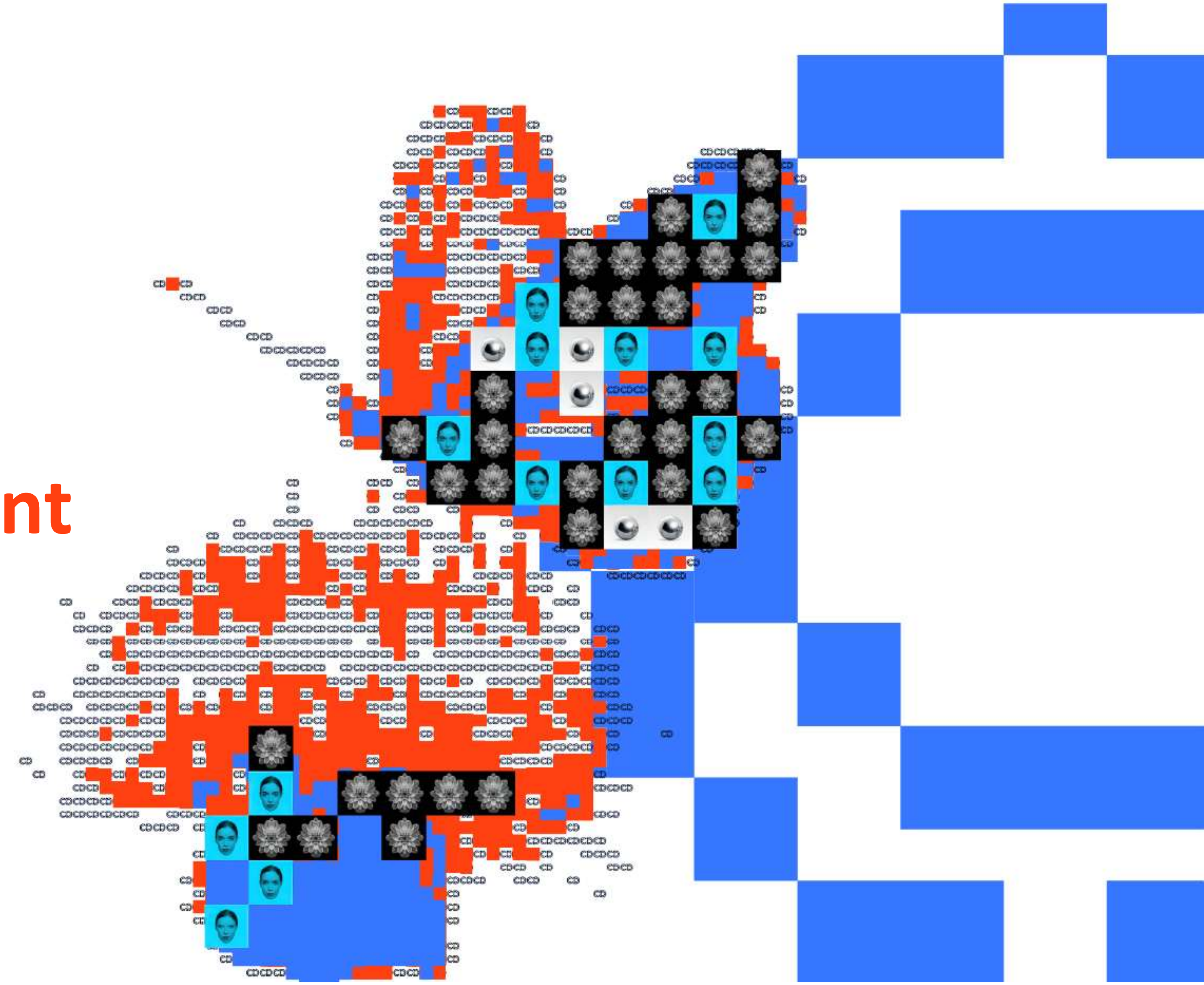
Asset Pricing & Portfolio Management

Prof. Dr. Jorge Miguel Bravo, PhD

MSc Data Science for Finance Program



Prof. Dr. Jorge Miguel Bravo

Short Biography

- Associate Professor NOVA IMS @ Universidade Nova de Lisboa, PhD Economics, Certified Actuary
- Director MSc in Risk Management, NOVA IMS
- Director Executive MSc in Data Science for Finance, NOVA IMS
- Director Executive MSc in Financial Markets and Risks, NOVA IMS & ISCTE-IUL
- Director MSc in Fintech, Digital and Decentralised Finance, NOVA IMS
- Director MSc Data Science for Business, NOVA IMS & Rabat Business School
- Invited Full Professor Université Paris-Dauphine PSL, Paris, France; NOVA FCT
- European University Institute Pension Reserve Fund Supervisory Board
- BBVA Bank Pensions Institute, Madrid
- SOCIEUX+ EU Expertise on Social Protection, Labour and Employment
- Statistics Portugal (INE), Bank of Portugal (Central Bank)
- SG Fundos Pensões Banco de Portugal, Fidelidade, Generalli, Tranquilidade, Açoreana, Advanced Care,…
- Tribunal de Contas, APFIPP – Portuguese Association of Investment Funds, Pensions and Asset Management,
- Ministry of Solidarity and Social Security of Portugal, Ministry of Finance, Svoucher,…

Dauphine | PSL
UNIVERSITÉ PARIS

FIDELIDADE

ISEG
Lisbon School of Economics & Management
Universidade de Lisboa

NOVA
NOVA SCHOOL OF SCIENCE & TECHNOLOGY

Rabat Business School
UIR Rabat Business School COLLEGE OF MANAGEMENT
AACSB ACCREDITED

UNIVERSIDADE DE ÉVORA

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TRIBUNAL DE CONTAS

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DO BANCO DE PORTUGAL, S.A.

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STATISTICS PORTUGAL

socieux+

REPÚBLICA PORTUGUESA
FINANÇAS

ANSR
AUTORIDADE NACIONAL SEGURANÇA RODOVIÁRIA

Asset Pricing & Portfolio Management

Course Outline

1. Introduction to Portfolio Theory

1.1. Primer on Financial Data; 1.2. Modeling the Returns; 1.3. Portfolio Basics

2. Heuristic Portfolios

3. Theory of Portfolio Optimization

3.1. The Markowitz framework

3.2. Capital asset pricing model (CAPM)

3.3. Portfolio optimization in the presence of a benchmark

3.4. Tactical asset allocation (TAA) – The Black-Litterman model

4. Risk-Based Portfolios

4.1. Global minimum variance portfolio (GMVP)

4.2. Inverse volatility portfolio (IVP)

4.3. Risk parity portfolio (RPP) or equal risk portfolio (ERP)

4.4. Most diversified portfolio (MDP)

4.5. Maximum decorrelation portfolio (MDCP)

4.6. Hierarchical Risk Parity Portfolio (HRPP)

Asset Pricing & Portfolio Management

Course Outline

5. Smart Beta, Factor Investing and Alternative Risk Premia

5.1. Linear Factor Model

5.2. Macroeconomic Factor Models

5.3. Fundamental Factor Models

- The Fama-French model
- The Carhart model
- BARRA Approach

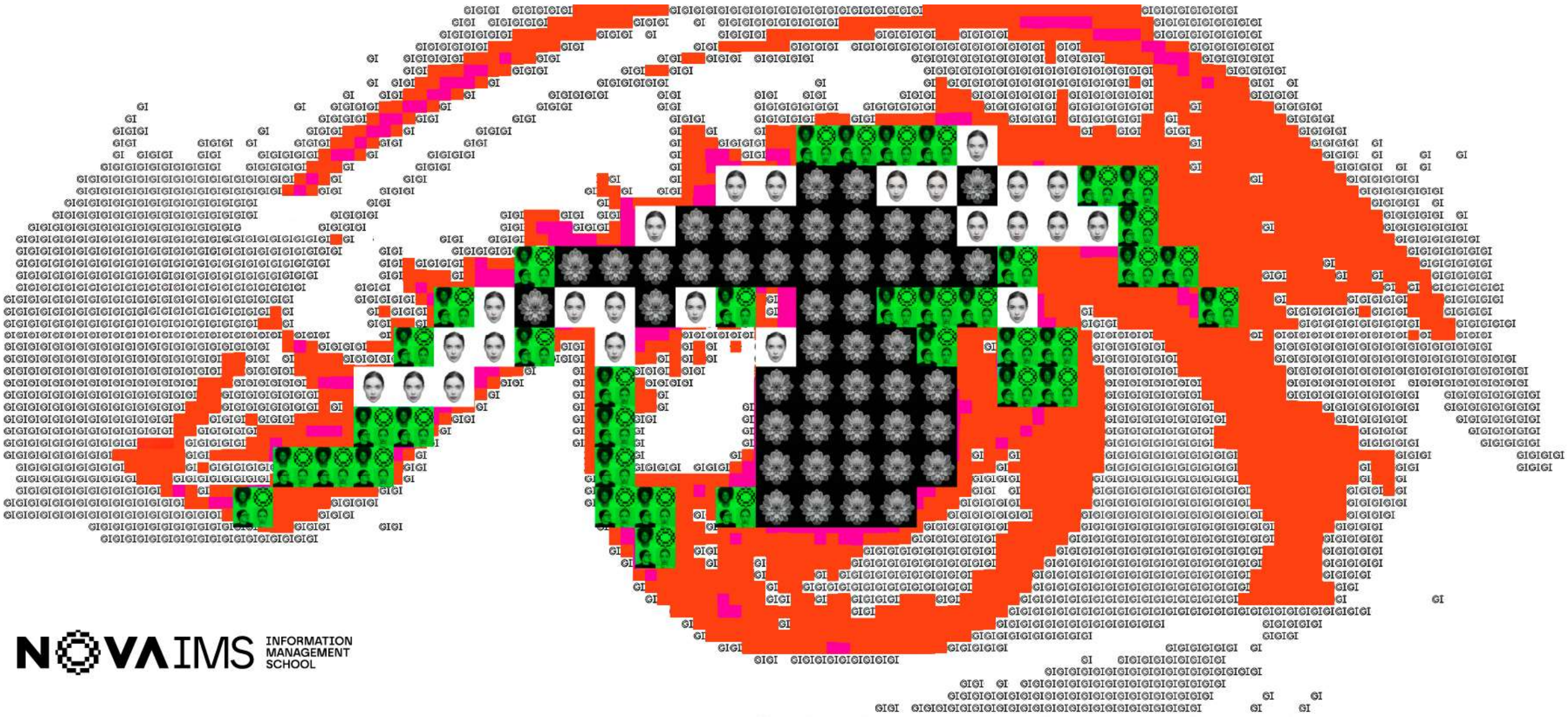
5.4. Principal Components Analysis

5.5. Statistical Factor Models

5.6. Risk-based Indexation

6. Green and Sustainable Finance, ESG Investing and Climate Risk

7. Machine Learning in Asset Management



NOVA IMS INFORMATION MANAGEMENT SCHOOL

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Asset Pricing & Portfolio Management

Primer on Financial Data

Jorge Miguel Bravo, PhD

NOVA IMS

Modern Portfolio Theory

- Markowitz (1952) showed that an efficient portfolio is the portfolio that maximizes the expected return for a given level of risk (the variance of portfolio return)
- Markowitz concluded that there is not only one optimal portfolio, but a set of optimal portfolios which is called the efficient frontier
- By studying the liquidity preference, Tobin (1958) showed that
  - optimal portfolios correspond to a combination of the risk-free asset and one particular efficient portfolio named the tangency portfolio
  - the process of investment choice can be broken down into two phases (Separation Theorem):
    - 1. the choice of a unique optimum combination of risky assets
    - 2. a separate choice concerning the allocation of funds between such a combination and a single riskless asset

Modern Portfolio Theory

- Sharpe developed the CAPM theory highlighting the relationship between the risk premium of the asset (the difference between the expected return and the risk-free rate) and its beta (the systematic risk with respect to the tangency portfolio)
- In equilibrium, the prices of assets are such that the tangency portfolio is the market portfolio, which is composed of all risky assets in proportion to their market capitalization
- This implies that we do not need assumptions about the expected returns, volatilities and correlations of assets to characterize the tangency portfolio
- led to the emergence of index funds and to the increasing development of passive management
- In the active management domain, fund managers used to use the Markowitz framework to optimize portfolios in order to take into account their views and to play their bets
- But the Markowitz framework has many limitations...

Contents

Readings: Palomar (2025), Ch. 2-5; Bodie et al. (2021), Ch. 5 ; Pfaff (2016), Ch. 3

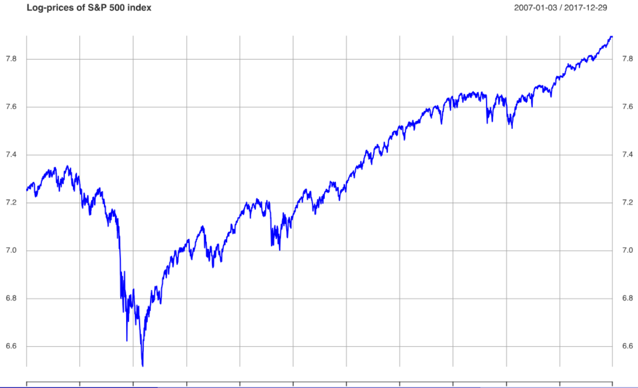
Recommended readings

- **Textbooks** (in English)
  - Palomar, Daniel (2025). Portfolio Optimization. Cambridge University Press.
  - Elton, E., G. Gruber, S. J. Brown, & W. N. Goetzmann (2014). Modern Portfolio Theory and Investment Analysis. 9th Edition. New York: Wiley.
  - Pfaff, B. (2016). Financial Risk Modelling and Portfolio Optimization with R. 2nd Edition, UK.: Wiley.
  - Roncalli, Thierry (2013). Introduction to Risk Parity and Budgeting, Chapman & Hall/CRC.
  - Roncalli, Thierry (2023). Handbook of Sustainable Finance Thierry. Author's edition.
  - Ruppert, David & Matteson, David S. (2015). Statistics and Data Analysis for Financial Engineering with R examples, 2nd Edition. Springer

Asset log-prices

- Let $P_{i,t}$ be the price of an asset i at (discrete) time index t.
- The fundamental model for the log-prices $y_{i,t} \triangleq \ln P_{i,t}$ is a random walk (μ : drift; ϵ_t : i.i.d. random noise)

$$y_{i,t} = \mu + y_{i,t-1} + \epsilon_t$$



The Efficient Market Hypothesis (E.M.H.)

Overview

- **Origin**: Developed by Eugene Fama in the 1960s and 1970s
- **Core Idea**: Financial markets are informationally efficient, implying that asset prices fully reflect all available information
- **Implication**: It is difficult, if not impossible, for investors to consistently outperform the market by exploiting publicly available information; It is impossible to systematically beat the market

The Efficient Market Hypothesis (E.M.H.)

Forms of Market Efficiency

- Definitions (Roberts, 1959, Fama, 1965 and 1970) of the three forms of (informational) efficiency:
- **Weak-Form Efficiency**: Asset prices fully reflect all past trading information, including historical prices and trading volume
  - Implication: Technical analysis cannot be used to consistently generate excess returns
- **Semi-Strong Form Efficiency**: Asset prices fully reflect all publicly available information, including past trading information and fundamental data
  - Implication: Neither technical analysis nor fundamental analysis can be used to consistently generate excess returns
- **Strong-Form Efficiency**: Asset prices fully reflect all information, both public and private, including "insider information" (e.g., such as an impending announcement of a take-over or a merger).
  - Implication: No investor, even with inside information, can consistently generate excess returns

The Efficient Market Hypothesis (E.M.H.)

Criticisms and Limitations

- **Criticisms:**
  - **Behavioral finance** argues that psychological biases can lead to irrational investor behavior and market inefficiencies
  - Empirical evidence of **market anomalies** and excess returns challenges the assumptions of market efficiency

- **Limitations:**
  - Does not account for the impact of **market frictions**, such as transaction costs and liquidity constraints
  - Assumes that all investors have equal access to information and can process it rationally and quickly

The Efficient Market Hypothesis (E.M.H.)

Remarks

- **Remark 1.** These notions of efficiency are included (an efficient market in the strong sense, is also efficient in the semi-strong and the weak form definition)
- **Remark 2.** An academic consensus: markets are efficient (at least in the weak form definition). However, most of practitioners spend their time trying to beat the market ("active" strategies)
- **Remark 3.** Logical consequences: strategies based on specific return predictions are useless (Technical analysis, fundamental analysis...). Returns are merely random (starting point within this seminar)

Asset returns

Returns Based on Prices

- Let P_t be the price of an asset i at (discrete) time index t
- Suppose the asset pays no dividends; the **simple single-period discrete rate of return** on an individual asset (a.k.a. **linear return** or **net return** or **profit rate**) from time $t-1$ to t is

$$R_{i,t} \triangleq \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} = \frac{P_{i,t}}{P_{i,t-1}} - 1 \tag{1}$$

- The quantity

$$(1 + R_{i,t}) = \frac{P_{i,t}}{P_{i,t-1}} \tag{2}$$

is referred to as **total return** or **gross return**

$$P_{i,t} = P_{i,t-1} \times (1 + R_{i,t}) \tag{3}$$

Asset returns

Returns Based on Prices

- If there exists dividend payments, $D_{i,t}$ at time t, the simple return is

$$R_{i,t} \triangleq \frac{P_{i,t} - P_{i,t-1} + D_{i,t}}{P_{i,t-1}} \tag{4}$$

which can be decomposed into capital gains and the dividend yield

$$R_{i,t} = \underbrace{\frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}}_{\text{Capital gain}} + \underbrace{\frac{D_{i,t}}{P_{i,t-1}}}_{\text{Dividend yield}} \tag{5}$$

- Total Return definition (dividends re-invested) is the general international benchmark.

Asset log-returns

Continuously compounded single-period rate of return

- Holding an asset i for one period from date $t-1$ to date t results in the continuously compounded net rate of return (realized log-return):

$$r_{i,t} \triangleq \ln\left(1+R_{i,t}\right) = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right) = y_{i,t} - y_{i,t-1} \tag{6}$$

where $y_t \triangleq \log\left(p_t\right)$ is is the log-price and log denotes the natural logarithm

$$P_{i,t} = P_{i,t-1} \times \exp\left(r_{i,t}\right)$$

- Observe that the log-return is "stationary" after detrending:

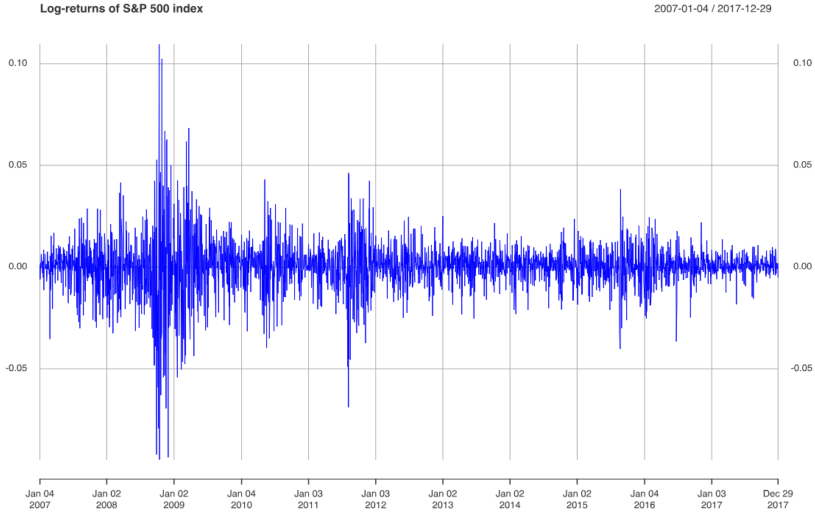
$$r_{i,t} \triangleq y_{i,t} - y_{i,t-1} = \mu + \epsilon_t$$

- If we take into account the dividend payment, the log-return becomes:

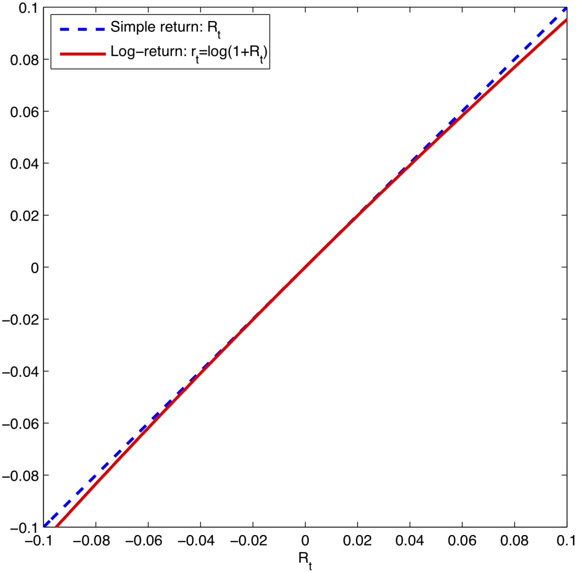
$$r_{i,t} \triangleq \ln\left(1+R_{i,t}\right) = \ln\left(\frac{P_{i,t} - P_{i,t-1} + D_{i,t}}{P_{i,t-1}}\right) \tag{7}$$

- Note that when $R_{i,t}$ is small around zero, i.e., the changes in $P_{i,t}$ are small, $r_{i,t} \approx R_{i,t}$

S&P 500 index - Log-returns



Log-returns vs Simple returns



Portfolios and Portfolio Returns

- Consider an investment in N assets and denote by V_j the dollar amounts invested in asset j
- We call such an investment in multiple assets a portfolio
- The total dollar amount invested in the portfolio is

$$V_p = \sum_{j=1}^{N} V_j \tag{8}$$

- Let $w \in \mathbb{R}^N$ be a vector with w_j denoting normalized capital (the share of dollars) invested into the j-th as

$$w_j = \frac{V_j}{V_p}, \text{ with } \sum_{j=1}^{N} w_j = 1$$

- Negative values for w_j are possible and represent short sales
- The collection of investment shares w_j and the initial wealth invested V_p defines a portfolio

Portfolio Return Aggregation

- The discrete one-period rate of return on a portfolio of N assets with investment shares w_j is the weighted average of the discrete rates of return of each asset in the portfolio, with portfolio relative proportions as weights, i.e.,

$$R_{p,t} = \sum_{j=1}^{N} w_j R_{j,t} \tag{9}$$

with the constraint (unitary sum of weights by construction due to the so-called Budget Constraint):

$$\sum_{j=1}^{N} w_j = 1$$

where w_j is the proportion of the portfolio value V_p invested at time t in the asset j, which can be positive, negative (i.e., short selling; see below) and greater than one (i.e., a leveraged portfolio).

Portfolio Return Aggregation

- For a portfolio of N assets with investment shares w_j, the corresponding one-period portfolio gross returns are defined as:

$$
\begin{aligned}
1 + R_{p,t} &= 1 + \sum_{j=1}^{N} w_j R_{j,t} \Leftrightarrow && (10) \\
1 + R_{p,t} &= \sum_{j=1}^{N} w_j \left(1 + R_{j,t}\right) && (11)
\end{aligned}
$$

with

$$\sum_{j=1}^{N} w_j = 1$$

Portfolio Return Aggregation

Continuous rate of return on a portfolio

- The log-return of a portfolio does not have the above additivity property
- The continuous rate of return on a portfolio is a non-linear combination of the individual continuous rates of return, since the log of a sum of elements is not the same as the sum of log-elements, that is:

$$\begin{aligned} r_{p,t} &\triangleq \ln\left(1+R_{p,t}\right) \\ &= \ln\left[\sum_{j=1}^{N} w_j + \sum_{j=1}^{N} w_j R_{j,t}\right] \\ &= \ln\left[\sum_{j=1}^{N} w_j \left(1+R_{j,t}\right)\right] \end{aligned} \tag{12}$$

- Note here that by the so-called Jensen's Inequality:

$$\ln\left(1+R_{p,t}\right) \geq \left[\sum_{j=1}^{N} w_j \ln\left(1+R_{j,t}\right)\right] \text{ i.e.,}$$

$$r_{p,t} \geq \sum_{j=1}^{N} w_j r_{j,t}$$

Portfolio Return Aggregation

Continuous rate of return on a portfolio

- However, for short intervals of time, since returns are close to zero (jumps excluded), the continuously compounded single-period return on a portfolio is close to the weighted average of the continuously compounded single-period returns on the individual assets, that is:

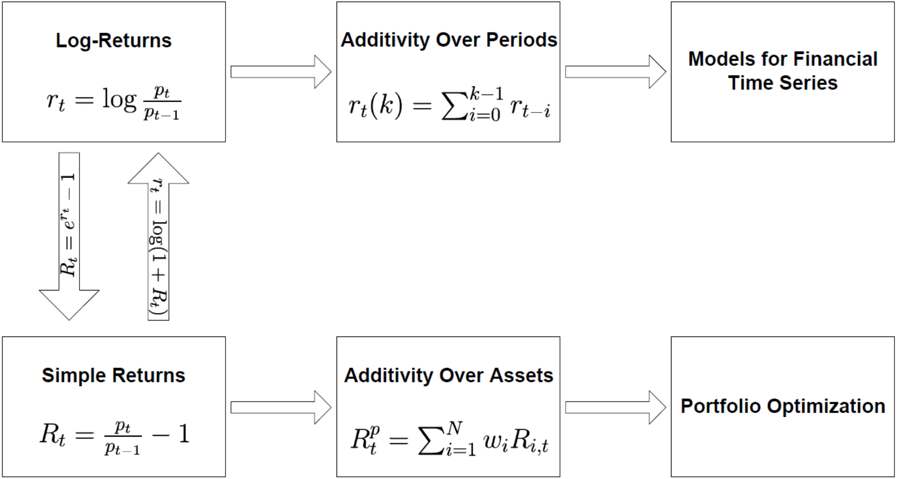
$$r_{p,t} \simeq R_{p,t} \tag{13}$$

since

$$\sum_{j=1}^{N} w_j r_{j,t} \simeq \sum_{j=1}^{N} w_j R_{j,t}$$

- Conclusion: The portfolio discrete (continuous) short one-period return is then (almost) a linear combination of the asset discrete (continuous) returns
- When some $R_{j,t}$ are significantly different from zero, using $\sum_{j=1}^{N} w_j r_{j,t}$ to approximate $R_{p,t}$ may introduce some serious errors

Simple returns versus log-returns



Portfolios and Portfolio Returns

- Computing multiperiod portfolio returns (cumulative return from inception to time t) depends on the assumptions made about the portfolio weights each period and the reinvestment of returns:
  - Portfolio weights from the initial portfolio are allowed to change over time as prices of the underlying assets change over time and/or returns are reinvested. In this case no rebalancing of the portfolio is done. This is called a buy-and-hold portfolio
  - Portfolio weights remain constant over time, which implies that the portfolio is rebalanced at every time period (associated with holding period) to maintain constant weights.
  - Portfolio weights from the initial portfolio are rebalanced at specific time intervals that are different than the holding period (e.g. rebalance quarterly for monthly holding periods). This is a hybrid of 1 & 2.
  - Portfolio weights are actively changed at each time period associated with the holding period. This is called an actively managed portfolio.

Multi-period Discrete Rate of Return

- Holding an asset or a portfolio for T periods from date t to date $t+T$ results in the $T-$period **Gross Compounded Rate of Return**:

$$
\begin{aligned}
(1+R_{t,t+T}) &= \left(\frac{P_{t,t+T}}{P_t}\right) \\
&= \left(\frac{P_{t+1}}{P_t} \times \frac{P_{t+2}}{P_{t+1}} \times \ldots \times \frac{P_{t+T}}{P_{t+T-1}}\right) \\
&= (1+R_{t+1}) \times (1+R_{t+2}) \times \ldots \times (1+R_{t+T}) \\
&= \prod_{k=1}^{T} (1+R_{t+k}) \qquad (14)
\end{aligned}
$$

from which we get

$$R_{t,t+T} = \left[\prod_{k=1}^{T} (1+R_{t+k}) - 1\right] \qquad (15)$$

where the $T-$period **Net Compounded Return** is given by

$$R_{t,t+T} = \frac{P_{i,t+T} - P_{i,t}}{P_{i,t}} \qquad (16)$$

Multi-period Continuous Rate of Return

- Holding an asset or a portfolio for T periods from date t to date $t+T$ results in the Multi-period Continuously Compounded Rate of Return

$$r_{t,t+T} = \ln\left(\frac{P_{t,t+T}}{P_t}\right)$$

which is equivalent to

$$\begin{aligned}
r_{t,t+T} &= \ln\left(\frac{P_{t+1}}{P_t} \times \frac{P_{t+2}}{P_{t+1}} \times \ldots \times \frac{P_{t+T}}{P_{t+T-1}}\right) \\
&= \ln\left[(1+R_{t+1}) \times (1+R_{t+2}) \times \ldots \times (1+R_{t+T})\right] \\
&= \ln(1+R_{t+1}) + \ln(1+R_{t+2}) + \ldots + \ln(1+R_{t+T}) \\
&= r_{t+1} + r_{t+2} + \ldots + r_{t+T} \\
&= \sum_{k=1}^{T} r_{t+k}
\end{aligned}$$

Annualizing the Discrete Rate of Return

Time Return Aggregation

- For making investments with different lengths comparable, returns are usually annualized (i.e., expressed on a yearly basis)
- For a $T-$year horizon investment, $T > 1$, that means:

$$R^A_{t,t+T} = \left[\prod_{k=1}^{T} (1 + R_{t+k})\right]^{1/T} - 1 \qquad (17)$$

- In financial markets, most of the common rates of return on the market are annualized.

Portfolios and Portfolio Returns using matrix algebra

- Denote the following $N \times 1$ column vectors containing the asset returns and portfolio weights

$$\mathbf{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix}, \qquad \mathbf{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix}$$

- The return on the portfolio using matrix notation is

$$\begin{aligned} R_{p,t} &= \mathbf{w}'\mathbf{R} = \begin{pmatrix} w_1 & \cdots & w_N \end{pmatrix} \cdot \begin{pmatrix} R_1 \\ \vdots \\ R_N \end{pmatrix} \\ &= w_1 R_1 + ... + w_N R_N \\ &= \sum_{i=1}^{n} w_i R_i = \mathbf{w}'\mathbf{R} \end{aligned}$$

Portfolio variance

- Basic properties of individual returns

$$\begin{aligned} \text{Expected return (Mean)} &= \mathbb{E}\left(R_j\right) = \mu_j \\ \text{Variance} &= Var\left(R_j\right) = \mathbb{E}\left[\left(R_j - \mu_j\right)\left(R_j - \mu_j\right)\right] = \sigma_j^2 \\ \text{Standard deviation} &= \sqrt{Var\left(R_j\right)} = \sigma_j \end{aligned}$$

- The portfolio variance is computed as

$$\begin{aligned} Var\left(R_p\right) &= \sum_{j=1}^{N} w_j^2 \sigma_j^2 + \sum_{j \neq i}^{N} w_j w_i \sigma_{ji} \\ &= \sum_{j=1}^{N} w_j^2 \sigma_j^2 + \sum_{j \neq i}^{N} w_j w_i \rho_{ji} \sigma_j \sigma_i \end{aligned} \tag{18}$$

- Two asset portofio case

$$Var\left(R_p\right) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho_{12i} \sigma_1 \sigma_2$$

Portfolio variance

- Let $\Sigma = \mathbb{E}\left[(R-\mu)(R-\mu)^{\top}\right]$ denote the variance-covariance matrix of asset returns

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$$

- The portfolio variance using matrix notation is

$$Var\left(R_p\right) = \mathbf{w}'\Sigma\mathbf{w} \tag{19}$$

Performance measures

- Expected return: $\mathbf{w}'\boldsymbol{\mu}$
- Volatility: $\sqrt{\mathbf{w}'\Sigma\mathbf{w}}$
- Sharpe Ratio (SR): expected excess return per unit of risk

$$SR = \frac{\mathbf{w}'\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}'\Sigma\mathbf{w}}}$$

  where r_f is the risk-free rate
- Information Ratio (IR): SR with respect to a benchmark (e.g., the market index):

$$IR = \frac{\mathbb{E}\left[\mathbf{w}'\mathbf{R} - r_{b,t}\right]}{\sqrt{Var\left[\mathbf{w}'\mathbf{R} - r_{b,t}\right]}}$$

The Return Characterization

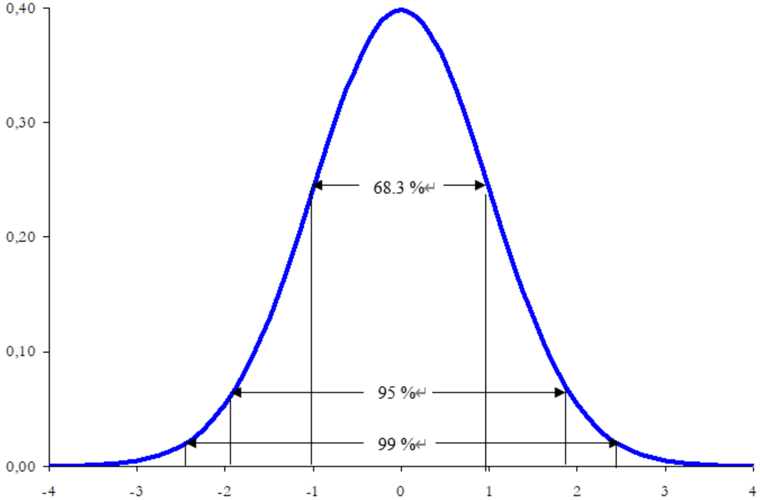
- Asset returns are the main object of study in asset pricing and portfolio theory
- One of the objectives is to try to describe expected returns via a probability distribution (e.g., Gaussian)
- The Gaussian distribution is commonly called "the normal distribution" and is often described as a "bell-shaped" curve.
- The variable $X_{i,t}$ is Independently and Identically (multivariate) Normally Distributed, that is $\forall i \ (i = 1, ..., N)$, $\forall t \ (t = 1, ..., T)$ and $X_{i,t} \in \mathbb{R}$

$$X_{i,t} \sim N\left(\mu_i, \sigma_i^2\right) = \frac{1}{\left(2\pi\sigma_i^2\right)^{1/2}} \exp\left[-\frac{1}{2}\left(\frac{X_{i,t} - \mu_i}{\sigma_i}\right)^2\right]$$

  where $N(\cdot)$ is the density probability function of a Normally distributed random variable with mean μ_i and standard deviation σ_i
- The Gaussian distribution is entirely characterized by its first two moments - the mean and the variance

The Return Characterization

Standard Normal Return Density Function



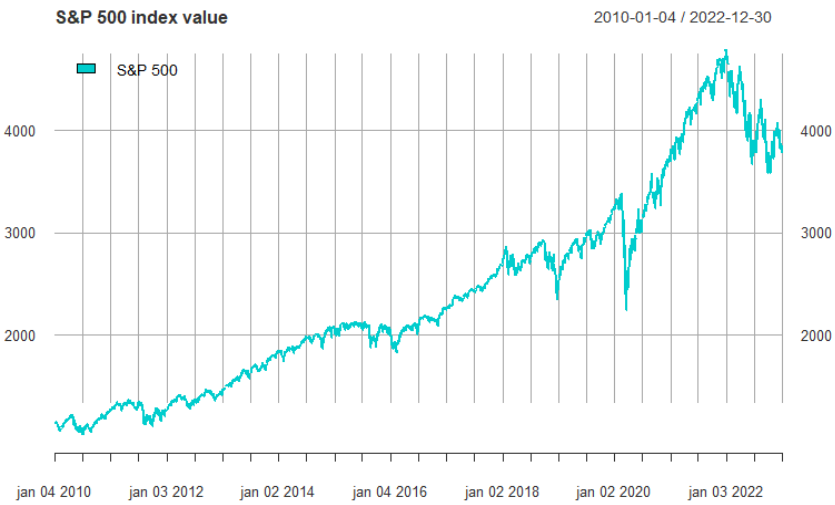
Empirical Properties of Asset Returns, stylized facts

- **Absence of autocorrelations**: (linear) autocorrelations of asset returns are often insignificant, except for very small intraday time scales
- **Heavy tails**: the (unconditional) distribution of returns possess heavy tails, i.e. the distribution has more mass in the tails than in the entre. Even if the precise form of the tails often is difficult to determine the normal distribution can be readily excluded
- **Gain/loss asymmetry**: it can observed in stock prices and stock market indices that upward movements tend to be smaller than, the often large, drawdowns
- **Aggregational normality**: as the time scale is increased over which returns are calculated, their distribution becomes more and more Gaussian; in particular the shape of the distribution varies across time scales
- **Intermittency**: returns display, at any time scale, a high degree of variability. This is quantified by the presence of irregular bursts in time series of volatility estimators
- **Volatility clustering**: many measures of volatility display a positive autocorrelation over several days, which quantifies the fact that high-volatility events tend to cluster in time

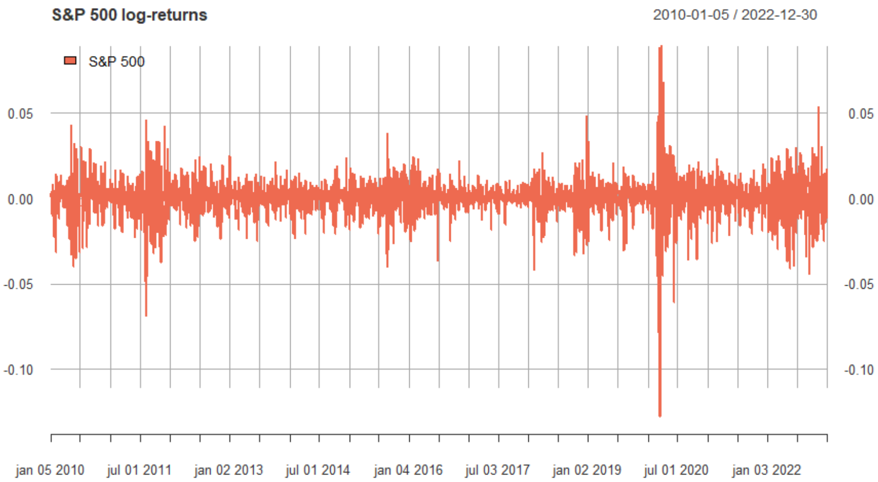
Empirical Properties of Asset Returns, stylized facts

- **Conditional heavy tails**: even after correcting returns for volatility clustering, e.g. via GARCH-type models, the residual time series still display heavy tails. However, the tails are less heavy than those of the unconditional distribution
- **Slow decay of autocorrelation in absolute returns**: the autocorrelation function of absolute returns decays slowly as a function of the time lag. This is sometimes interpretedas a sign of long-range dependence
- **Leverage effect**: most measures of volatility of an asset are negatively correlated with the returns of that asset
- **Volume/volatility correlation**: trading volume is correlated with all measures of volatility
- **Asymmetry in time scales**: coarse-grained measures of volatility predict more granular volatility better than the other way around

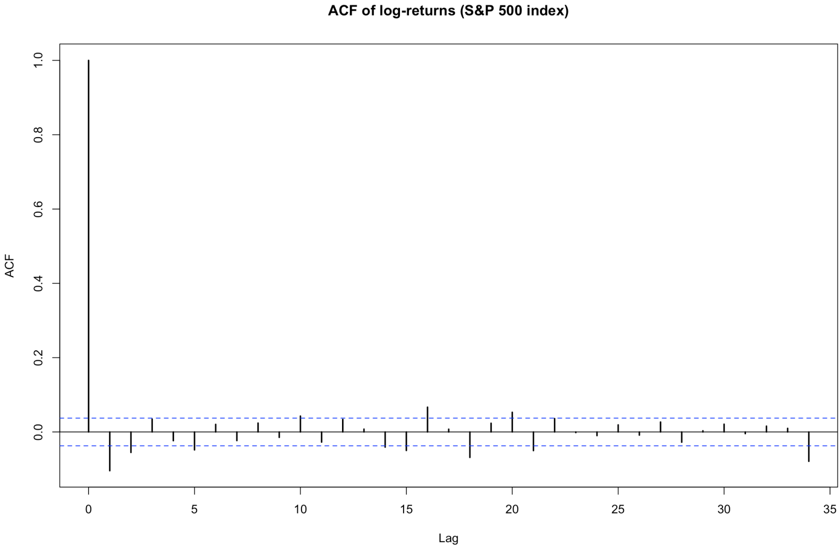
S&P 500 index values



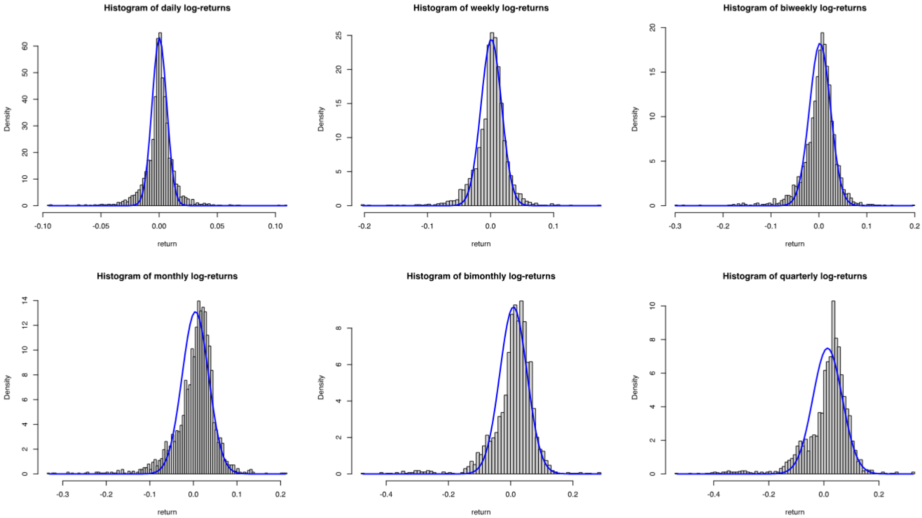
S&P 500 index - Log-returns



Autocorrelation

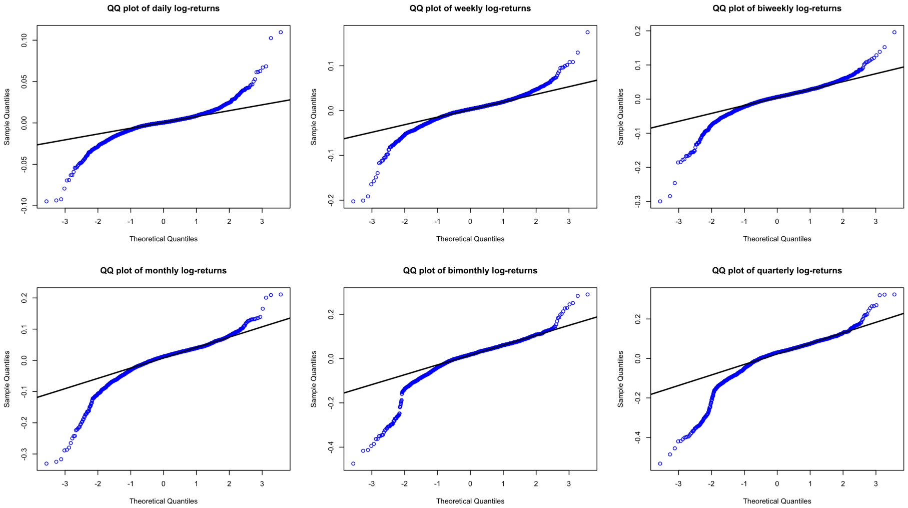


Non-Gaussianity and asymmetry

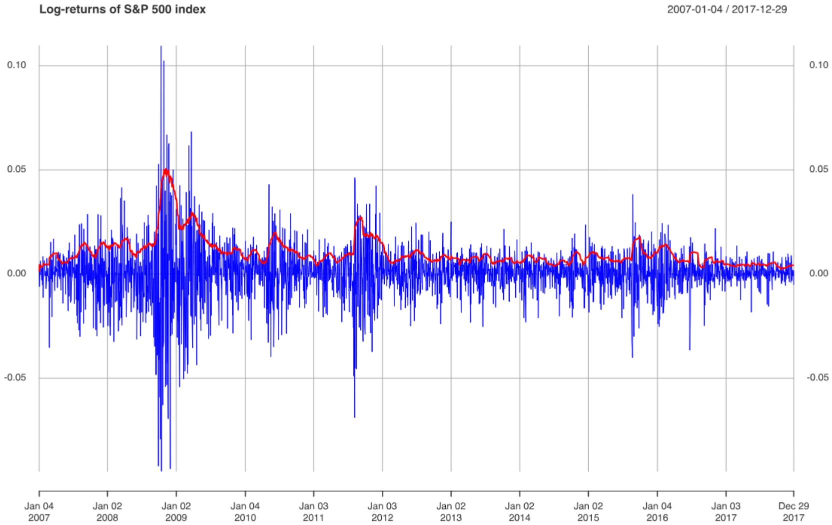


Heavy-tails

QQ plots of S&P 500 log-returns

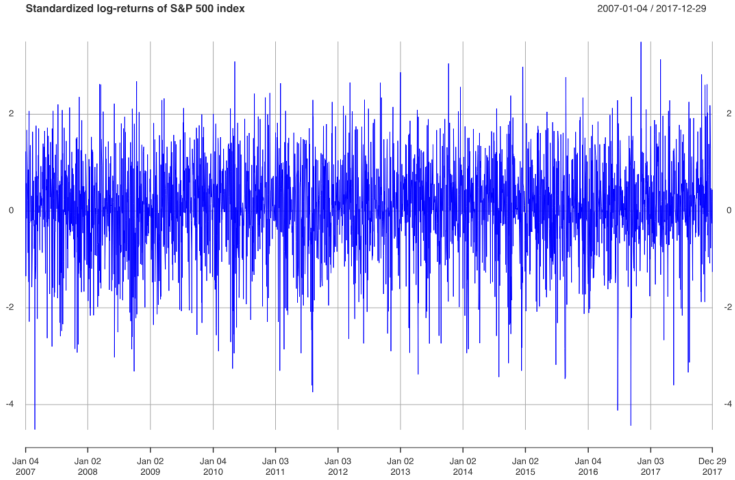


Volatility clustering



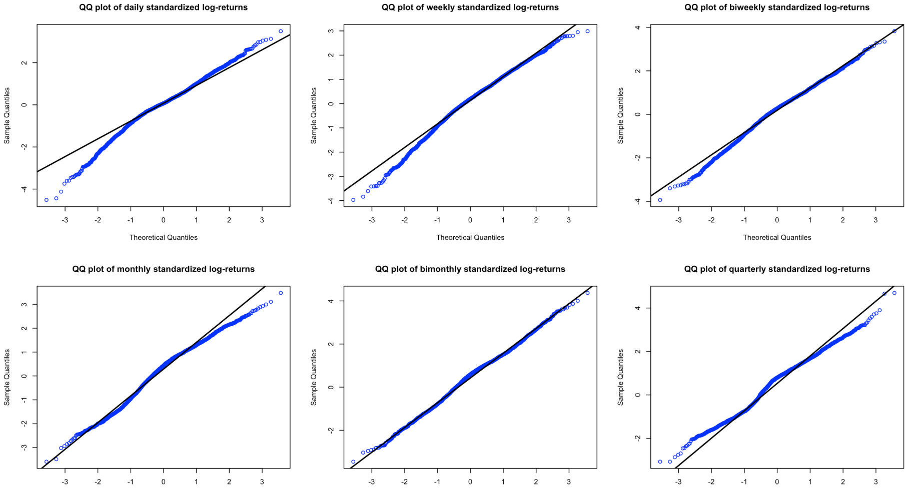
Volatility clustering removed

Standardized S&P 500 log-returns:



Conditional heavy-tails

QQ plots of standardized S&P 500 log-returns (conditional heavy-tailness aka aggregational Gaussianity):



Frequency of data and asset return distribution

- Low frequency (weekly, monthly): Gaussian distributions approximate reality after correcting for volatility clustering (except for the asymmetry), but the nonstationarity is a big issue
- Medium frequency (daily): heavy tails persist even after correcting for volatility clustering, as well as asymmetry
- High frequency (intraday, 30min, 5min, tick-data): below 5min the noise microstructure starts to reveal itself

Stylized facts for asset returns: summary

- Univariate time series
  - Time series data of returns, in particular daily return series, are in general not independent and identically distributed (iid)
  - The volatility of return processes is not constant with respect to time
  - The absolute or squared returns are highly autocorrelated
  - The distribution of financial market returns is leptokurtic. Extreme events occur more often than predicted by normal distribution
  - Extreme returns are observed closely in time (volatility clustering)
  - The empirical distribution of returns is skewed to the left; negative returns are more likely to occur than positive ones
- Multivariate time series
  - The absolute value of cross-correlations between return series is less pronounced and contemporaneous correlations are the strongest
  - The absolute or squared returns do show high cross-correlations.
  - Contemporaneous correlations are not constant over time
  - Extreme observations in one return series are often accompanied by extremes in the other return series

Heuristic portfolios

- Heuristic portfolios are not formally derived from a sound mathematical foundation. Instead, they are intuitive and based on common sense and practicioner behavior

- We will overview the following simple and heuristic portfolios:
  - Buy & Hold (B&H)
  - Buy & Rebalance (B&R)
  - Equally Weighted Portfolio (EWP) or $1/N$ portfolio
  - Quintile portfolio (QP)
  - Global Maximum Return Portfolio (GMRP).

Buy & Hold/Rebalance

- The simplest investment strategy consists of selecting just one asset, allocating the whole budget B to it:
  - Buy & Hold (B&H): chooses one asset and sticks to it forever.
  - Buy & Rebalance (B&R): chooses one asset but it reevaluates that choice regularly.
- The belief behind such investment is that the asset will increase gradually in value over the investment period.
- There is no diversification in this strategy.
- One can use different methods (like fundamental analysis or technical analysis) to make the choice.
- Mathematically, it can be expressed as

$$\mathbf{w} = e_i$$

where e_i denotes the canonical vector with a 1 on the ith position and 0 elsewhere.

Equally weighted portfolio

- One of the most important goals of quantitative portfolio management is to realize the goal of diversification across different assets in a portfolio
- A simple way to achieve diversification is by allocating the capital equally across all the assets
- This strategy is called equally weighted portfolio (EWP), $1/N$ portfolio, uniform portfolio, or maximum deconcentration portfolio:

$$\mathbf{w} = \frac{1}{N}$$

- It has gained much interest due to superior historical performance and the emergence of several equally weighted ETFs (DeMiguel et al. 2009). For example, Standard & Poor's has developed many S&P 500 equal weighted indices.

Quintile Portfolio

- The quintile portfolio is widely used by practitioners.
- Two types: long-only quintile portfolio and long-short quintile portfolio.
- Basic idea:
  - 1) rank the N stocks according to some criterion,
  - 2) divide them into five parts, and
  - 3) long the top part (and possibly short the bottom part).
- One can rank the stocks in a multitude of ways
- If we restric to price data, three common possible rankings are according to:
  - $\boldsymbol{\mu}$
  - $\boldsymbol{\mu}/\mathit{diag}\left(\Sigma\right)$
  - $\boldsymbol{\mu}/\sqrt{\mathit{diag}\left(\Sigma\right)}$

Global maximum return portfolio (GMRP)

- Another simple way to make an investment from the N assets is to only invest on the one with the highest return
- Mathematically, the global maximum return portfolio (GMRP) is formulated as

$$\begin{array}{ll} \underset{w}{\text{maximize}} & \mathbf{w}'\boldsymbol{\mu} \\ \text{subject to} & \mathbf{1}'\mathbf{w} = 1, \quad \mathbf{w} \geq 0 \end{array}$$

- The solution is trivial: allocate all the budget to the asset with maximum return
- However, this seemingly good portfolio lacks diversification and performs poorly because past performance is not a guarantee of future performance

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Cofinanciado por

Co-funded by the European Union

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