Exercise 1 Show by induction:

a) $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

b) For any $n \geq 7$, we have that $n! > 3^n$

Exercise 2 Show that the following recursive algorithm computes correctly the factorial of a number. Assume that $n \geq 0$.

```
Function F(n)
  if n = 0 then
    s = 1
  else
    s = n · F(n − 1)
  return s
```

Exercise 3 Show that the following insertion sort algorithm is able to sort a list A of n numbers in nondecreasing order.

```
Function IS(n, A)
  if n ≥ 2 then
    IS(n − 1, A)
    i = n − 1
    while i ≥ 1 and A[n] < A[i] do
      i = i − 1
    i = i + 1
    p = A[n]
    j = n − 1
    while j ≥ i do
      A[j + 1] = A[j]
      j = j − 1
    A[i] = p
```

Exercise 4 Read the problem *A new chess game* in Mooshak. Consider a recursive approach to solve it.

Base Case $(n=0)$

$0! = 1$ ✓

Inductive Hypothesis

Lets assume the algorithm properly calculates the factorial of $(k-1)$

$(k-1) \cdot F(k-2) = (k-1)!$

then

$k! = k(k-1)! = k(k-1)F(k-2) = k\,F(k-1)$ c.q.d

1 a) $1^2+2^2+\dots+n^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad 2k^2+k+2k+1$

$\Rightarrow 2k^2+3k+1$

Caso Base $(n=1)$:

$\sum_{k=1}^{n} k^2 = 1$

CA

$\frac{-3 \pm \sqrt{9-8}}{4} \Rightarrow \frac{-3+1}{4} \vee \frac{-3-1}{4}$

$\Rightarrow -\frac{1}{2} \vee -1$

Hypótese Indutiva:

$$1^2+2^2+\dots+(k-1)^2 = \frac{k(k-1)\cdot(2k-1)}{6}$$

Passo Indutivo:

$$\underbrace{1^2+2^2+\dots+(k-1)^2}+k^2 = \frac{k(k+1)(2k+1)}{6}$$

$$\Leftrightarrow \frac{k\ (k-1)(2k-1)}{6} + \frac{6k^2}{6} = \text{'' ''}$$

$$\Rightarrow k(k-1)(2k-1)+6k^2 = k(k+1)(2k+1)$$

$$\Rightarrow (k^2-k)(2k-1)+6k^2 = \text{'' ''}$$

$$\Rightarrow 2k^3-k^2-2k^2+k+6k^2 = \text{'' ''}$$

$$\Rightarrow 2k^3+3k^2+k = \text{'' ''}$$

$$\Rightarrow k(2k^2+3k+1) = \text{'' ''}$$

$$\Rightarrow k(k+1)(2k+1) = k(k+1)(2k+1) \quad \text{cqd}$$

1 b) For any $n \geq 7$: $n! > 3^n$

$7 \times 6 = 42 \times 5 = 210$

Base Case ($n=7$):

$7! > 3^7$

$5040 > 2187$

$\underbrace{\overbrace{3 \times 3}^{9} \times 3}_{27} \times 3 \times 3 \times 3 \times 3$

$= 27$
$\times 3$
81
$\times 3$
243
$\times 3$
729
$\times 3$
2187

210
$\times 4$
840
$\times 3$
2520
$\times 2$
5040

Inductive Hypothesis:

Lets assume (for $k-1$) that:

$(k-1)! > 3^{(k-1)}$

Inductive Step:

$k! = k(k-1)!$ given k positive:

$k(k-1)! > k\, 3^{k-1}$

Temos que $k \geq 7$ em particular:

$k! > 3^k$

$k\, 3^{k-1} \geq 7\, 3^{k-1}$

$k! > k\, 3^{k-1} \geq 7\, 3^{k-1} > 3\, 3^{k-1} = 3^k$ eqd