

**Exercise 1** Show by induction:

- a)  $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- b) For any  $n \geq 7$ , we have that  $n! > 3^n$

**Exercise 2** Show that the following recursive algorithm computes correctly the factorial of a number. Assume that  $n \geq 0$ .

**Function**  $F(n)$

```
if  $n = 0$  then
   $s = 1$ 
else
   $s = n \cdot F(n - 1)$ 
return  $s$ 
```

**Exercise 3** Show that the following insertion sort algorithm is able to sort a list  $A$  of  $n$  numbers in nondecreasing order.

**Function**  $IS(n, A)$

```
if  $n \geq 2$  then
   $IS(n - 1, A)$ 
   $i = n - 1$ 
  while  $i \geq 1$  and  $A[n] < A[i]$  do
     $i = i - 1$ 
   $i = i + 1$ 
   $p = A[n]$ 
   $j = n - 1$ 
  while  $j \geq i$  do
     $A[j + 1] = A[j]$ 
     $j = j - 1$ 
   $A[i] = p$ 
```

**Exercise 4** Read the problem *A new chess game* in Mooshak. Consider a recursive approach to solve it.