

Decision Tree Induction

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

q87 Info(D)

$$\text{Info}(D) = I(9,5) = -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right) = 0.70$$

q87 Info_{age}(D)

$$\text{Info}_{\text{age}}(D) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2)$$

$$I(2,3) = -\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{3}{3} \log_2\left(\frac{3}{3}\right) = 0.991$$

$$I(4,0) = \frac{4}{4} \log_2\left(\frac{4}{4}\right) - \frac{0}{4} \log_2\left(\frac{0}{4}\right) = 0$$

$$I(3,2) = -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) = 0.991$$

$$\text{unun} \quad \text{Info}_{\text{age}}(D) = \frac{5}{14} (0.991) + \frac{4}{14} (0) + \frac{5}{14} (0.991) = 0.674$$

q87 Gain_{age})

$$\text{Gain}_{\text{age}}) = 0.99 - 0.674$$

$$= 0.246$$

q87 Info_{income}(D)

$$\text{Info}_{\text{income}}(D) = \frac{4}{14} I(2,2) + \frac{6}{14} I(4,2) + \frac{6}{14} I(3,1)$$

$$I(2,2) = -\frac{2}{4} \log_2\left(\frac{2}{4}\right) - \frac{2}{4} \log_2\left(\frac{2}{4}\right) = ,$$

$$I(4,2) = -\frac{4}{6} \log_2\left(\frac{4}{6}\right) - \frac{2}{6} \log_2\left(\frac{2}{6}\right) = 0.918$$

$$I(3,1) = -\frac{3}{6} \log_2\left(\frac{3}{6}\right) - \frac{1}{6} \log_2\left(\frac{1}{6}\right) = 0.899$$

$$\text{unun} \quad \text{Info}_{\text{income}}(D) = \frac{4}{14} (1) + \frac{6}{14} (0.918) + \frac{6}{14} (0.899) = 0.911$$

an Gain(income)

$$\begin{aligned} \text{Gain(income)} &= 0.94 - 0.991 \\ &= 0.029 \end{aligned}$$

an Info_{student}(D)

$$\text{Info}_{\text{student}}(D) = \frac{7}{14} I_{(6,1)} + \frac{7}{14} I_{(3,0)}$$

$$I_{(6,1)} = -\frac{6}{7} \log_2\left(\frac{6}{7}\right) - \frac{1}{7} \log_2\left(\frac{1}{7}\right) = 0.592$$

$$I_{(3,0)} = -\frac{3}{7} \log_2\left(\frac{3}{7}\right) - \frac{4}{7} \log_2\left(\frac{4}{7}\right) = 0.985$$

$$\text{unus } \text{Info}_{\text{student}}(D) = \frac{7}{14}(0.592) + \frac{7}{14}(0.985) = 0.989$$

an Gain(cstudent)

$$\begin{aligned} \text{Gain(cstudent)} &= 0.94 - 0.789 \\ &= 0.151 \end{aligned}$$

an Info_{credit-rating}(D)

$$\text{Info}_{\text{credit-rating}}(D) = \frac{8}{14} I_{(6,2)} + \frac{6}{14} I_{(3,1)}$$

$$I_{(6,2)} = -\frac{6}{8} \log_2\left(\frac{6}{8}\right) - \frac{2}{8} \log_2\left(\frac{2}{8}\right) = 0.899$$

$$I_{(3,1)} = -\frac{3}{6} \log_2\left(\frac{3}{6}\right) - \frac{3}{6} \log_2\left(\frac{3}{6}\right) = 1$$

$$\text{unus } \text{Info}_{\text{credit-rating}}(D) = \frac{8}{14}(0.899) + \frac{6}{14}(1) = 0.892$$

an Gain(credit-rating)

$$\begin{aligned} \text{Gain(credit-rating)} &= 0.94 - 0.872 \\ &= 0.048 \end{aligned}$$

mn Gain

$$\text{Gain}(age) = 0.216$$

$$\text{Gain}(income) = 0.029$$

$$\text{Gain}(student) = 0.159$$

$$\text{Gain}(credit_ratio) = 0.049$$

(?) Gain für unabhängige Variablen (nicht Gain age)

age ($C = 30$)

in Info(D) vs age ($C=30$)

$$\text{Info}(D) \text{ vs age } (C=30) = I(2,3) = 0.991$$

in Info(income(D)) vs age ($C=30$)

$$\text{Info}_{\text{income}}(D) \text{ vs age } (C=30) = \frac{2}{3} I(0,2) + \frac{2}{3} I(1,1) + \frac{1}{3} I(1,0)$$

$$I(0,2) = -\frac{0}{2} \log_{2}(0) - \frac{2}{2} \log_{2}(1) = 0$$

$$I(1,1) = -\frac{1}{2} \log_{2}\left(\frac{1}{2}\right) - \frac{1}{2} \log_{2}\left(\frac{1}{2}\right) = 1$$

$$I(1,0) = -\frac{1}{1} \log_{2}(1) - \frac{0}{1} \log_{2}(1) = 0$$

$$\text{Innun } \text{Info}_{\text{income}}(D) \text{ vs age } (C=30) = \frac{2}{3} (0) + \frac{2}{3} (1) + \frac{1}{3} (0) = 0.4$$

in Gain(income) vs age ($C=30$)

$$\text{Gain}(income) \text{ vs age } (C=30) = 0.991 - 0.4 = 0.591$$

in Info(student(D)) vs age ($C=30$)

$$\text{Info}_{\text{student}}(D) \text{ vs age } (C=30) = \frac{2}{3} I(2,0) + \frac{1}{3} I(0,3)$$

Student Yes \rightarrow Yes (buy-computer), No \rightarrow No (buy-computer)

Student wird nun in 2 Kategorien unterteilt

age (>40)

in Info(D) vs age (>40)

$$\text{Info}(D) \text{ vs age } (>40) = I^{Y^R}_{C3,2} = 0.991$$

in Info(income(D)) vs age (>40)

$$\text{Info(income}(D) \text{ vs age } (>40) = \frac{3}{5} I^{Y^R}_{C2,1} + \frac{2}{5} I^{Y^R}_{C1,1}$$

$$I^{Y^R}_{C2,1} = -\frac{2}{3} \log_2(2)(\frac{1}{3}) + \frac{1}{3} \log_2(2)(\frac{1}{3}) = 0.918$$

$$I^{Y^R}_{C1,1} = 1$$

$$\text{inward Info(income}(D) \text{ vs age } (>40) = \frac{3}{5}(0.918) + \frac{2}{5}(1) = 0.959$$

in Gain(income) vs age (>40)

$$\text{Gain(income) vs age } (>40) = 0.991 - 0.959 = 0.032$$

in Info(student(D)) vs age (>40)

$$\text{Info(student}(D) \text{ vs age } (>40) = \frac{2}{3} I^{Y^R}_{C2,1} + \frac{1}{3} I^{Y^R}_{C1,1}$$

$$I^{Y^R}_{C2,1} = -\frac{2}{3} \log_2(2)(\frac{2}{3}) - \frac{1}{3} \log_2(2)(\frac{1}{3}) = 0.918$$

$$I^{Y^R}_{C1,1} = 1$$

$$\text{inward Info(student}(D) \text{ vs age } (>40) = \frac{2}{3}(0.918) + \frac{1}{3}(1) = 0.959$$

in Gain(student) vs age (>40)

$$\begin{aligned} \text{Gain(student) vs age } (>40) &= 0.991 - 0.959 \\ &= 0.032 \end{aligned}$$

$$\text{un } \text{Info}_{\text{credit-rating}}(D) \text{ un } \text{age } (> 40)$$

$$\text{Info}_{\text{credit-rating}}(D) \text{ un } \text{age } (> 40) = \boxed{\frac{3}{5} I(3,0)} + \boxed{\frac{2}{5} I(0,2)}$$

fair \rightarrow Yes (buy-computer), excellent \rightarrow No (buy-computer)
 (un)fair credit-rating unabhängig davon!

arg

