Exercise 1

Sorting Problem

To reduce the median problem to the sorting problem we shall use the sorting problem to sort the array and then pick the corresponding element of the array.

This takes $O(n \log n)$ for the sorting + $O(1)$ to pick the element.

Median Problem

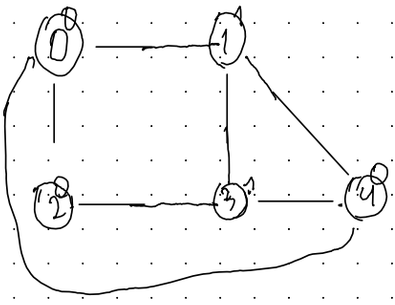
$A = \{1\ 2\ (3)\ 4\ 5\}$ $n = 5$

$A\left[\frac{n+1}{2}\right]$ if n is odd

$1\ 2\ 3\ 4$ $n = 4$

$\frac{A[n/2] + A[n/2+1]}{2}$ if n is even

Exercise 2 → Problema de decisão (k cores)



Given an adjency list:

`vector<vector<int>> adj`

where: `adj[v]` has the child nodes of the v vertex.

```
func violations(v, adj):
    visited[v] = 1
    int n = 0;
    for u in adj[v]:
        if !visited[u]:
            if (color[u] == color[v]):
                n++;
            violations(u, adj);
    visited[v] = 0
    return n
```

The time complexity is $O(|E|) \times O(1)$

$O(|E|)$: Number of states; $O(1)$: Time per state

Just set a very high "k" and binary search it to find the minimum.

The number of calls is $\log(V)$ because in the worst case we need V colors to color the graph (all nodes are connected).

Exercise 3

Generate all triplets of points, calculate the slopes and check if they match.

Time complexity: $O(N^3)$

b) Considerando $y = f(x)$

$P_1(x_1, f(x_1))$ $P_2(x_2, f(x_2))$ $P_3(x_3, f(x_3))$

Podemos converter os inteiros para pontos num plano.

Sabendo que:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2} \Leftrightarrow$$

$$\Leftrightarrow (y_2 - y_1)(x_3 - x_2) = (y_3 - y_2)(x_2 - x_1)$$

$$\Leftrightarrow (x_2 - x_1)(x_2 + x_1)(x_3 - x_2) = (x_3 - x_2)(x_3 + x_2)(x_2 - x_1)$$

Seja $f(x) = x^2$

$$\Leftrightarrow x_1 + x_2 = x_3 + x_2$$

$$\Leftrightarrow x_1 - x_3 = 0$$

$$\Leftrightarrow$$