

## Resolução Ficha TP1

Resumen:

- Fase 1:  $H(X|Y) = 1.35 \text{ bits}$
- Los estados  $\{1, 2\}$  tienen la misma probabilidad:  $P(1) = P(2) = 0.5$
- Los estados  $\{1, 2\}$  tienen la misma entropía:  $H(1) = H(2)$
- Fase 2:  $H(Y|X) = 0.2 \text{ bits}$
- Fase 3: Entropía POCO

Ejemplo:  $H(X|Y) = \sum_x P(x)H(x)$

Entropía:  $H(X,Y) = H(X) + H(Y|X)$

Diagrama de Venn:

Diagrama de árbol:

```

graph LR
    Root(( )) --> Node1(( ))
    Root --> Node2(( ))
    Node1 --> Leaf1(( ))
    Node1 --> Leaf2(( ))
    Node2 --> Leaf3(( ))
    Node2 --> Leaf4(( ))
    Leaf1 --> Leaf1_1(( ))
    Leaf1 --> Leaf1_2(( ))
    Leaf2 --> Leaf2_1(( ))
    Leaf2 --> Leaf2_2(( ))
    Leaf3 --> Leaf3_1(( ))
    Leaf3 --> Leaf3_2(( ))
    Leaf4 --> Leaf4_1(( ))
    Leaf4 --> Leaf4_2(( ))
    Leaf1_1 --> Leaf1_1_1(( ))
    Leaf1_1 --> Leaf1_1_2(( ))
    Leaf1_2 --> Leaf1_2_1(( ))
    Leaf1_2 --> Leaf1_2_2(( ))
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```

Entropía:  $H(X,Y) = H(X) + H(Y|X)$

Diagrama de Venn:

es 1.35 (b)   
 Label   
 Entrevista   
 Ficha 1   
 Ejercicio P00

Ficha TP1 | Formato:  
 a b  
 c d

$P(x=1) = \frac{a}{2}$     $P(x=2) = 1-a$     $P(x=3) = a/2$

a)  $\frac{a}{2} = 1-a \Rightarrow \frac{3}{2}a = 1 \Rightarrow a = \frac{2}{3}$  (ad)

b)  $H(x) = \sum_{x \in A_X} p_{xi} \log_2 \frac{1}{p_{xi}} = a \log_2 \left(\frac{2}{3}\right) + (1-a) \log_2 (1-a)$   
 c)

c) (a), (c)  
 d)  $I(X;Y) = H(X) - H(X|Y) = H(X) \quad (\text{a})$   
 e) (d)

I(X;Y)  $H(Y|X)$

Tomorrow:

- Fazer ex 2.33 (União)
- Ler Slides
- Ler 2º tema
- Fazer Atividade 1
- Fazer Exame POO

Obs: PARA O DESCODIFICADOR TER SUCESSO É PRECISO QUE PELO MENOS 2 BITS SEJAM TRANSMITIDOS CORRETAMENTE

$E[f(x)] = \sum_x P(x)f(x)$

$H(x,y) = H(x) + H(Y|x)$

$P_{\text{erro}} = 0,1 \quad P_{\text{acerto}} = 1 - 0,1 = 0,9$

Saja  $X = \text{"MENSAGEM TRANSMITIDA INCORRETAMENTE"}$

$P(x) = P(\text{transmitida corretamente}) \quad 0 \text{ bits} \leftarrow (\text{acertos}) \quad 3 \rightleftharpoons 2$

$P(x) = P(\text{transmitida incorretamente}) \quad 1 \text{ bit} \leftarrow (\text{erros}) \quad 1 \rightleftharpoons 2$

$= 0,1^3 + \binom{3}{1} 0,1^2 0,9 = 0,028$

(a)  $P_{\text{heads}} = a \quad P_{\text{tails}} = 1-a$  Lançar  $n$  heads (ex: TTTT)

a) Sejam  $x$  os lançamentos até termos heads:

$$P = (1-a)^{x-1} \cdot a \quad H = -(1-a)^{x-1} \cdot a \log_2 \left[ (1-a)^{x-1} \cdot a \right]$$

b) Se a moeda for equilibrada:  $P_{\text{heads}} = P_{\text{tails}} = 0,5$

$H = \frac{1}{2} \left[ \left( \frac{1}{2}^{x-1} \cdot \frac{1}{2} \right) \log_2 \left( \frac{1}{2}^x \right) \right] = \frac{1}{2} \left( \frac{1}{2}^x \right) \log_2 \left( \frac{1}{2} \right)^x$

$= \frac{1}{2} x \log_2 2^x = x \ln \left( \frac{1}{2} \right)^x = \frac{x}{\left( \frac{1}{2} \right)^x} = \frac{\frac{x}{2}}{\frac{1}{2^x}} = \frac{x}{2} = \frac{4}{2} = 2 \text{ bits}$

2.35 (Avançado)   
 1.000   
 em Ficha 1   
 em Exame P00

$X = \sum f(x_i) P(x_i)$   
 $H(X) = H(X) + H(Y|X)$

(1)  $X$  → Probabilidade entre os 2 jogos | 7 jogos à melhor de 4  
 $\Omega_X = \{AAAA, BBB, ABBB, BABB, ..., AABBB, ABABBB, \dots\}$   
 → Número de jogos  
 $A = \{1, 3, 5, 6, 7\}$   
 Jogos independentes //  
 $H(X) = \sum P(x_i) \log_2 \frac{1}{P(x_i)}$   
 $= \frac{1}{35} \log_2 \frac{33}{35} + \frac{4}{35} \log_2 \left(\frac{33}{35}\right) +$   
 $+ \frac{2}{7} \log_2 \left(\frac{3}{7}\right) + \frac{4}{7} \log_2 \left(\frac{3}{7}\right) \approx 1,482$   
 $H(X) = \sum_{k=1}^{20} P(x=k) \log_2 \left(\frac{1}{P(x=k)}\right)$   
 $= 2 \times 0,5^4 \log_2 \frac{1}{2 \times 0,5^4} + 8 \times 0,5^5 \log_2 \frac{1}{8 \times 0,5^5} + 20 \times 0,5^6 \log_2 \frac{1}{20 \times 0,5^6} + 40 \times 0,5^7 \log_2 \frac{1}{40 \times 0,5^7}$   
 $\approx 1,924$

Seja  $K = n^{\text{a}} \text{ de jogos} :$   
 $K=4: \text{ existem } 2 \text{ possibilidades} = x_4$   
 $K=5: \sim 2 \times (5c_2) - \cancel{x_4} = \cancel{+} x_5 = 8$   
 $K=6: 2 \times 7c_2 - x_5 - x_4 = 20$   
 $K=7: 2 \times 7c_3 - x_6 - x_5 - x_4 = 40$   
 TOTAL = 70

$P(X=k) = \frac{1}{70}$   
 $P(X=4) = \frac{2}{70} = \frac{1}{35}$   
 $P(X=5) = \frac{8}{70} = \frac{4}{35}$   
 $P(X=6) = \frac{20}{70} = \frac{2}{7}$   
 $P(X=7) = \frac{40}{70} = \frac{4}{7}$   
 $P(AAAA) = 0,5^4$   
 $P(x_1, x_2, \dots, x_n) = 0,5^n$

$H(X|Y) = H(X) - H(X|Y)$   
 $H(X|Y) = 1,482 - 0 = 1,482$   
 $1,482 = H(X) - H(X|Y) \Leftrightarrow H(X|Y) = 1,924 - 1,482 = 0,442$