



**Observe que:**

Exame com consulta

Qualquer tentativa de fraude conduzirá à anulação da prova para todos os intervenientes.

1 - Consider the set  $S = \{(7,Y), (14,G), (2,Y), (18,G), (53,G), (23,Y), (33,Y), (8,G)\}$  of data points belonging to classes Y and G. Provide the output of the first iteration of the binning of S using the entropy-based method. Please provide all elementary steps.

2 - Let the time series  $y(t)$  be defined by:

$t$	0	0.1	0.2	0.3	0.4	0.4	0.5	0.7	0.8	0.9
$y$	1	1.69	2.17	2.30	2.08	1.65	1.23	1.63	1.27	1.87

$t$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$y$	2.71	3.59	4.27	4.62		4.48	4.36	4.52	5.10	6.10	7.39

Using a linear estimation technique of order 3, please provide the matrixes required to estimate  $y(1.4)$  and determine its value.

3- Consider the following data table:

BP (mmHg)	160	210	150	60	70	60	240	155	230
W(kg)	81	65	135	45	55	35	165	140	89
HR (bpm)	100	145	180	160	55	45	78	120	155
LVET (ms)	267	400	450	225	123	100	302	279	389
PEP (ms)	75	89	80	45	123	89	99	100	67
Paciente	N	N	N	HF	HF	HF	N	N	HF

- If we assume that the centers of classes of classes N e HF are defined by  $C=(BP, W, HR, LVET \text{ e } PEP)$  such that  $C1=(0,0,0,0,0)$  e  $C2=(10,7.25,13,21,51)$ , please comment if there is any possibility that the k-means might not be able to cluster this dataset?
- Determine the classes allocated to each data point by the k-means algorithm using the conditions described in a).
- Determine the three most relevant principal components that describe the *feature space*. Please provide the calculations to obtain the required matrixes.
- What can you state about the 3rd principal component? Please, provide the mathematical details.

4 - Consider the signal

$$x(t) = \cos(20\pi t + \pi)$$

- Show its angular frequency, fundamental frequency, fundamental period, and phase at the origin.

- b) Write the expression of a discrete signal  $y[n]$ , sampled from  $x(t)$  with a sampling frequency  $f_s = 3 f_0$ . Show its angular frequency and period.

5 – The Fourier Transform of a non-periodic signal  $x(t)$  is given by:

$$X(\omega) = \begin{cases} 0, & \omega < -20\pi \vee \omega > 20\pi \\ 1 - \frac{\omega}{20\pi}, & \omega > -20\pi \vee \omega < 20\pi \end{cases}$$

- a) Find the smallest sampling frequency needed to reconstruct  $x(t)$  from a sampled signal  $x[n]$ , without aliasing.