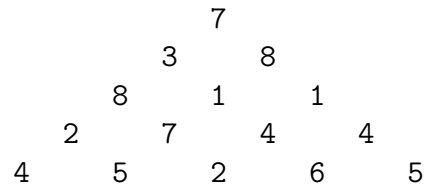


Exercise 1 Consider the following problem: Given a "pyramid" of positive integer numbers, as shown in the figure below, find the path from top to bottom with the largest sum of the numbers belonging to the path. It is only allowed to turn left or right.

- a) Sketch the pseudo-code of a top-down dynamic programming approach (with memoizing) that solves the problem, indicating the subproblem, recurrence relation and basis cases. Show that your approach is correct by using the optimal substructure property, and discuss its time complexity.



- b) Sketch the pseudo-code of a bottom-up dynamic programming approach. Is it possible to improve the space complexity if there is no need to reconstruct the optimal path?

Exercise 2 Given a square matrix of size $n \times n$ find the number of monotonic paths from position $(1, 1)$ to (n, n) . In a monotonic path, one can only turn down or right. Discuss a dynamic programming approach to the problem as well as its time complexity.

Exercise 3 Read the problem *Radical Winter Games* in EA2025_PL in Mooshak and solve it using the techniques discussed above.