# Programming-Exam Numerical Linear Algebra
## Winter 2022/23

## Assignment

Minimum: implement the QR-method following Algorithm 1.4.17 in the notes and verify your implementation.

Gold standard: implement the Francis-QR-iteration with double shifts and deflation in Algorithm 1.5.15.

## Dates

The program(s) should be submitted electronically before the last week of classes. Details in early January.

## Minimal requirements

- The program must run with several example matrices without crashing and you must be able to change parameters like the matrix size

- The program must be subdivided into functions of well-defined purpose

- You must be able to describe how you verify the correctness of your program

If your program following algorithm 1.4.17 works for several matrices and you can explain it, you will pass the exam, but only with the minimal passing grade (4,0).

## Improving your grade

You can improve your grade by putting more effort into the implementation. You can consider items from the following list, but you can also think of other improvements:

1. Use implicit single shifts to accelerate convergence (Algorithm 1.5.6)

2. Use double shifts to avoid complex arithmetic (Theorem 1.5.13)

3. Use deflation when the subdiagonal element in the last row is sufficiently small

4. Use deflation, if any other subdiagonal element is small (somewhat more difficult)

5. Compute not only eigenvalues but also eigenvectors

6. Well-chosen tests for correctness

7. Investigation into convergence

8. Write very well structured code

9. Well-prepared jupyter notebooks with code, accompanying text, and results

10. Object-oriented programming: it is possible to implement algorithm 1.4.17 in a way, that $H$ is either Hessenberg or tridiagonal, either real or complex, but encapsulate the differences inside the QR-decompositions applied in every step.

Note that combining for instance options 1 and 2 is not really helpful. Choose wisely!

If you implement the QR iteration based on Algorithm 1.5.15 observing items 2, 3, 4, 6, 7, and 8 of the previous list, and you can explain your work, the result will be a top grade (1,0). But, going all the way may not be necessary.

## Additional rules

- You can prepare the assignment by yourself or in a team of two students.

- There will be short oral tests with the authors of each program in order to verify authorship and help determining a grade. You will have to demonstrate the running program. Both students of a team will have to contribute equally!

- Please submit at the oral test a signed declaration: "I/We have prepared the assignment myself/ourselves and I/we have only used the sources declared in comments to the program".