Aprendizagem Computacional em Biologia
Inteligência Geoespacial
Reconhecimento de Padrões

2017/2018

Exame Normal 20 June 2023 Duration: 2h30

Name: **Number:**

WARNING/AVISO

The Exam has a duration of 2h30m. The test is composed by five questions. The last question is a practical question. Each question must be answered in the framed box below (and following) it. Questions may be answered in Portuguese or English. This is a closed book test. You may use only 1 A4 manuscript with your 'own' notes. You are allowed to use a calculator machine. Violation of the rules ends up with exam cancellation, course failure and eventually you may be subject to disciplinary procedure. If you have any questions, you may ask. Good Luck!

Question	pts	Results	Graded by:
1)	20		
2)	20		
3)	20		
4)	25		
5)	15		

Graded by:

Question 1 - PCA

☐ **20pts**

Consider the datasets in Figure 1.

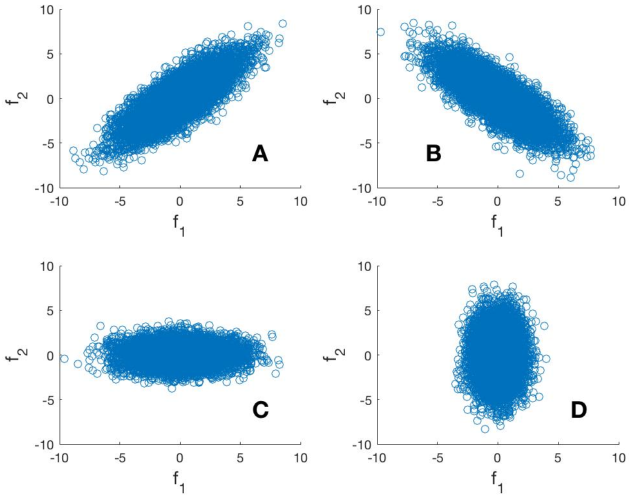

Figure 1: Data Distributions

a) Relate, justifying your choices, the following covariance matrices with each one of the data distributions:

$$C_1 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

b) What is the variance of f_1 and f_2 in each case? Justify why.

c) If PCA is applied what will be the eigenvectors for each dataset? Justify.

Your answer to 1):

Cont. your answer to 1):

Question 2 - Minimum Distance Classifiers (MDC)

☐ **20 pts**

(a) Describe how a MDC works, in which consists its training, and given new patterns how they are labeled.

(b) Given the data available in Figure 2:

- Develop a Mahalanobis MDC for the data. To which class the classifier will assign the pattern $\mathbf{x} = [11 \ \ 0.5]^T$?
- Compute the decision boundary $d_{12}(\mathbf{x})$ and an equation for the separation hyperplane.
- Explain why for this dataset a Mahalanobis MDC is equal to an Euclidean MDC? Justify.

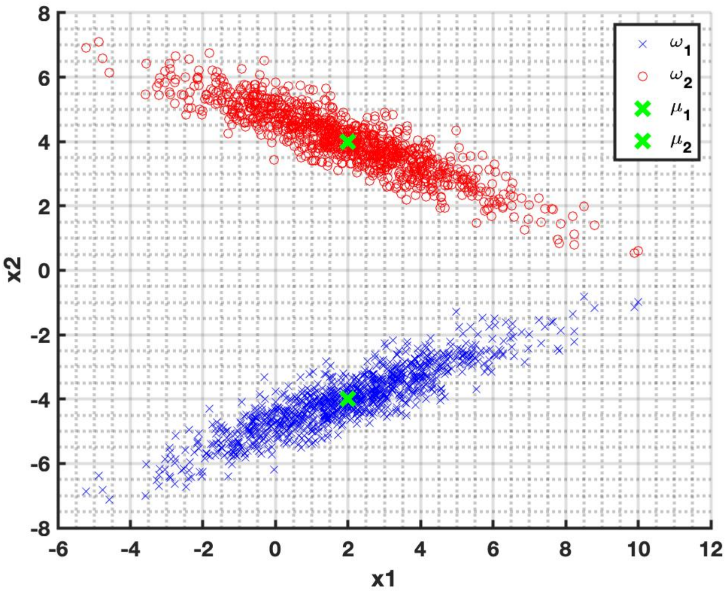

Figure 2: Binary classification problem described by features x1 and x2

Calculus support:

Mean vectors: $\mu_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Covariance matrices: $\mathbf{C}_1 = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ $\quad \mathbf{C}_2 = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

Inverse covariance matrix: $\mathbf{C}^{-1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$

Your answer 2):

Cont. your answer to 2):

Cont. your answer to 2):

Question 3 - Multivariate Normal Bayesian Classification

☐ **20 pts** Consider the conditional probability density functions in Figure 3 that are generated by a multivariate Gaussian process. The mean vectors for both classes are: $\mu_1 = [-1 \quad -1]^T$ and $\mu_2 = [1 \quad 1]^T$. The covariance matrix is the identity matrix for both classes. Consider also that patterns from ω_1 and ω_2 appears in equal prevalence.

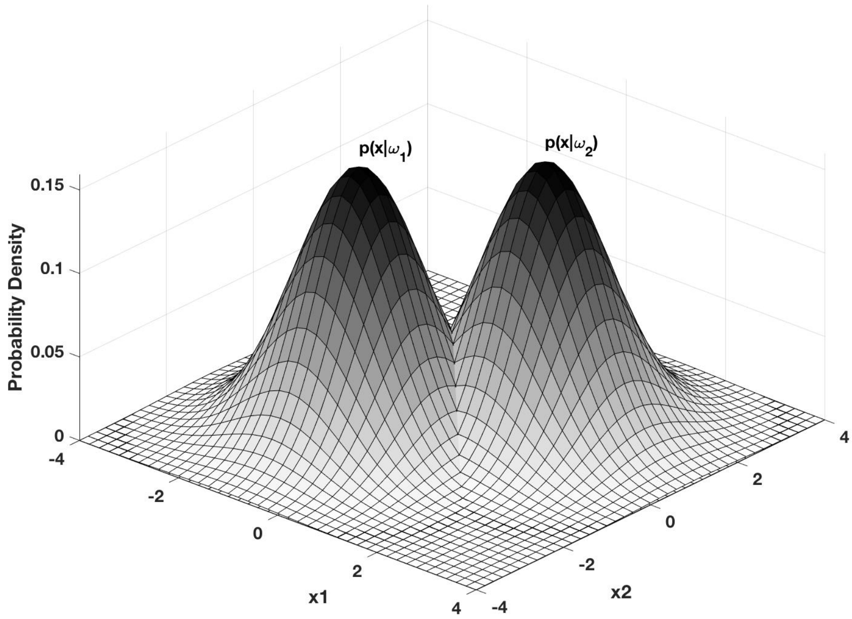

Figure 3: Conditional probability density functions defined in a space spanned by features x1 and x2.

a) What is the problem dimensionality? Justify.
b) Starting with the Bayes rule develop the Likelihood Ratio Test (LRT).
c) Develop the generic decision function of the form $d_{12}(\mathbf{x}) = \mathbf{w}^T\mathbf{x} + w_0$. Specify all the steps.
d)Specify $d_{12}(\mathbf{x})$ for the concrete values of this exercise. What is the relation between this classifier and an Euclidean MDC? Justify.

Your answer to 3):

Cont. your answer to 3):

Cont. your answer to 3):

Question 4 - Support Vector Machines

☐ **25 pts**

Consider a linear SVM trained for a binary problem, where ω_1 is the positive class.The training results are listed in the following table:

Lagrange Multipliers	1	1
Support Vectors	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 3 \\ 1 \end{bmatrix}$
Class	ω_1	ω_2

(a) Describe how SVMs are trained.
(b) Compute the decision function $d(\mathbf{x})$.
(c) Could the sample $\mathbf{x} = [3 \;\; 3]^T$ be a support vector? If yes, what is the class to which it bellongs? Justify.
(d) What is the accuracy of the following testing patterns $\mathbf{x} = [x1 \;\; x2]^T$ are considered?

$x1$	2	3	0
$x2$	0	0	3
True class(ω_k)	1	2	1

Your answer 4):

Cont. your answer to 4):

Cont. your answer to 4):

Question 5 - kNN classifiers

□ **15 pts**

Write two functions to training and testing kNN classifiers. In Matlab the functions should have the following prototypes:

- **function model=kNN_training(Xtr,Ttr,k)**

- **function** [ss,sp]=**kNN_testing(Xte,Tte,model)**

Where:

- **model** is a structure with fields that contain the kNN parameters needed for testing;
- **Xtr** is a matrix with dimensions D x Ptr, being D the problem dimensionality and Ptr the number of samples in the training data;
- **Ttr** is the target vector in the training with dimensiona 1 x Ptr, and with "1" labeling positive patterns and "2" labeling negative patterns;
- **k** is the number of neighbors ;
- **Xte** is a matrix with dimensions D x Pte, being D the problem dimensionality and Pte the number of samples in the testing data;
- **Tte** is the target vector in the testing data with dimensiona 1 x Pte, and with "1" labeling positive patterns and "2" labeling negative patterns;
- **ss** is the sensitivity on the testing data;
- **sp** is the specificity on the testing data;

Important note: You are advised to not use any built-in function from STPRtool, Matlab (e.g. knnrule.m, knnclass.m, fitcknn.m and predict.m) or any built-in function from any other language.

Help:

Consider that you have available the following functions:

IDX = knnsearch(X,Y,k): Finds the k nearest neighbor in X for each point in Y by considering the Euclidean distance. X is an MX-by-N matrix and Y is an MY-by-N matrix. Rows of X and Y correspond to patterns and columns correspond to features. IDX is matrix with dimensions MY-by-k. Each row in IDX contains the indexes of the k nearest neighbor in X for the corresponding row in Y. If Y is a single pattern, IDX will be a vector with dimensions 1-by-k.

S = struct('field1',VALUES1,'field2',VALUES2,...): Creates a structure with the specified fields and values.
For example St=struct('Vector',[3;4],'Matrix',[1 2;2 1]) creates a structure with two fields one containing a vector and other a matrix. For instance, the matrix can be obtained by doing St.Matrix.

I = find(X): Returns the linear indices corresponding to the nonzero entries of the array X. X may be a logical expression.

Your answer to 5):

Cont. your answer to 5):