Bayes' Theorem: Basics

- Total probability Theorem:

$$p(B) = \sum_i p\,(B|A_i)p(A_i)$$

- Bayes' Theorem:

P → test data < A ? B ?

$$\boxed{p(H|\mathbf{X})} = \frac{p(\mathbf{X}|H)P(H)}{\underbrace{p(\mathbf{X})}_{\text{เท่ากันเสมอ}}} \propto \boxed{p(\mathbf{X}|H)}\,\boxed{P(H)}$$

posteriori probability	likelihood	prior probability
What we should choose	What we just see	What we knew previously

- **X:** a data sample ("*evidence*")
- H: X belongs to class C

ไปเจอว่าเป็น class อะไร เท่ากับเท่าไหร่

Prediction can be done based on Bayes' Theorem:

Classification is to derive the maximum posteriori

Naïve Bayes Classifier: Making a Naïve Assumption

- Practical difficulty of Naïve Bayes inference: It requires initial knowledge of many probabilities, which may not be available or involving significant computational cost
- A Naïve Special Case
  - Make an additional $\boxed{\text{assumption}}$ to simplify the model, but achieve comparable performance.

attributes are conditionally independent
(i.e., no dependence relation between attributes)

$$p(X|C_i) = \boxed{\prod_k p(x_k|C_i) = p(x_1|C_i) \cdot p(x_2|C_i) \cdots\cdots \boxed{p(x_n|C_i)}}$$

  - Only need to count the class distribution w.r.t. features

Naïve Bayes Classifier: Categorical vs. Continuous Valued Features

- If feature $\mathrm{x_k}$ is categorical, $\mathrm{p(x_k = v_k|C_i)}$ is the # of tuples in $\mathrm{C_i}$ with $\mathrm{x_k = v_k}$, divided by $|C_{i,\,D}|$ (# of tuples of C_i in D)

$$\mathrm{p}(\mathrm{X}|C_i) = \prod_{\mathrm{k}} \mathrm{p(x_k|C_i) = p(x_1|C_i) \cdot p(x_2|C_i) \cdots\cdots p(x_n|C_i)}$$

- If feature $\mathrm{x_k}$ is continuous-valued, $\mathrm{p(x_k = v_k|C_i)}$ is usually computed based on Gaussian distribution with a mean μ and standard deviation σ

$$\mathrm{p(x_k = v_k|C_i)} = N\left(\mathrm{x_k}\middle|\mu_{\mathrm{C_i}}, \sigma_{\mathrm{C_i}}\right) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{C_i}}} e^{-\frac{\left(x-\mu_{C_i}\right)^2}{2\sigma^2}}$$

Naïve Bayes Classifier: Training Dataset

Class:

C1:buys_computer = ‘yes’

C2:buys_computer = ‘no’

Data to be classified:

X = (age <=30, Income = medium,

Student = yes, Credit_rating = Fair)

classified ทำคนที่มีลักษณะแบบนี้จะซื้อคอมพิวเตอร์หรือไม่

Pop

$P = (\overset{Y}{H} \mid \hat{X}) = ?$

$P = (\overset{N}{H} \mid \hat{X}) = ?$

training data

$P = (X \mid \overset{Y}{H})\, P(\overset{Y}{H}) \rightarrow \frac{9}{14}$

มีโอกาส yes

x | y

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Yes = 9

No = 5

Training data

ทุกๆ ฟีเจอร์ไม่มีความสัมพันธ์กัน

Naïve Bayes Classifier: An Example

- $P(C_i)$: P(buys_computer = "yes") = 9/14 = $\boxed{0.643}$

  P(buys_computer = "no") = 5/14= 0.357

- Compute $P(X|C_i)$ for each class

  P(age = "<=30" | buys_computer = "yes") = 2/9 = 0.222

  P(age = "<= 30" | buys_computer = "no") = 3/5 = 0.6

  P(income = "medium" | buys_computer = "yes") = 4/9 = 0.444

  P(income = "medium" | buys_computer = "no") = 2/5 = 0.4

  P(student = "yes" | buys_computer = "yes) = 6/9 = 0.667

  P(student = "yes" | buys_computer = "no") = 1/5 = 0.2

  P(credit_rating = "fair" | buys_computer = "yes") = 6/9 = 0.667

  P(credit_rating = "fair" | buys_computer = "no") = 2/5 = 0.4

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

- **X = (age <= 30 , income = medium, student = yes, credit_rating = fair)**

$P(X|C_i)$: P(X|buys_computer = "yes") = 0.222 x 0.444 x 0.667 x 0.667 = $\boxed{0.044}$

P(X|buys_computer = "no") = 0.6 x 0.4 x 0.2 x 0.4 = 0.019

0.044 0.643

$P(X|C_i)*P(C_i)$: P(X|buys_computer = "yes") * P(buys_computer = "yes") = 0.028

P(X|buys_computer = "no") * P(buys_computer = "no") = 0.007

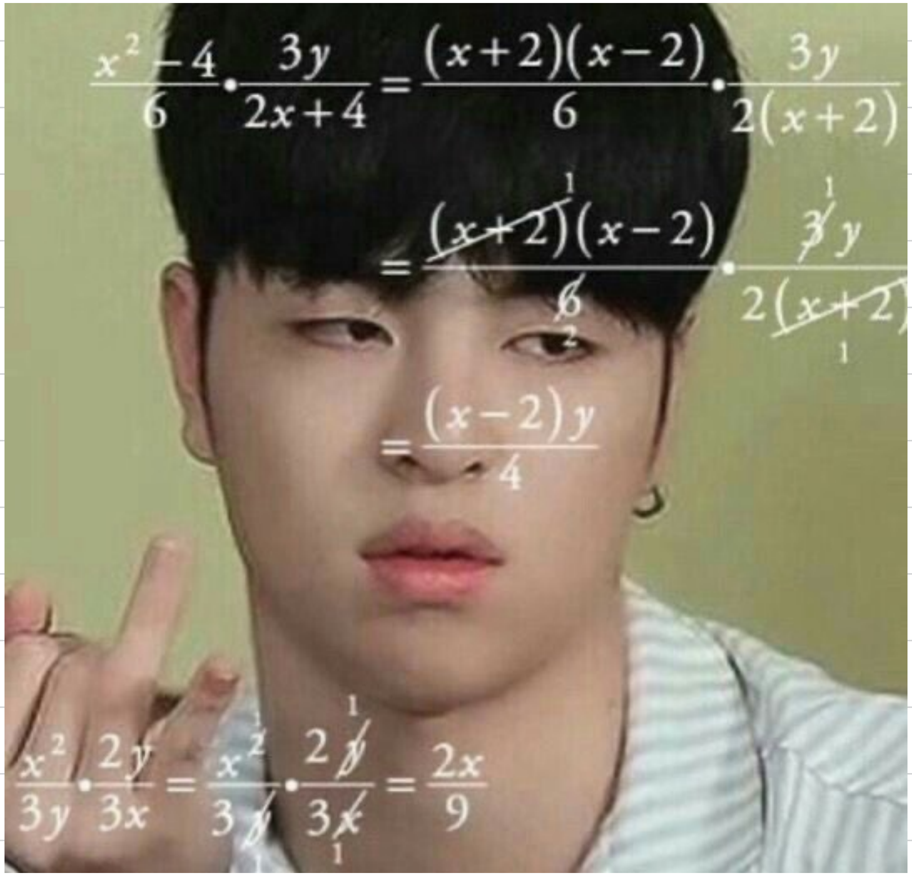
Therefore, X belongs to class ("buys_computer = yes")

$\hat{x}$ = age = 42, student = yes ?

$P = (H \mid \hat{x}) = ?$ 42 Yes 2

$P(H=y \mid (age = 42, student = yes)) = P(age = 42 \mid popy)\, P(student \mid popy)\, P(popy)$

$\frac{3}{9} \times \frac{6}{9} \times \frac{9}{14}$



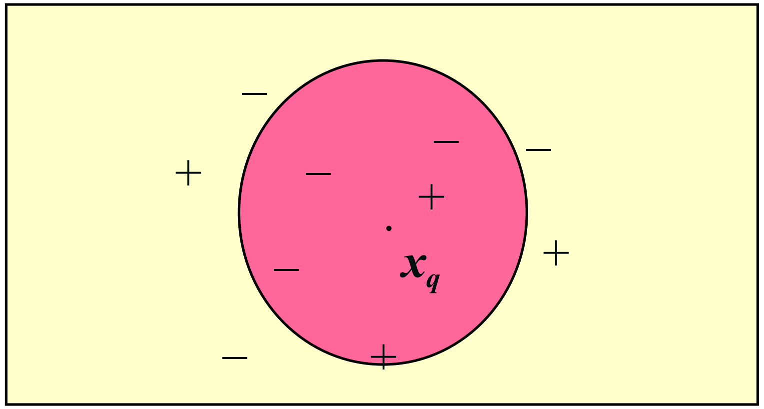
Lazy Learner: Instance-Based Methods

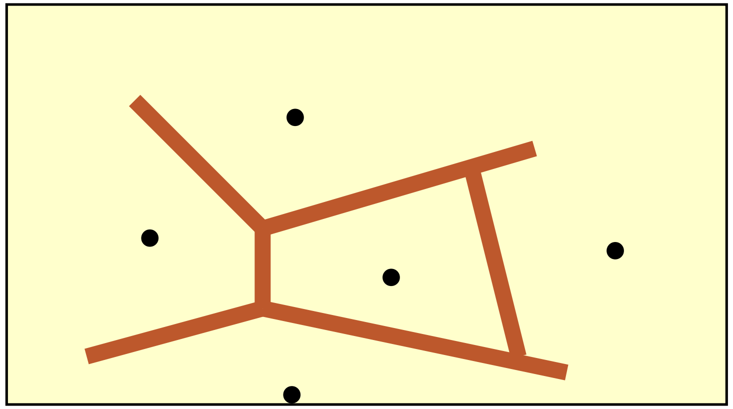
เมื่อได้ข้อมูล train มาเก็บเอาไว้เลย เมื่อมี data เข้ามาถึงจะทำ เก็บไว้จนสุดท้ายว่าต้องทำ

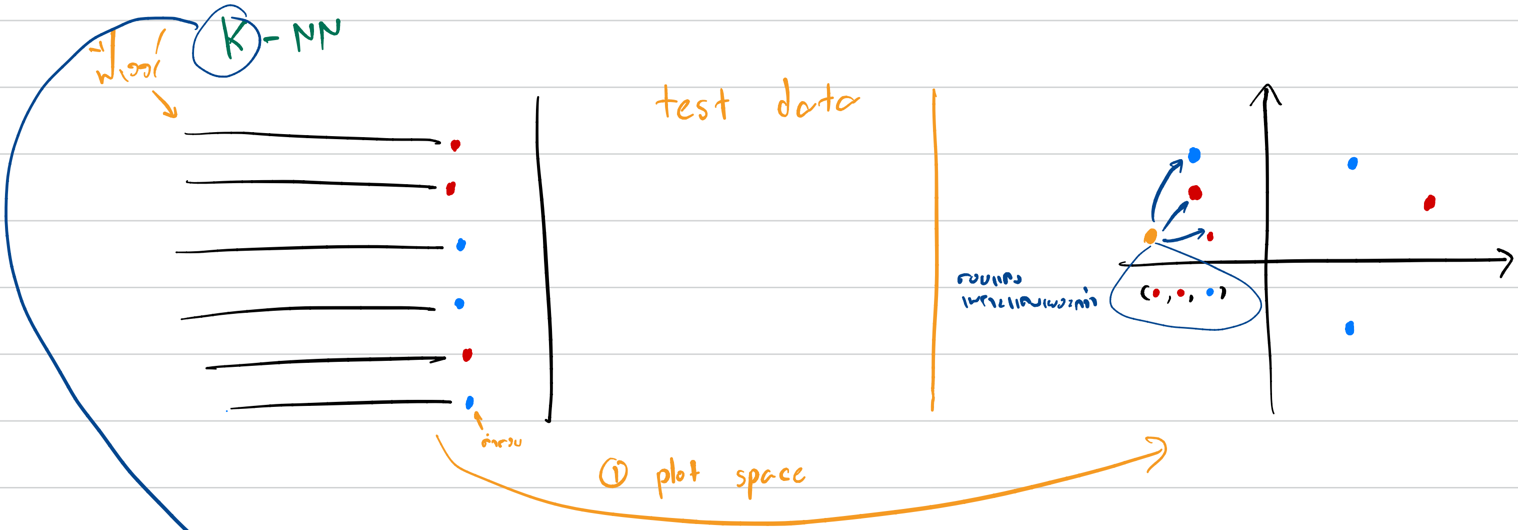
- Instance-based learning:
  - Store training examples and delay the processing (“lazy evaluation”) until a new instance must be classified
- Typical approaches
  - <u>*k*-nearest neighbor approach</u>
    - Instances represented as points in a Euclidean space.
  - <u>Locally weighted regression</u>
    - Constructs local approximation
  - <u>Case-based reasoning</u>
    - Uses symbolic representations and knowledge-based inference

The *k*-Nearest Neighbor Algorithm

- All instances correspond to points in the n-D space
- The nearest neighbor are defined in terms of Euclidean distance, dist($\mathbf{X}_1$, $\mathbf{X}_2$)
- Target function could be discrete- or real- valued
- For discrete-valued, *k*-NN returns the most common value among the *k* training examples nearest to x_q
- Vonoroi diagram: the decision surface induced by 1-NN for a typical set of training examples







ถาม 3 คน ได้แดง
5 คน ได้น้ำเงิน
เปลี่ยนไปกับค่า k มากๆ
ดูเพื่อนบ้านที่อยู่ใกล้ก่อน

② ถามเพื่อนบ้าน k คน
คนที่ 1 แดง
คนที่ 2 แดง
คนที่ 3 น้ำเงิน