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The Black-Litterman Model in Modern Portfolio Management: A Systematic Approach to Integrating Investor Preferences

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Abstract

This study explores the integration of investor preferences into portfolio management through the Black-Litterman model, a significant advancement beyond traditional asset allocation methods. Developed originally for fixed income portfolios, this approach was expanded to include equities, offering a disciplined framework to incorporate unique market views. By blending market equilibrium data with investor insights, the model adjusts asset allocations systematically, enhancing the strategic alignment with evolving market conditions. This project employs a decade-spanning dataset to explore the framework and examine its potential to refine investment strategies through a combination of theoretical finance and practical application.

Keywords

Black-Litterman, Portfolio Management, Asset Allocation, Quantitative Strategy

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Introduction

Traditional asset allocation models, while foundational, fall short in accommodating real-time market sentiments or specific investor views, leading to suboptimal portfolio performance.

The challenge persists in systematically and accurately investor views into the allocation process. This thesis proposes an innovative enhancement to the Black-Litterman model by incorporating investor views derived from the Institutional Brokers' Estimate System (I/B/E/S), a comprehensive database of analyst forecasts. This integration leverages real-time, systematically-grounded expectations to refine the asset allocation process.

The utilization of I/B/E/S data allows for a dynamic incorporation of market sentiments as expressed by professional analysts. These views provide forward-looking insights which are typically not captured by traditional models that rely heavily on historical data. By embedding these insights into the Black-Litterman framework, the model adapts to new information, reflecting more accurate future expectations and thereby potentially increasing the robustness of portfolio returns.

This approach is not only innovative but also highly relevant in today's rapidly changing financial markets, where the ability to quickly adapt to new information represents a critical advantage. The following sections will detail the methodology, implementation, and outcomes of this integration, illustrating its impact on the strategic asset allocation process.

1. Literature review

The Black-Litterman model, a cornerstone in the field of portfolio management, represents a significant stride beyond traditional asset allocation methods. This innovative approach, developed by Fisher Black and Robert Litterman during their tenure at Goldman Sachs in the early 1990s, sought to address some of the practical limitations of the Markowitz mean-variance optimization framework (Black, F., & Litterman, R. (1992). "Global Portfolio Optimization," Financial Analysts Journal).

Fischer Black's tenure at Goldman Sachs, spanning just over a decade, was marked by his unique blend of theoretical finance and practical market strategies.

Black, already renowned for co-developing the Black-Scholes Option Pricing Model, continued his path in financial theory at Goldman Sachs. In 1986, he brought Bob Litterman, a PhD in economics from the University of Minnesota, into the Goldman Sachs Fixed Income Research team. This collaboration culminated in 1990 with the creation of the Black-Litterman model, initially aimed at aiding clients in diversifying their global bond portfolios. Originally developed for fixed income portfolios, the Black-Litterman model's scope quickly expanded, encompassing equities by 1991.

The model's publication in the Journal of Fixed Income in September 1991 highlighted its ability to generate balanced portfolios that aligned with investor views without imposing arbitrary portfolio composition constraints (Goldman Sachs, "Innovative Black-Litterman Global Asset Allocation Model Is Developed").

The Black-Litterman model can be considered a breakthrough, designed to provide a quantitative and disciplined approach to portfolio structuring, allowing portfolio managers to incorporate their unique market views in a coherent and practical manner (Goldman Sachs, "Innovative Black-Litterman Global Asset Allocation Model Is Developed").

The framework significantly influenced the strategies for optimal portfolio allocation, particularly in international equity, fixed income, and currency markets (Goldman Sachs, "Innovative Black-Litterman Global Asset Allocation Model Is Developed").

Key works by Black and Litterman themselves, along with subsequent analyses by He and Litterman (1999) in "The Intuition Behind Black-Litterman Model Portfolios," provide essential insights into the model's conceptual and mathematical framework. By contrasting it with the pioneering yet sometimes restrictive Markowitz model (Markowitz, H. (1952). "Portfolio Selection," Journal of Finance), this review highlights the Black-Litterman model's advancements in dealing with real-world portfolio optimization challenges.

2. Methodology

2.1 Methodology Overview

The methodology employed in this study is a combination of the classical Black-Litterman (BL) model for posterior return estimation with the incorporation of analyst price targets derived from the I/B/E/S academic database. This integration aims to leverage the strengths of both: the Black-Litterman model's ability to incorporate investor views with market equilibrium, and the forward-looking insights provided by analyst price targets.

The dataset utilized for analysis spans a decade, covering the period from January 1, 2013, to January 1, 2024. This timeframe is segmented into two parts. The first segment, from January 1, 2013, to December 31, 2017, is as an out-of-sample estimation period, whereas the second segment, from January 1, 2018, to January 1, 2024, serves as a validation period. The initiation of the portfolio construction process occurs on the first trading day of 2018. For this initial portfolio creation, all available data up to that day is utilized for the BL input estimation. A key feature of this methodology is the annual rebalancing, coinciding with the expanding estimation period. Each subsequent year on the first trading day, the portfolio is rebalanced, taking into account the additional year of data now available. This, paired with the continuously expanding dataset, allows for the portfolio to adapt and evolve in response to new market information and changing investment landscapes.

2.2 The asset universe

The process of portfolio construction within the framework of the Black-Litterman model begins with a crucial step: defining the asset universe. This refers to selecting a specific set of assets for consideration in the investment portfolio. For the purpose of this study, the asset universe is composed of 50 large cap US stocks. This selection is underpinned by several practical considerations that make these securities particularly suitable for quantitative analysis and investment strategy formulation.

Firstly, large cap US stocks are well-documented. They are covered extensively by financial analysts and researchers, providing a rich and complete dataset for analysis.

Secondly, these stocks exhibit high liquidity, which is critical in ensuring that the portfolio can be adjusted quickly in response to changing market conditions or investor preferences. The ability to buy and sell with minimal impact on the price is essential for implementing a strategy that requires rebalancing, especially when aligning with the dynamic views of analysts as modeled in this framework.

Additionally, trading on open exchanges, these stocks offer transparency in pricing and availability. This transparency is key for this quantitative study, as it relies on clear, accessible market data to integrate investor views with the market equilibrium.

Broad market is represented by SPX, a numerical value that represents the level of the S&P 500 index. The S&P 500 covers approximately 80% of available market capitalization, meaning it includes a large portion of the total value of the stock market.

In assessing the risk-free rate, I employ the 10-year bond return as a benchmark. Government bonds, particularly those with a 10-year maturity, are widely recognized for their extremely low default risk.

This metric was derived as an annualized average value over the period under analysis, utilizing data available from Professor Robert Shiller's website.

2.3 Equilibrium returns

Equilibrium returns are a starting point for the BL framework. In "Modern Investment Management – An Equilibrium Approach" by Litterman et al. (2003), the authors explore the concept of the equilibrium approach. Litterman describes equilibrium as a theoretical state where demand and supply are balanced. Importantly, this state is not practically achievable in financial markets. These returns or prior can be seen as the "default" estimate, in the absence of any information. Black and Litterman (1991).

Literature introduces several methods of equilibrium estimation, including CAPM, historical mean or market capitalization weight. Following the modern approach presented in Idzorek (2002), I proceed using a reverse optimization method. Using the below formula, I extract the prior returns for every asset, on every rebalancing date using all known information.

$$\boldsymbol{\Pi} = \boldsymbol{\lambda\Sigma}\, \mathbf{w}_{mkt}$$

Where:

$\boldsymbol{\Pi}$ is the vector of implied excess returns;

$\boldsymbol{\lambda}$ is the risk aversion coefficient, that reflects the tradeoff between expected return and risk. It determines how much return an investor is willing to sacrifice for reduced variance. In reverse optimization, λ scales the estimated excess returns; a higher λ means more excess return per unit of risk, increasing the estimated returns (Idzorek 2002).

The literature review suggests that this parameter can be either calculated, or asigned an arbitrary value. I follow the "original" approach introduced by He and Litterman in their 1999 study, where market-implied risk premium, is the market's (SPX) excess return divided by its variance.

$\boldsymbol{\Sigma}$ is the covariance matrix of excess returns. I estimate the covariances using the method introduced by Ledoit & Wolf (2003). It is a widely-used technique in statistical modeling, particularly in contexts involving high-dimensional data or when the number of observations is limited. The method involves computing an optimally weighted average of two estimators: the sample covariance matrix and the single-index covariance matrix. This method systematically draws extreme coefficients towards central values, effectively reducing estimation error in crucial areas (Ledoit, Wolf 2003)

$\mathbf{w}_{mkt}$ is the market capitalization weight of the assets. Straight forward calculation method involves assigning weight to each asset in a portfolio based on its market capitalization relative to the total market capitalization of all assets in the portfolio.

2.3 Investor views

In the Black-Litterman model, the concept of views is pivotal, representing an investor's specific expectations or opinions about the performance of various assets or asset classes. The uniqueness of the Black-Litterman model lies in its ability to systematically blend these subjective investor views with objective market data.

In this study, a unique systematic approach is employed to define these views using analysts' forecasts available in the Institutional Brokers' Estimate System (I/B/E/S). The process is quantified using:

$$\boldsymbol{Q} = \frac{\boldsymbol{T}}{\boldsymbol{P}} - \mathbf{1}$$

Where:

Q is the view vector, expressed as 12-month expected return

T is s vector of target prices, calculated as a mean of all 12-month forecasts available in I/B/E/S.[1]

P is a vector of adjusted close prices on the first trading day of the year.

[1] Using 12 month horizon price targets, released not later than 30 days before the rebalance date.

This method provides an economically grounded and systematic way to generate investor views. By leveraging the aggregate wisdom encapsulated in the I/B/E/S forecasts, it offers a practical and data-driven approach to forming views. These views can then be integrated into the Black-Litterman model, enhancing its ability to create a portfolio that not only aligns with market equilibrium but also reflects the nuanced expectations based on professional analysts' insights.

Its notable that my methodology represents a simplification in the application of the primary Black-Litterman model. The orginal work by Black and Litterman (1990), does consider employing a more sophisticated method, allowing views to be expressed in relative terms as well. Relative views have potential to more closely approximate the way investment managers feel about different assets and overall broad market dynamics

2.4 Scalar Tuning constant (τ)

τ is a parameter used to adjust the level of confidence in the equilibrium returns estimated from the market portfolio. This constant essentially scales the uncertainty of these equilibrium returns. A smaller τ indicates greater trust in the market's implied returns, whereas a larger τ suggests more reliance on the investor's specific views. I follow the methodology of Black and Litterman (1990) , where the scalar tuning constant is determined as follows:

$$\boldsymbol{\tau} = \frac{\mathbf{1}}{\boldsymbol{sqrt}(\boldsymbol{T})}$$

Where:

$\boldsymbol{\tau}$ is the scalar tuning constant

$\boldsymbol{T}$ is the number of data periods, in this case 252, the number of trading days in a year

2.5 Picking matrix (P)

The picking matrix is a tool designed to align investor views with the asset allocation process.

This aspect of the model is particularly essential in practice, because it acknowledges a realistic scenario in portfolio management: not every asset in a portfolio will have a corresponding investor view each year.

Each row in the matrix represents a distinct view held by the investor, columns on the other hand correspond to an assets in the portfolio. The intersection of a row and a column in this matrix is where the specific view about a particular asset is represented. If a view pertains to a single asset, only one column in that row will have a non-zero value (1 or -1, depending on the nature of the view).

2.6 Posterior Returns and Covariances

The Black-Litterman model, produces key outputs in the form of posterior returns and covariances. These outputs are derived from what is often referred to as the "Black-Litterman Master Formula." This formula encompasses a set of equations that integrate market equilibrium with the investor's unique views, as expressed through the picking matrix.

In the context of the Black-Litterman model, posterior expected returns represent a refined estimation of asset returns after accounting for both the market equilibrium and the investor's views. As articulated in the foundational paper by Black and Litterman in the "Financial Analysts Journal" (1992), this posterior estimate is an updated forecast that arises from the combination of the prior (the market equilibrium returns) and the investor's specific views, using Bayesian statistics to integrate these perspectives. The computation of posterior returns within this model, is not a simple arithmetic average; it is a sophisticated mathematical process that weights the investor's views and market equilibrium by their respective uncertainties, as detailed by He and Litterman in "The Intuition Behind Black-Litterman Model Portfolios" (1999). The role of the covariance matrix of returns and the Tau (τ) parameter is absolutely critical.

The posterior expected returns are derived using the following formula:

$$E(R) = [(\tau\Sigma - 1 + PT\Omega P] - 1\ [(\tau\Sigma) - 1\Pi + PT\Omega Q]$$

where:

- $E(R)$ represents the posterior expected returns.
- τ is the scaling factor for the uncertainty of the prior.
- $\backslash sigma$ is the covariance matrix of the asset returns
- \(P \) is a matrix that identifies the assets involved in the views.
- \(\Omega \) is the diagonal matrix of the confidence in each view.
- \(\Pi \) represents the vector of equilibrium returns.
- \(Q \) is the vector of the investor's views on the assets.

Since the final stage of the portfolio allocation process involves "feeding" the posterior estimates into an optimizer, returns alone are not sufficient. The Black-Litterman model addresses this by providing a mechanism to estimate the covariance matrix as well, allowing for a comprehensive assessment of both the returns and the inter-asset dependencies.

This adjusts the prior market covariances to include the investor's unique views, thereby aligning the covariance matrix with the expected returns. Within the Black-Litterman framework, the posterior covariance matrix is derived by appropriately scaling the original market covariance matrix (denoted by ΣΣ) using the Tau (τ) parameter to moderate the impact of the market's estimates with the investor's confidence in their own views.

For covariances:

$$\Sigma^{\{BL\}} = \Sigma + \left([\tau\Sigma]^{\{-1\}} + P'\Omega^{\{-1\}P}\right)^{\{-1\}}$$

2.7 Portfolio Allocation

Mean-Variance Optimization (MVO) is a quantitative tool used to construct investment portfolios that aim to maximize expected return for a given level of risk, or equivalently, minimize risk for a given level of expected return. Developed by Harry Markowitz in the 1950s, this foundational concept of modern portfolio theory uses the expected returns and the covariance matrix of the asset returns to find the optimal portfolio weights.

When applying MVO in the context of the Black-Litterman framework, I use the posterior expected returns and the adjusted covariance matrix as inputs to determine the optimal weights for a portfolio. These posterior estimates, which blend market equilibrium with the investor's views, aim to provide a more reliable foundation for the optimization process than raw historical data alone.

To further enhance the robustness of the MVO, L2 regularization is introduced as an objective function. This method adds a penalty term for large portfolio weights, effectively shrinking them towards zero.

This enhanced MVO approach is grounded in academic literature, where regularization techniques have been shown to improve portfolio optimization. Notable works include "Portfolio Selection with Parameter and Model Uncertainty: A Multi-Prior Approach" by Tu and Zhou (2011), which demonstrates the benefits of incorporating parameter uncertainty into the optimization process. Similarly, "Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?" by DeMiguel, Garlappi, and Uppal (2009) compares various optimization strategies, underscoring the effectiveness of regularized approaches over naive diversification.

3. Practical application and result interpretation

This section is dedicated to a thorough discussion of the results obtained from the model. Calculation and its outcomes are analyzed to elucidate their implications in the context of investment decision-making and strategy formulation.

For readers interested in the technical specifics of the implementation, the detailed Python code utilized in this study is provided as an appendix.

3.1 Asset universe, price data and correlations

The practical application commences with the collection of stock price data for the 50 companies with the highest market capitalization from Yahoo Finance. The data spans from January 1, 2013, to February 2, 2024. The initial segment of the analysis covers the period from January 1, 2013, to December 31, 2017, which is utilized as the out-of-sample period for the construction of the first Black-Litterman portfolio, established on January 1, 2018. Subsequently, the time frame from January 1, 2018, to January 2, 2023, is designated as an additional out-of-sample period for further evaluation. The risk-free rate is extracted from Professor Shiller's website.

For each year out-of-sample year, I calculate a distinct covariance matrix `. These matrices will be estimated using data available up to the respective rebalance date, employing a methodology introduced by the Ledoit & Wolf (2003). To visualize the data, correlationmatrices are plotted and presented in the figure below.

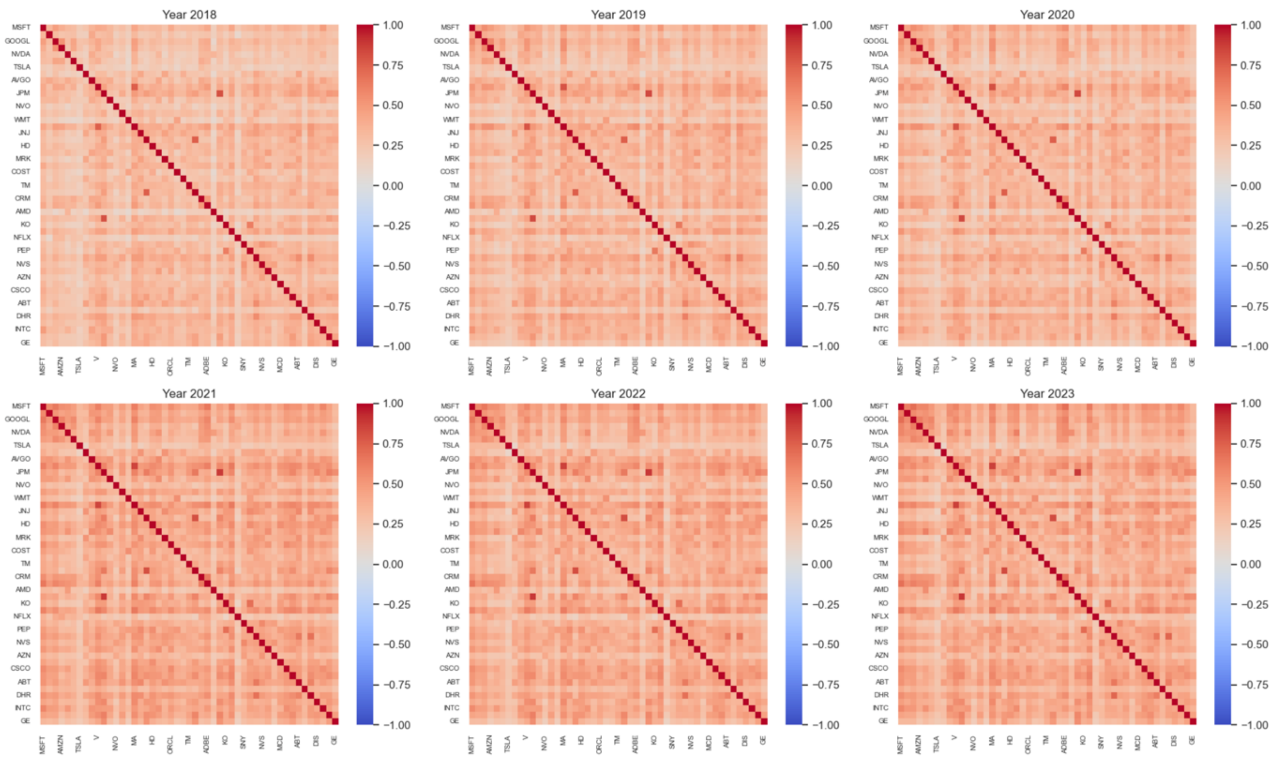


Figure 1: Correlation matrices for years 2018-2023

High correlation values indicate strong linear relationships between the returns of the respective stock pairs. Here are some observations and comments on these correlations:

Financial Sector Correlations: Pairs like JPM - BAC, V - MA, BRK-B - JPM, and BRK-B - BAC show high correlations. This is expected as these stocks belong to the financial sector and are influenced by similar macroeconomic factors, interest rates, and regulatory changes.

Energy Sector Correlation: XOM - CVX exhibits a high correlation, which is unsurprising since both companies operate in the energy sector and are influenced by similar factors such as oil prices, geopolitical events, and global energy demand.

Technology Sector Correlations: MSFT - ADBE, MSFT - GOOGL, and MSFT - AAPL reflect the strong interdependencies within the technology sector. These companies often move in tandem due to industry trends, product launches, and technological innovations.

Consumer Goods Correlations: KO - PEP shows a high correlation, which is common among stocks in the consumer goods sector. These companies produce similar products and face comparable market dynamics, consumer trends, and competitive pressures.

Healthcare Sector Correlation: TMO - DHR and ABT - DHR exhibit notable correlations, reflecting the interconnectedness within the healthcare sector. These companies may share customers, research interests, and regulatory challenges.

Industrial Sector Correlations: BRK-B - UNP, BRK-B - LIN, and ACN - LIN suggest correlations within the industrial sector. Companies in this sector often have correlated performance due to common supply chain dependencies, economic cycles, and infrastructure investments.These correlations highlight the sectoral similarities and market dynamics that drive the co-movements of stocks. However, it's essential to remember that correlation does not imply causation, and other factors, such as company-specific news, earnings reports, and global events, can also influence stock movements.

3.2 Delta ie.the risk aversion coefficient

I utilize Black-Litterman module of PyPortolfioOpt to calculate deltas. To make sure the model captures current market dynamics, I use a distinct delta every time I rebalance the portfolio. The parameter is calculated using the entire in sample market data up to the rebalance date.

Obtained deltas can be seen below:

year	delta
2018	6.37
2019	3.54
2020	5.05
2021	3.11
2022	3.81
2023	2.32

Figure 2: BL deltas for years 2018-2023

The results suggest, that there has been significant variation in the deltas. This implies that the performance of the S&P 500, when adjusted for risk, has not been consistent and has varied d-epending on the specific timeframe or conditions under which it was analyzed.

3.3 Equilibrium Returns

I estimate equilibrium returns for each stock, across a series of predefined rebalance dates. This process is designed to determine the market-implied equilibrium returns, which reflect the consensus expected returns of the market participants based on available market data. For each year, I begin by extracting market capitalizations for each stock from a structured collection. These values are organized into a pandas Series, which is then carefully aligned with the corresponding year's covariance matrix of stock returns by reindexing.

With the market capitalizations, adjusted covariance matrix, and risk aversion coefficient at hand, the BL model computes the market-implied prior returns for each stock. For each rebalance date, we generate a distinct set of equilibrium returns, ensuring that the model's outputs are specifically tailored to the financial landscape and market conditions prevalent at each point in time. This unique set of returns for each date provides a granular insight into the expected market behavior, allowing investors to make informed decisions based on the temporal shifts in market dynamics and investor sentiment.

The equilibrium returns calculated in the preceding section are further visualized and analyzed in subsequent sections of this document. There, they are compared against posterior returns to provide a comprehensive understanding of their implications and accuracy.

3.4 Investor views

In this study I use analysts' price targets to calculate expected returns (investor views). Historical targets are sourced from the IBES database. Initially, price targets denominated in currencies other than USD are identified and converted into USD using historical exchange rates to ensure uniformity in data comparison. Subsequently, the analysis is refined by selecting only those price targets issued in December of each year, which have a projection horizon of 12 months.

Despite encountering instances of incomplete data- specifically, 7 missing values for 2017, 8 for 2018, 6 for 2019, 3 for 2020, 4 for 2021, and 3 for an unspecified year—the decision is made to proceed with the available dataset without imputation of missing values. This approach is justified by the robustness of the Black-Litterman (BL) framework in handling input incompleteness specifically related to investor views. The framework's capacity to integrate available views and generate reliable outputs ensures that the analysis can continue effectively even with gaps in the data.

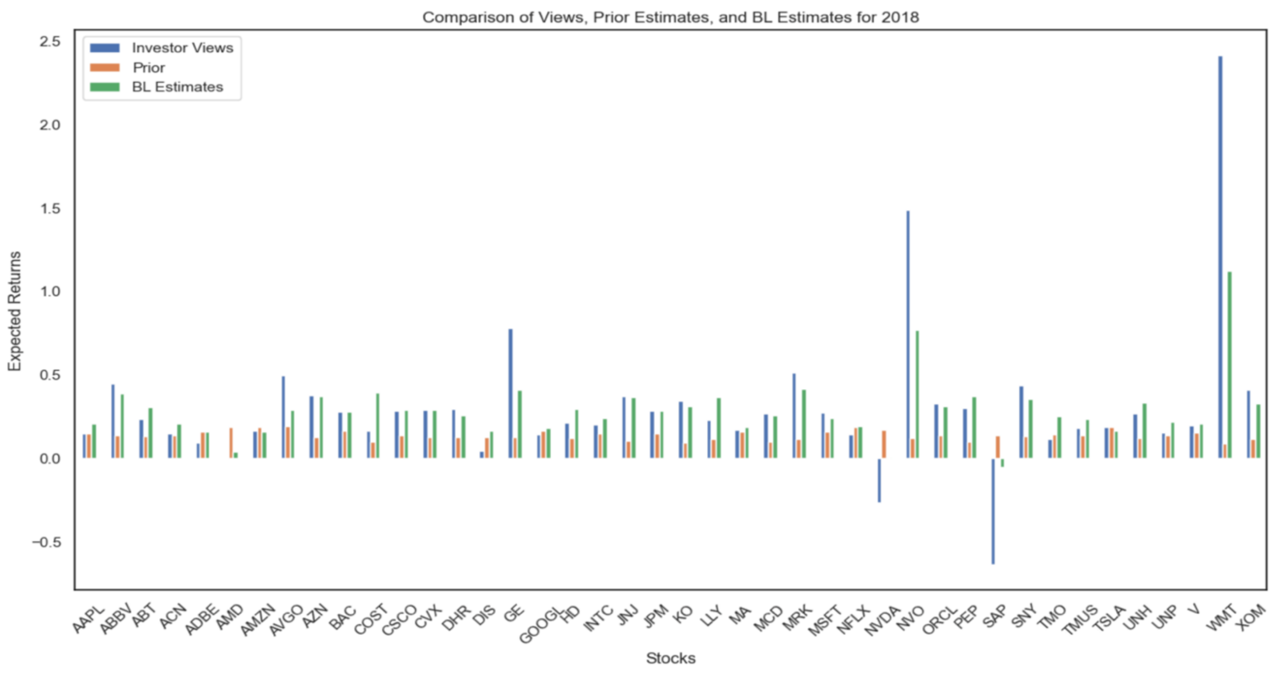
For ‘highly covered’ stocks, exhibiting more than one estimate per year, the methodology involves calculating the average value of all available estimates. This approach is adopted to streamline the analysis by consolidating multiple viewpoints into a single representative figure per stock per year. Such aggregation is crucial for maintaining analytical consistency and ensuring that the derived insights are reflective of the collective market sentiment rather than being influenced by outlier estimates.I calculate 'investor expectations,' or the return expected by analysts, by determining the simple return over a 12-month period. This process yields a distinct set of views for each year, expressed in percentage terms. This methodology ensures that the expectations derived from the analysts' forecasts are quantitatively assessed and clearly defined, facilitating an annual comparison of anticipated market movements based on expert insights. *Similarly to the equilibrium returns, these are visualized and interpreted in the following section.*

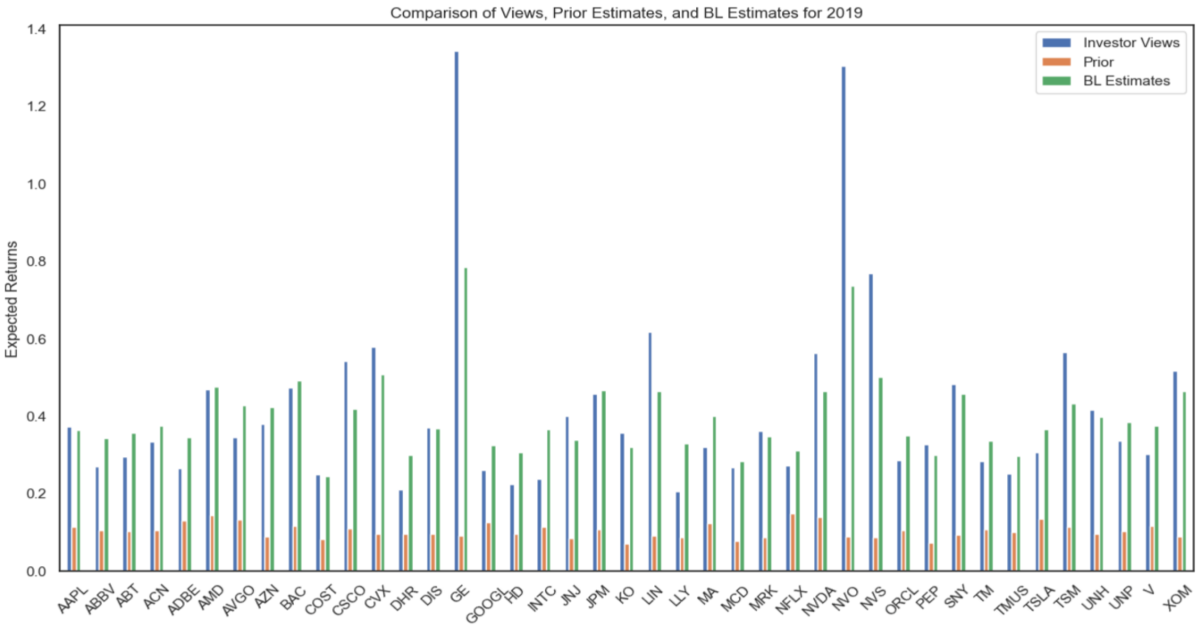
3.5 Scalar tuning constant

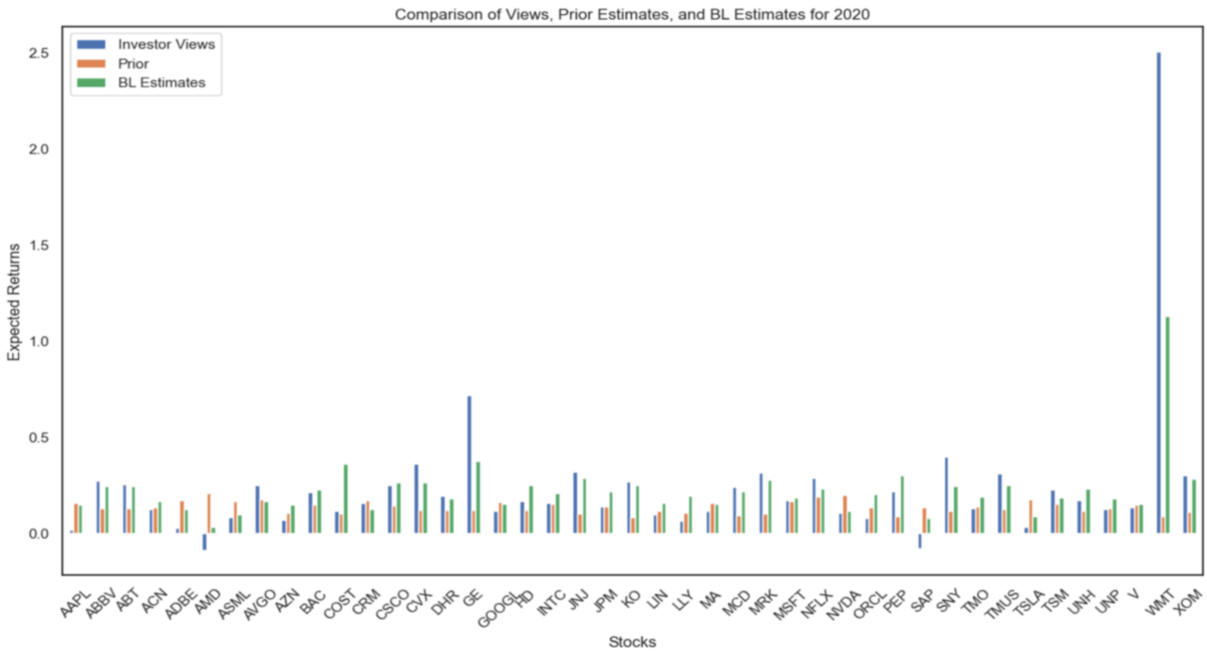
I employ a widely adopted method for setting this parameter, which involves using the formula 1/sqrt(T), where T represents the number of data periods. T is set to 250, applying this formula results in 1/sqrt(250). This computation yields an approximate value of **0.063**.

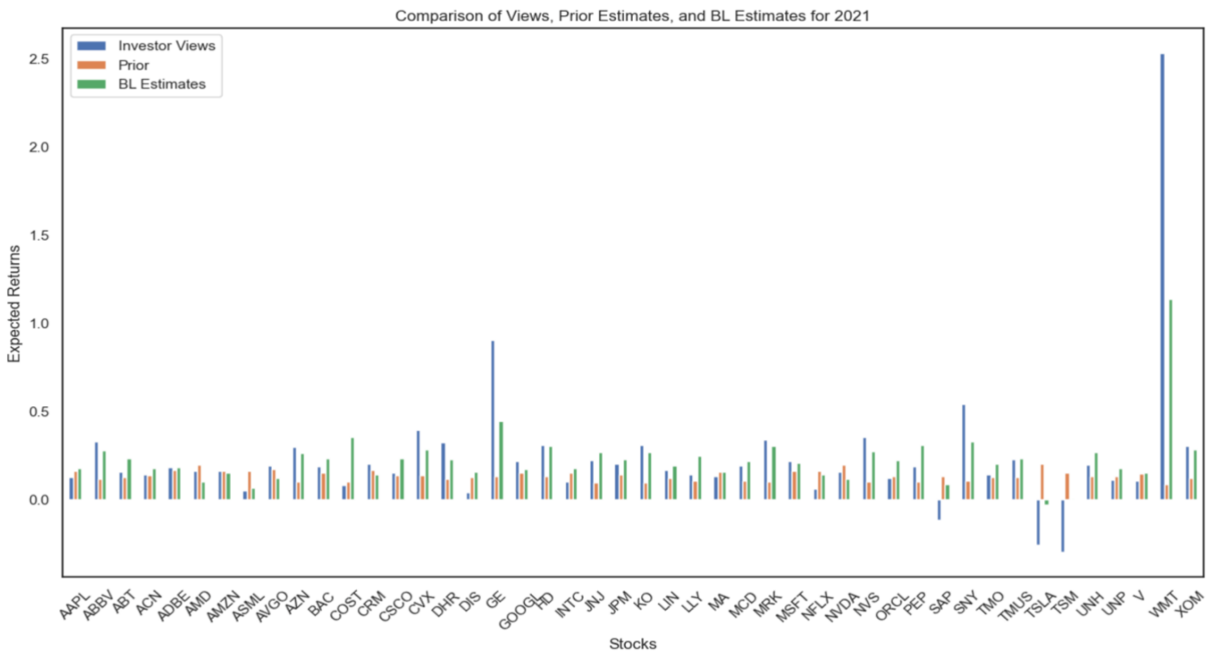
3.6 Posterior returns

I employ the PyPortfolioOpt library, along with the previously mentioned inputs, to calculate the posterior returns, which represent the primary output of the Black-Litterman model. These, with investor views and equilibrium estimates, are presented below for each rebalance date during the out-of-sample period. The chart presents a comparative analysis of expected returns for an array of stocks in 2018, incorporating data from three distinct perspectives: Investor Views, Prior Estimates, and Black-Litterman (BL) Estimates. The investor views, denoted in blue, reflect the aggregated analyst forecasts and serve as a subjective assessment of future performance. Prior estimates, illustrated in orange, signify the equilibrium returns derived from market data and represent an objective baseline expectation. The Black-Litterman estimates, shown in green, are the outcome of combining the investor views with the equilibrium estimates, adjusted for confidence levels. This amalgamation of market data with analyst forecasts via the BL model provides a nuanced and theoretically sophisticated forecast of returns, highlighting the blend of subjective and objective expectations within the investment landscape. The chart serves to visually demonstrate the varying degrees of optimism or conservatism embodied within each set of returns and how the BL model mediates between them for each stock in the investment universe.




Comparison of Views, Prior Estimates, and BL Estimates for 2019
Investor Views
Prior
BL Estimates
Expected Returns
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
AAPL ABBV ABT ACN ADBE AMD AVGO AZN BAC COST CSCO CVX DHR DIS GE GOOGL HD INTC JNJ JPM KO LIN LLY MA MCD MRK NFLX NVDA NVO NVS ORCL PEP SNY TM TMUS TSLA TSM UNH UNP V XOM


Comparison of Views, Prior Estimates, and BL Estimates for 2020
Investor Views
Prior
BL Estimates
Expected Returns
2.5
2.0
1.5
1.0
0.5
0.0
AAPL ABBV ABT ACN ADBE AMD ASML AVGO AZN BAC COST CRM CSCO CVX DHR GE GOOGL HD INTC JNJ JPM KO LIN LLY MA MCD MRK MSFT NFLX NVDA ORCL PEP SAP SNY TMO TMUS TSLA TSM UNH UNP V WMT XOM
Stocks


Comparison of Views, Prior Estimates, and BL Estimates for 2021
Investor Views
Prior
BL Estimates
Expected Returns
2.5
2.0
1.5
1.0
0.5
0.0
AAPL ABBV ABT ACN ADBE AMD AMZN ASML AVGO AZN BAC COST CRM CSCO CVX DHR DIS GE GOOGL HD INTC JNJ JPM KO LIN LLY MA MCD MRK MSFT NFLX NVDA NVS ORCL PEP SAP SNY TMO TMUS TSLA TSM UNH UNP V WMT XOM
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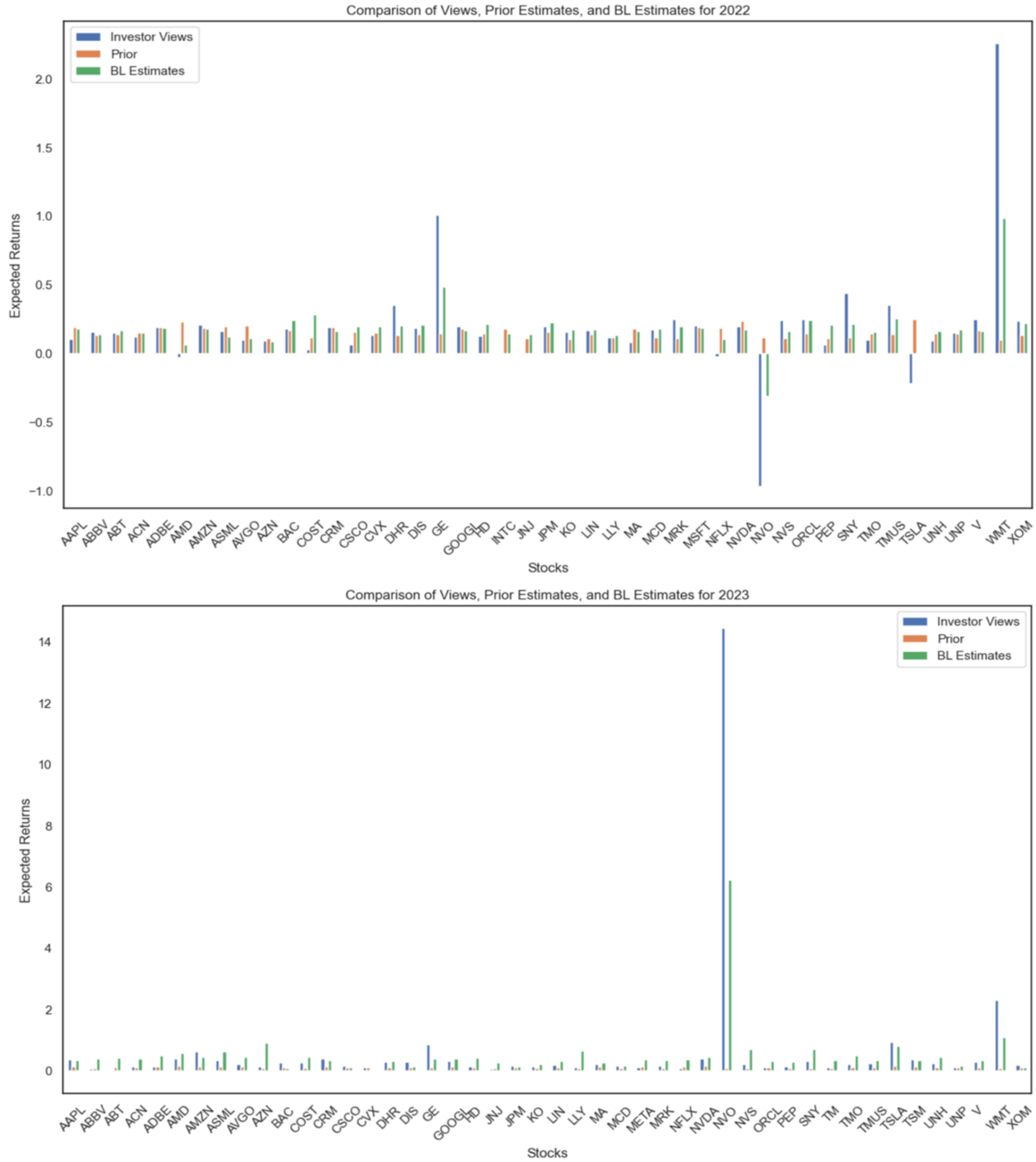

Figure 3: Views, equilibrium (prior) estimates and BL posterior return estimates for years 2018-2023

The charts illustrate the key metrics from the Black-Litterman (BL) model study: the prior estimates, the investor views, and the resultant BL estimates. The posterior BL distribution, as depicted, characteristically positions itself intermediate to the prior and the views. This intermediate positioning is inherent to the BL model's Bayesian foundation, effectively balancing between market equilibrium and the subjective views, as noted in the seminal paper by Black and Litterman (1992).

The BL framework exhibits a desirable characteristic of mitigating extreme estimates. When either the posterior or equilibrium returns generate an outlier value, the BL framework appears to reconcile this value by blending it with the information from the other component. This behavior aligns with the existing literature and seems to exert a positive influence on the allocation process, regardless of the chosen optimization algorithm.

3.7 Portfolio Allocation

Subsequent portfolio optimization, especially when combined with BL-derived covariance matrices, can yield efficient portfolios that are both theoretically sound and practically aligned with an investor's risk and return objectives. Such optimization may involve traditional mean-variance techniques, or more contemporary approaches like Black-Litterman-Tikhonov regularization, which mitigates estimation error through the BL model's inherent shrinkage properties.

I use BL posterior returns and covariance estimates to construct a portfolio using mean-variance optimization. MVO aims to find the optimal allocation of assets in a portfolio to maximize return for a given level of risk or minimize risk for a given level of return. It does this by considering the expected returns and the covariance matrix of asset returns.

L2 regularization is added as an objective function to the MVO problem. It introduces a penalty term that discourages the optimization process from assigning excessively large weights to individual assets. This helps to prevent extreme asset allocations that might lead to overfitting or unstable portfolios.

year	**MSFT**	**AAPL**	**GOOGL**	**AMZN**	**NVDA**	**META**	**TSLA**	**LLY**	**AVGO**	**V**	**JPM**	**TSM**
2018	0,00886	0,00468	0,00158	0	0	0	0	0,03265	0,01388	0,00473	0,01628	0,00382
2019	0,01238	0,01376	0,00948	0,0065	0,01287	0,0098	0,00608	0,01861	0,01541	0,01522	0,02626	0,02456
2020	0,00803	0,00143	0,00311	0,00854	0	0	0	0,01678	0,00208	0,00242	0,0152	0,01253
2021	0,01105	0,00656	0,00589	0,00354	0	0	0	0,02492	0	0,00139	0,01513	0
2022	0,01326	0,01258	0,01082	0,01439	0,00141	0	0	0,00921	0	0,00934	0,0224	0,01699
2023	0,01403	0,00124	0,0061	0,01012	0,0005	0,00178	0,02759	0,03069	0,00511	0,00345	0	0,00446

year	**NVO**	**UNH**	**WMT**	**MA**	**JNJ**	**XOM**	**HD**	**ASML**	**MRK**	**ORCL**	**COST**	**ABBV**
2018	0,09821	0,02577	0,1584	0	0,0373	0,02765	0,02014	0	0,04439	0,02288	0,04001	0,03636
2019	0,07229	0,02294	0,01214	0,01545	0,02389	0,03194	0,01048	0,02316	0,02392	0,01592	0,01023	0,01874
2020	0,01312	0,02181	0,21867	0,00028	0,03984	0,03317	0,02491	0	0,03682	0,0152	0,05247	0,02656
2021	0,02248	0,02563	0,20434	0	0,03235	0,0306	0,03298	0	0,03858	0,01671	0,04714	0,03018
2022	0	0,0122	0,22797	0,00678	0,01306	0,02764	0,02368	0	0,02782	0,03173	0,04596	0,00987
2023	0,48393	0,01268	0,075	0	0,00485	0	0,01068	0,01907	0,00907	0,00296	0,01908	0,01028

year	**TM**	**CVX**	**CRM**	**ADBE**	**AMD**	**BAC**	**KO**	**ACN**	**NFLX**	**SNY**	**PEP**	**TMO**
2018	0,02448	0,01869	0	0	0	0,01364	0,02935	0,00495	0,00243	0,03435	0,03728	0,01337
2019	0,01989	0,03343	0,00642	0,00785	0,01312	0,0264	0,0216	0,01634	0,00564	0,03609	0,01776	0,016
2020	0,022	0,02662	0	0	0	0,01531	0,03192	0,00555	0,01736	0,03066	0,04124	0,01326
2021	0,01637	0,02722	0	0,00487	0	0,01358	0,03249	0,0067	0	0,04289	0,03932	0,01409
2022	0,01793	0,01813	0,00654	0,01194	0	0,02403	0,02162	0,00557	0	0,03169	0,02858	0,01111
2023	0,00996	0	0	0,01007	0,00813	0	0,00301	0,00677	0,00064	0,03519	0,00569	0,01505

year	**NVS**	**SAP**	**AZN**	**MCD**	**CSCO**	**LIN**	**ABT**	**TMUS**	**DHR**	**DIS**	**INTC**	**UNP**	**GE**
2018	0,03069	0	0,03658	0,0204	0,0188	0,01083	0,0221	0,01393	0,0157	0,00137	0,0077	0,0094	0,03635
2019	0,0437	0,02555	0,0316	0,01459	0,02365	0,03124	0,01786	0,01207	0,01158	0,01776	0,01159	0,01992	0,0663
2020	0,02183	0	0,00949	0,02474	0,0272	0,007	0,02465	0,02743	0,01271	0,01753	0,01194	0,01069	0,04788
2021	0,03361	0	0,03045	0,02164	0,01777	0,01238	0,01958	0,02169	0,02028	0,0048	0,00449	0,00881	0,0575
2022	0,02011	0,01029	0,00018	0,02248	0,0179	0,01543	0,01396	0,0366	0,02395	0,02203	0,00207	0,01401	0,08678
2023	0,03771	0,0324	0,04909	0	0	0,00411	0,00875	0,00697	0,0038	0	0	0	0,00996

Figure 4: Portfolio weights in years 2018-2023

The tabulated data presented above showcases portfolio allocations across a spectrum of stocks, delineated by year. These weights have been optimized through the application of the Efficient Frontier technique within the context of the Black-Litterman (BL) framework.

Note that certain stocks in specific years have been assigned zero weight. This absence of allocation can be attributed to the lack of investor views pertaining to these stocks during those years. Absence of investor views, logically results in a zero allocation for the affected stocks within the optimized portfolio.

In the years where investor views are present, the BL model utilizes these views to adjust the initial market equilibrium returns (priors), leading to a posterior distribution of expected returns. The Efficient Frontier algorithm then exploits these posterior expected returns and the BL covariance matrix to allocate weights that aim to maximize the Sharpe ratio, thus striving for an optimal balance between expected return and risk. The resultant portfolio compositions reflect a strategic blend of positions tailored to each rebalance year, factoring in both market information and specific investor insights. It is noteworthy that the allocations are not static but dynamic, adjusting annually to changes in both market conditions and investor sentiment as captured by the BL model.

3.8 Portfolio out-of-sample performance

I use the obtained weights to construct the portfolio on the 1st trading day of 2018 and rebalance the portfolio every year to adjust to strategic weights presented in figure 4. The graph depicted below illustrates the theoretical growth trajectory of a single dollar invested at the beginning of 2018 in three distinct portfolios: the BL portfolio and two benchmark strategies (Equal Weight Portfolio and Value Weight Portfolio).

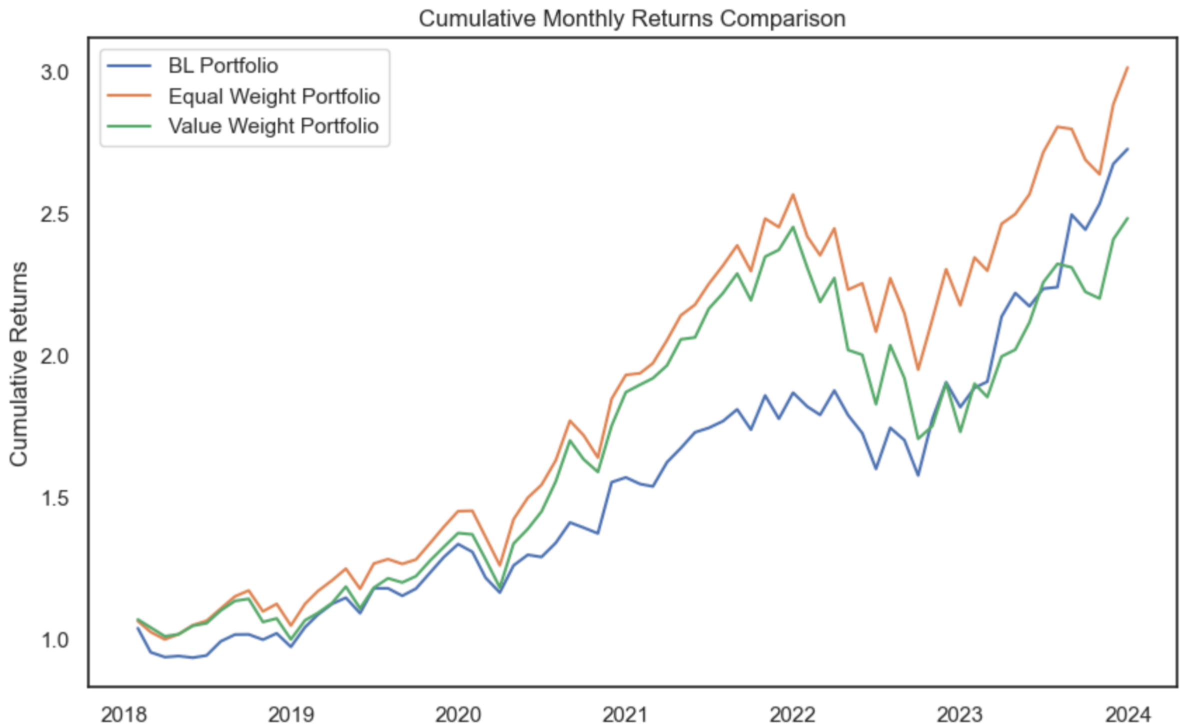

Figure 5: Cumulative portfolios' growth 2018-2024

In this theoretical scenario, the dollar invested in the BL portfolio would have appreciated to $2.73 by the beginning of 2024. To gain a more comprehensive understanding of this performance, I analyze additional performance metrics:

Metric	BL portfolio	Equal weight portfolio	Value weighted portfolio
Total Return	273%	299%	248%
Annualized Return	25%	26%	23%
Annualized Volatility	17%	18%	19%
Annualized Downside Volatility	8%	9%	11%
Maximum Drawdown	-16%	-24%	-30%
Sharpe Ratio	1,24	1.23	1.01
Sortino Ratio	2,64	2.54	1.71
Calmar Ratio	1,54	1.08	0.76

Figure 6: Portfolios' performance statistics 2018-2023

Surprisingly, the straightforward, the least sophisticated equal-weight portfolio managed to outperform in the context of cumulative return, with a total return of 299%, compared to 273% for the BL portfolio and 248% for the value-weighted portfolio. In terms of annualized returns, the difference was also noticeable, with the equal-weight portfolio achieving 26% against the BL's 25% and the value-weighted's 23%. This slight edge came at the cost of a higher annualized volatility, 18% (equal weighted) compared to 17% for BL and 19% for value-weighted, indicating a risk-return trade-off where higher risks associated with the equal-weight portfolio were compensated with better returns.

Both the BL and equal-weight portfolios showed similar sensitivity to adverse market movements, with nearly matching downside volatilities. However, the resilience of each strategy under stress differed, as evidenced by their maximum drawdowns: -24% for the equal-weight versus -16% for the BL portfolio and -30% for the value-weighted portfolio, underscoring the BL portfolio's better performance during market stress.

When examining risk-adjusted performance through the lenses of Sharpe, Sortino, and Calmar ratios, the BL portfolio demonstrated more effective risk management and recovery capabilities. With a Sharpe Ratio of 1.24, a Sortino Ratio of 2.64, and a Calmar Ratio of 1.54, it confirmed that the BL portfolio managed risks better and provided more favorable returns per unit of risk, particularly in managing downside risks and recovering from drawdowns. The value-weighted portfolio , struggled in comparison, with a Sharpe Ratio of only 1.01, a Sortino Ratio of 1.71, and a Calmar Ratio of 0.76

The overperformance of the BL portoflio is unexpactedly marginal, and noticeable only in the risk-adjusted perspective. My understanding is that the benchmark strategy, with equal distribution across assets, avoids the potential biases and errors in forecasting that can affect the view reliant BL model.

Importantly, I don't see the results observed in this performance comparison as a shortcoming of the Black-Litterman (BL) framework itself, which, as noted in the section above, did successfully incorporate the views into the allocation process. In my view, these outcomes point more specifically to potential limitations in the very particular approach I chose to implement, especially in how views were generated and integrated into the model.

The Black-Litterman model is highly dependent on the accuracy and reliability of the input views. The problem here seems to be rooted in the methodology chosen for view generation, which might not have captured the nuances or the future potential of the assets under consideration.

4. Limitations and considerations for further research

While the project provided a rich understanding of the BL framework, certain limitations were encountered. Notably, a subset of "important" high-capitalization stocks lacked analyst coverage in the IBES database for specific years. This data gap could potentially bias the overall composition of the analysis towards companies with more consistent coverage.

This limitation could be partially addressed by utilizing alternative sources of price targets, such as analyst reports or crowdsourced data, to fill the missing inputs.

The model could also potentially benefit from the incorporation of relative view metrics. The current framework's reliance on systematically retrieved data from an academic database limits this possibility. Implementing a qualitative approach to incorporate relative valuation metrics would necessitate a trade-off. While such an approach could offer valuable insights, the resulting dataset would likely be less comprehensive than the one employed in this study. This limitation would likely manifest in two ways: a reduced number of assets considered and a shorter timeframe covered.

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