EN 2019

$$d_{12}(x) = w^T x + w_0$$

① 1) $d_{12}(x) = (m_1 - m_2)^T (x - 0,5(m_1 + m_2))$

$$= (-0,47 \quad -0,12) \begin{pmatrix} x_1 - 1,205 \\ x_2 - 0,96 \end{pmatrix} =$$

$$= -0,47 x_1 + 0,566 - 0,12 x_2 + 0,115 =$$

$$= -0,47 x_1 - 0,12 x_2 + 0,681$$

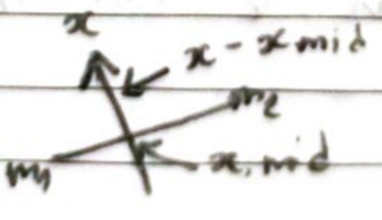
2) Decision hyperplane $d_{12}(x) = 0$

$x_{mid} = 0,5(m_1 + m_2)$ is the mid point between the segment, so we can write the decision boundary as:

$$d_{12}(x) = (m_1 - m_2)(x - x_{mid}) = 0$$

the dot product of two vectors is only 0 if they are orthogonal.

$(m_1 - m_2)$ is the vector connecting the means, so the condition only holds when $\underbrace{(x - x_{mid})}_{\text{vector connecting midpoint to any } x \text{ on the plane}}$ is orthogonal with that vector.



vector connecting midpoint to any x on the plane

3)

$d_{12}\left(\begin{pmatrix}1\\1\end{pmatrix}\right) = -0,47 - 0,12 + 0,681 = 0,091$

Positive, classify as ω_1 (apples)

(2)

(a) Map data into a higher-dimensional space. $\phi(x)$ projects x into a feature space. this can be done through a kernel function $K(x,y)$ that computes the dot product of the data points in that feature space without explicitly calculating the transformation.

(b) In this case, given $\phi(x) = \begin{pmatrix}x_1^2\\x_2\end{pmatrix}$, the Kernel is:

$$K(x,y) = \phi(x)^T \phi(y)$$
$$K(x,y) = (x_1^2, x_2)(y_1^2\ y_2)^T$$
$$K(x,y) = x_1^2 y_1^2 + x_2 x_2$$

(c) 1- Transform space (every point's x_1 value is squared.

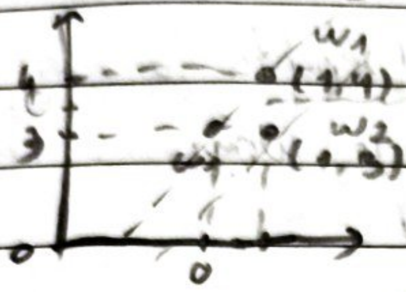
2- Find SVs (points from diff classes that are closest to each other) in the transformed space)

Possible: $(1,4)\ \omega_1$ and $(1,3)\ \omega_2$

$(0,3)\ \omega_1$ and $(1,3)\ \omega_2$

3- Develop decision function ($w_1 z_1 + w_2 z_2 + b = 0$)

Such that it separates these points with max margin.



Let's try a line of slope 1 that passes in the midpoint between (1,4) and (1,3) (which is (1, 3,5)).

$w_2 z_2 = w_1 z_1 + b$ with slope 1:

$$\begin{cases} 3,5 = 1 + b \Rightarrow b = 2,5 \\ z_2 = z_1 + 2,5 \end{cases}$$

Test if it's equidistant to every SV:

Distance from a point (z_1, z_2) to a line given by $A x_1 + B x_2 + C = 0$ is:

$$D = \frac{|A z_1 + B z_2 + C|}{\sqrt{A^2 + B^2}}$$

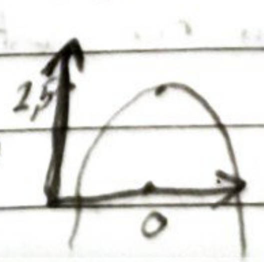
→ To $(1,4) \in w_1$: $D = \left|\frac{0,5}{\sqrt{2}}\right|$ → Positive side

→ To $(0,3) \in w_1$: $D = \left|\frac{0,5}{\sqrt{2}}\right|$ → Positive side

- To $(1,3) \in w_2$: $D = \left|\frac{-0,5}{\sqrt{2}}\right|$ → Negative side

To sketch, substitute $z_1 = x_1^2$:

$x_2 = x_1^2 + 2,5$ →



③ ⓐ Likelihood Ratio Test:

$$\Lambda(x) = \frac{P(I|G)}{P(I|F)} > \frac{P(F)}{P(G)}$$ decide G if this holds

ⓑ

Loss function ($\lambda(\alpha_i, w_j)$):

Function that assigns a cost/penalty to taking action α_i (e.g., deciding Genuine) when the true state is w_j (e.g. Imposter)

Risk (R):

Expected loss for a specific decision given an input I: (Loss over all possible true states weighted by their probabilities).

$$R(\alpha_i | I) = \sum_j \lambda(\alpha_i, w_j) P(w_j | I)$$

Bayes Rule:

Select action α with the lowest risk $R(\alpha | I)$.

Bayes Risk:

Minimum possible risk achievable for a given problem (obtained when Bayes Rule is followed)

Loss function for equal errors (zero-one loss):

$$\lambda(\alpha_i, w_j) = \begin{cases} 0 & \text{if } i = j \quad \text{Correct} \\ 1 & \text{if } i \neq j \quad \text{Miss} \end{cases}$$

(c)

Strategies:

Generative: Estimate prob distributions $P(\mathbb{I}|\omega)$ and $P(\omega)$ then apply Bayes theorem.

Discriminative: Learn decision boundary or the posterior prob directly without underlying distributions

Such as with SVM,..

$P(\omega|\mathbb{I})$

Generalization: Good in unseen data

Memorization: Overfit

4 (a) Product rule: combine output of K classifiers by assuming they are statistically indep.

It assigns x to ω_i that maximizes the product of the posterior probs of each classifier

i) if $\prod_{k=1}^{K} P(\omega_j|x)_k > \prod_{k=1}^{K} P(\omega_i|x)_k$ assign ω_j

else, assign ω_i

(b) $\prod P(O|x) = 0{,}8 \times 0{,}95 \times 0{,}91 \approx 0{,}69$

$\prod P(B|x) = 0{,}75 \times 0{,}85 \times 0{,}9 \approx 0{,}57$

$\Rightarrow$ Assign to orange

5 Answered in ER 2024