z = \boxed{\frac{3}{2}\ln(2) - i\left(\frac{3}{4} + 2K\right)\pi,\ K \in Z}$

$\beta = 2 + i$

$\alpha = \rho_\alpha e^{i\left(0 + \frac{\pi}{4}\right)}$

$\rho_\alpha 4 = 6 \Leftrightarrow \rho_\alpha = \frac{3}{2}$

$\alpha = \frac{3}{2} e^{i\frac{\pi}{4}} = \frac{3}{2\sqrt{2}} e^{i\frac{\pi}{4}} = \frac{3\sqrt{2}}{4} e^{i\frac{\pi}{4}}$

$f(z) = \left(\frac{3\sqrt{2}}{4} e^{i\frac{\pi}{4}}\right) z + 2 + i$