## Scientific papers on 'Taper functions'

prof. Roberto Scotti \* 07 set 2018

### Introduction

#### 2018 Forest Inventory course - First results of the collective assignement

Students, as homework, were asked to search for scientific papers presenting 'taper functions' and to compile a collective Rmarkdown document shared using GIT.

Rearranging their work, this document lists their findings.

### Analysed articles

##Article ID 1: (Scolforo, McTague, Raimundo, et al., 2018) Comparison of taper functions applied to eucalypts of varying genetics in {Brazil}: application and evaluation of the penalized mixed spline approach

Comparison of taper functions applied to eucalypts of varying genetics in
Brazil: Application and evaluation of the penalized mixed spline approach
Scolforo, H.F., McTague, J.P., Raimundo, M.R., Weiskittel, A., Carrero, O.,
Scolforo, J.R.S.
2017
Eucalypts
http:
//www.nrcresearchpress.com/doi/10.1139/cjfr-2017-0366#.W2Sb6Lhx02w
NA
NA

## ##Article ID 2: (Warner, Jamroenprucksa, and Puangchit, 2016) Development and evaluation of teak ({Tectona} grandis {L}.f.) taper equations in northern {Thailand}

Title.student	Development and evaluation of teak (Tectona grandis L.f.) taper equations in northern Thailand,
Authors.student	Andrew J. Warner, Monton Jamroenprucksa, Ladawan Puangchit,
Year.student	2016
Species	Tectona grandis L.f.
Base.URL	https://www.sciencedirect.com/science/article/pii/S2452316X16302459?via%
	3Dihub
Paper.local.file	1-s2.0-S2452316X16302459-main.pdf

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$$d_{ub} = (H - h) \left( S + \beta_3 \left( h - h_1 \right) + D_{ub} / [H - h_1] \right) \tag{2}$$
 where  $S = \beta_1 \beta_2^2 \left( h_1 - h \right) / \left[ \left( 1 + \beta_2 h \right) \left( 1 + \beta_2 h_1 \right) \left( 1 + \beta_2 H \right) \right]$  
$$\beta_1 = c_0 + c_1 H + c_2 H^2 + c_3 \left( D_{ub} / 10 \right)^2$$
 
$$\beta_2 = d_0 + d_1 H + d_2 / H$$
 Equations 
$$\beta_3 = f_0 + f_1 H + f_2 / H + f_3 \left( D_{ub} / 10 \right) + f_4 \left( D_{ub} / 10 \right)^2$$

##Article ID 3 : (Tang, Pérez-Cruzado, Fehrmann, et al., 2016) Development of a {Compatible} {Taper} {Function} and {Stand}-{Level} {Merchantable} {Volume} {Model} for {Chinese}

Title.student

Development of a Compatible Taper Function and Stand-Level Merchantable Volume Model for Chinese Fir Plantations

Authors.student

Xiaolu Tang, César Pérez-Cruzado, Lutz Fehrmann, Juan Gabriel Álvarez-Gonzalez, Yuanchang Lu, and Christoph Kleinn,

Year.student

Species

Cunninghamia lanceolata [Lamb.] Hook

Base.URL

Paper.local.file

Development of a Compatible Taper Function and Stand-Level Merchantable

Volume Model for Chinese Fir Plantations

Xiaolu Tang, César Pérez-Cruzado, Lutz Fehrmann, Juan Gabriel

Álvarez-Gonzalez, Yuanchang Lu, and Christoph Kleinn,

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$$d = c_1 \sqrt{H^{(k-b_1)/b_1 \bullet (1-q)^{(k-\beta)/\beta} \bullet \alpha_1^{I_1 + I_2} \bullet \alpha_2^{I_2}}$$
 (2)

where  $I_1 = 1$ , if  $p_1 \le q \le p_2$ ; 0 otherwise;

 $I_2 = 1$ , if  $p_2 \le q \le 1$ ; 0 otherwise

 $p_1$  and  $p_2$  are the relative height from the ground level where the two inflection points assumed in the model occur.

$$\begin{split} \beta &= b_1^{1-(I_1+I_2)} \bullet b_2^{I_1} \bullet b_3^{I_2}, \ a_1 = (1-p_1)^{\frac{(b_2-b_1) \bullet k}{b_1 \bullet b_2}}, \ a_2 = (1-p_2)^{\frac{(b_3-b_2) \bullet k}{b_2 \bullet b_3}} \\ \\ r_0 &= (1-h_{st}/H)^{\frac{k}{b_1}}, \ r_1 = (1-p_1)^{\frac{k}{b_1}}, \ r_2 = (1-p_2)^{\frac{k}{b_2}} \end{split}$$
 
$$c_1 = \sqrt{\frac{a_0 \bullet D^{a_1} \bullet H^{a_2-k/b_1}}{b_1 \bullet (r_0-r_1) + b_2 \bullet (r_1-\alpha_1 \bullet r_2) + b_3 \bullet \alpha_1 \bullet r_2}}$$

Equations

Title.student

{Fir} {Plantations}

##Article ID 4: (Corral-Rivas, Vega-Nieva, RodrÃguez-Soalleiro, et al., 2017) Compatible {System} for {Predicting} {Total} and {Merchantable} {Stem} {Volume} over and under {Bark}, {Branch} {Volume} and {Whole}-{Tree} {Volume} of {Pine} {Species}

Compatible System for Predicting Total and Merchantable Stem Volume over and under Bark, Branch Volume and Whole-Tree Volume of Pine Species"

Authors.student		Daniel Jose Vega-Nieva, Roque Rodríguez-Soalleiro, chez, Christian Wehenkel, Benedicto
		riel Álvarez-González and Ana Daría Ruiz-Gonzalez.
Year.student	2017	
Species	Pinus cooperi, Pinus durar	~
Base.URL	http://www.mdpi.com/199	99-4907/8/11/417
Paper.local.file	forests-08-00417-v2.pdf	
	(1) Over bark taper func	tion:
		$d_{ob} = c_1 \sqrt{H^{(k-b_1)/b_1} (1-q)^{(k-\beta)/\beta} \alpha_1^{I_1 + I_2} \alpha_2^{I_2}}$
	where $q = h/H$ and	
		$\int I_1 = 1 \text{ if } p_1 \leq q \leq p_2$ ; 0 otherwise
Equations		$\begin{cases} I_1 = 1 \text{ if } p_1 \le q \le p_2; \text{ 0 otherwise} \\ I_2 = 1 \text{ if } p_2 < q \le 1; \text{ 0 otherwise} \end{cases}$
•	(4) Under bark taper fu	nction
		$d_{ub} = c_2 \sqrt{H^{(k-b_1)/b_1} (1-q)^{(k-\beta)/\beta} \alpha_1^{I_1 + I_2} \alpha_2^{I_2}}$
	where	
		$c_2 = \sqrt{\frac{e_0 D^{e_1} H^{e_2 - k/b_1}}{b_1(r_0 - r_1) + b_2(r_1 - \alpha_1 r_2) + b_3 \alpha_1 r_2}}$
Equations		$b_1(r_0-r_1)+b_2(r_1-\alpha_1r_2)+b_3\alpha_1r_2$

# ##Article ID 5 : (Sun, Liang, Liang, et al., 2016) Deriving {Merchantable} {Volume} in {Poplar} through a {Localized} {Tapering} {Function} from {Non}-{Destructive} {Terrestrial} {Laser} {Scanning}

Title.student	Deriving Merchantable Volume in Poplar through a Localized Tapering
	Function from Non-Destructive Terrestrial Laser Scanning
Authors.student	Yuan Sun, Xinlian Liang, Ziyu Liang, Clive Welham and Weizheng Li
Year.student	2016
Species	Populus $\times$ canadensis Moench cv.
Base.URL	http://www.mdpi.com/1999-4907/7/4/87/htm
Paper.local.file	forests-07-00087.pdf

$$d^2 = a_0 D^{a_1} \frac{(H-h)^{a_2}}{H^{a_3}}$$
 (2)

Schumacher and Hall, 1933 [23]

Equations

# ##Article ID 6 : (Martins, Debastiani, Pelissari, et al., 2017) Estimativa do {Afilamento} do {Fuste} de {AraucÃ;ria} {Utilizando} {Técnicas} de {InteligÃancia} {Artificial}

Title.student	Araucaria Stem Taper or Use of Artificial Intelligence Techniques
Authors.student	Ana Paula Marques Martins, Aline Bernarda Debastiani, Allan Libanio
	Pelissari, Sebastião do Amaral Machado, Carlos Roberto Sanquetta

 Year.student
 2017

 Species
 Araucaria angustifolia

 Base.URL
 http://www.scielo.br/scielo.php?script=sci\_arttext&pid= \$2179-80872017000100152

 Paper.local.file
 2179-8087-floram-24-e20160234.pdf

 Equations
 NA

## ##Article ID 7: (Silva, Rodriguez, Caixeta Filho, et al., 2006) Fitting a taper function to minimize the sum of absolute deviations

Title.student	Fitting a taper function to minimize the sum of absolute deviations
Authors.student	Lana Mirian Santos da Silva, Luiz Carlos Estraviz Rodriguez, José Vicente
	Caixeta Filho; Simone Carolina Bauch
Year.student	2006
Species	Eucalyptus
Base.URL	http://www.scielo.br/scielo.php?script=sci_arttext&pid=
	S0103-90162006000500007
Paper.local.file	31406.pdf
Equations	NA

# ##Article ID 8 : (Arnoni Costa, Guimarães Finger, Schneider, et al., 2016) {FUNÇÃfO} {DE} {AFILAMENTO} {E} {SORTIMENTOS} {DE} {MADEIRA} {PARA} {Araucaria} angustifolia

Title.student	Taper function and timber assortments for Araucaria angustifolia
Authors.student	Emanuel Arnoni Costa, César Augusto Guimarães Finger, Paulo Renato
	Schneider, André Felipe Hess
Year.student	2016
Species	Araucaria angustifolia
Base.URL	http://www.redalyc.org/articulo.oa?id=53446151016
Paper.local.file	$53446151016.\mathrm{pdf}$
Equations	NA

# ##Article ID 9 : (Souza, Chassot, Finger, et al., 2008) Modelos de afilamento para o sortimento do fuste de {Pinus} taeda {L}

Title.student	Taper function for assortment of Pinus taeda L. stem
Authors.student	Carlos Alberto Martinelli de Souza, Tatiane Chassot, César Augusto
	Guimarães Finger, Paulo Renato Schneider, Frederico Dimas Fleig
Year.student	2008
Species	Pinus taeda L
Base.URL	http://www.scielo.br/scielo.php?script=sci_arttext&pid=
	S0103-84782008000900014
Paper.local.file	a14v38n9.pdf
Equations	NA

##Article ID 10 : (Arias-Rodil, Castedo-Dorado,  $C\tilde{A}_{i}$ mara-Obreg $\tilde{A}^{3}$ n, et al., 2015) Fitting and {Calibrating} a {Multilevel} {Mixed}-{Effects} {Stem} {Taper} {Model} for {Maritime} {Pine} in {NW} {Spain}

Title.student	Fitting and Calibrating a Multilevel Mixed-Effects Stem Taper Model for Maritime Pine in NW Spain
Authors.student	Manuel Arias-Rodil, Fernando Castedo-Dorado, Asunción Cámara-Obregón,
Year.student	Ulises Diéguez-Aranda 2015
Species	Pinus pinaster Ait.
Base.URL	http://europepmc.org/backend/ptpmcrender.fcgi?accid=PMC4668033& accid=PMC4668033& accid=PMC4680804&
	blobtype = pdf
Paper.local.file	pone.0143521.pdf
Equations	NA

## ##Article ID 11: (RodrÃguez, Lizarralde, and Bravo, 2015) Comparison of stem taper equations for eight major tree species in the {Spanish} {Plateau}

Title.student	Comparison of stem taper equations for eight major tree species in the Spanish Plateau
Authors.student	Francisco Rodríguez1, Iñigo Lizarralde1 and Felipe Bravo
Year.student	2015
Species	Various
Base.URL	http://revistas.inia.es/index.php/fs/article/view/6229
Paper.local.file	6229-27194-1-PB.pdf
Equations	NA

# ##Article ID 12 : $(N\tilde{A}_{j}var, Rodr\tilde{A}guez\text{-}Flores, and Dom\tilde{A}nguez\text{-}Calleros, 2013)$ Taper functions and merchantable timber for temperate forests of northern {Mexico}

Title.student	Taper functions and merchantable timber for temperate forests of northern
	Mexico
Authors.student	J. Návar, F. de Jesús Rodríguez-Flores, P.A. Domínguez-Calleros
Year.student	2013
Species	P.pseudostrobus, P. hartwegii, P. cooperi, P. ayacahuite, Q. spp, P.
	durangensis, P. leiophylla, P. teocote, P. arizonica, Quercus spp
Base.URL	http://www.editurasilvica.ro/afr/56/1/navar.pdf
Paper.local.file	navar.pdf
Equations	NA

# ##Article ID 13 : $(\tilde{A}-z\tilde{A}\S elik\ and\ Dirican,\ 2017)$ Stem taper and volume models for natural cedar and {Taurus} fir mixed stands in {Bucak} {District}

Title.student	Individual taper models for natural cedar and taurus fir mix stands of Bucak
	region, Turkey
Authors.student	Ramazan Özçelik, Osman Dirican
Year.student	2017
Species	Cedrus libani A. Rich., Abies cilicica Carr.
Base.URL	http://dergipark.gov.tr/download/article-file/330518
Paper.local.file	10.17099-jffiu.290845-330518.pdf

$$d = c_1 \sqrt{H^{(k-b_4)/b_4} (1-Z)^{(k-b)/b}} \alpha_1^{I_1+I_2} \alpha_2^{I_2}$$
 where:  $k = \pi/40,000$ ,  $Z = h/H$ ,  $\begin{cases} I_1 = 1 \text{ if } p_1 \leq Z \leq p_2; 0 \text{ otherwise} \\ I_2 = 1 \text{ if } p_2 < Z \leq 1; 0 \text{ otherwise} \end{cases}$ ,  $p_1 = h_1/H$  and  $p_2 = h_2/H$  ( $h_1$  and  $h_2$  are the heights from ground level where the two inflection points assumed in the model occur),  $b = b_4^{1-(I_1+I_2)} b_5^{I_1} b_6^{I_2}$ , Fang et al. (2000) 
$$\alpha_1 = (1-p_1)^{(b_5-b_4)k/b_4b_5}, \quad \alpha_2 = (1-p_2)^{(b_6-b_3)k/b_5b_6}, \quad r_0 = ((1-h_{st})/H)^{k/b_4}, \quad r_1 = (1-p_1)^{k/b_4}, \quad r_2 = (1-p_2)^{k/b_5},$$
 
$$c_1 = \sqrt{\frac{b_1 D^{b_2} H^{b_3-k/b_4}}{b_4(r_0-r_1) + b_5(r_1-\alpha_1r_2) + b_6\alpha_1r_2}},$$
 The compatible models for merchantable ( $\nu$ ) and total volume ( $\nu$ ) from stump height are:  $\nu = c_1^2 H^{k/b_4} \left(b_4 r_0 + (I_1 + I_2)(b_5 - b_4)r_1 + I_2(b_6 - b_5)\alpha_1r_2 - \beta(1-Z)^{k/\beta}\alpha_1^{I_1+I_2}\alpha_2^{I_2}\right)$   $V = b_1 D^{b_2} H^{b_3}$ 

Equations Area geografica: Abstract:

Regione del Bucak, Turchia

In this study, we assessed the performance of different types of taper equations for predicting tree diameters at specific heights and total stem volumes for mixed stands of Taurus cedar (Cedrus libani A. Rich.) and Taurus fir (Abies cilicica Carr.). We used data from mixed stands containing a total of 131 cedar and 124 Taurus fir trees. We evaluated six commonly used and well-known forestry taper functions developed by a variety of researchers (Biging (1984), Zakrzewski (1999), Muhairwe (1999), Fang et al. (2000), Kozak (2004), and Sharma and Zhang (2004)). To address problems related to autocorrelation and multicollinearity in the hierarchical data associated with the construction of taper models, we used appropriate statistical procedures for the model fitting. We compared model performances based on the analysis of three goodness-of-fit statistics and found the compatible segmented model of Fang et al. (2000) to be superior in describing the stem profile and stem volume of both tree species in mixed stands. The equation used by Zakrzewski (1999) exhibited the poorest fitting results of the three taper equations. In general, we found segmented taper equations to provide more accurate predictions than variable-form models for both tree species. Results from the non-linear extra sum of squares method indicate that stem tapers differ among tree species in mixed stands. Therefore, a different taper function should be used for each tree species in mixed stands in the Bucak district. Using individual-specific taper equations yields more robust estimations and, therefore, will enhance the prediction accuracy of diameters at different heights and volumes in mixed stands.

##Article ID 14 : (Machado, Urbano, and Conceiç£o, 2005) Comparação de métodos de estimativa de volume para {Pinus} oocarpa em diferentes idades e diferente iregimes de desbastes

Title.student	Comparação de Métodos de Estimativa de Volume para Pinus oocarpa em
	Diferentes Idades e Diferentes Regimes de Desbastes
Authors.student	Sebastião do Amaral Machado, Edilson Urbano, Marcio Barbosa da Conceição
Year.student	2005
Species	Pinus oocarpa
Base.URL	https://pfb.cnpf.embrapa.br/pfb/index.php/pfb/article/view/242/193
Paper.local.file	242-1027-1-PB.pdf
Equations	NA

##Article ID 15: (Duan, Zhang, Et al., 2016) Development of a stem taper equation and modelling the effect of stand density on taper for {Chinese} fir plantations in {Southern} {China}

Title.student	Development of a stem taper equation and modelling the effect of stand
	density on taper for chinese fir plantations in southern China
Authors.student	Alguo Duan, Sensen Zhang, Xiongqing Zhang, Jianguo Zhang
Year.student	2016
Charing	Abete sinese (Cunninghamia langualeta)

Species Abete cinese (Cunninghamia lanceolata)
Base.URL https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4860307
Paper.local.file peerj-04-1929.pdf

Based on dimensional analysis, *Sharma & Oderwald (2001)* developed a dimensionally compatible one-parameter taper equation:

$$d^{2}(h) = D^{2}\left(\frac{h}{1.3}\right)^{2-b_{1}}\left(\frac{H-h}{H-1.3}\right) \tag{4}$$

where, D: diameter at breast height.

Sharma & Zhang (2004) assumed that the  $b_1$  in Eq. (4) could be expressed in terms of the relative height (z), and resulted in a variable-exponent taper equation, i.e.,

$$d^{2}(h) = b_{1}D^{2}\left(\frac{h}{1.3}\right)^{2-(b_{2}+b_{3}z+b_{4}z^{2})}\left(\frac{H-h}{H-1.3}\right)$$
 (5)

where,  $z = \frac{h}{H}$ ,  $b_4$  is parameter.

Kozak (2004) developed a variable-exponent taper equation as

$$d = b_1 D^{b_2} H^{b_3} \left[ \frac{1 - z^{1/3}}{1 - p^{1/3}} \right]^{[b_4 z^4 + b_5 (1/\exp(D/H)) + b_6 (\frac{1 - z^{1/3}}{1 - p^{1/3}})^{0.1} + b_7 (1/D) + b_8 H^{1 - z^{1/3}} + b_9 (\frac{1 - z^{1/3}}{1 - p^{1/3}})]}$$
(6)

where,  $b_4$ ,  $b_5$ ,  $b_6$ ,  $b_7$ ,  $b_8$ ,  $b_9$  and p are parameters.

Equations Area geografica Abstract:

Contea di Fenyi, provincia di Jiangxi, Sud della Cina

Chinese fir (Cunninghamia lanceolata) is the most important commercial tree species in southern China. The objective of this study was to develop a variable taper equation for Chinese fir, and to quantify the effects of stand planting density on stem taper in Chinese fir. Five equations were fitted or evaluated using the diameter-height data from 293 Chinese fir trees sampled from stands with four different densities in Fenyi County, Jiangxi Province, in southern China. A total of 183 trees were randomly selected for the model development, with the remaining 110 trees used for model evaluation. The results show that the Kozak's, Sharma/Oderwald, Sharma/Zhang and modified Brink's equations are superior to the Pain/Boyer equation in terms of the fitting and validation statistics, and the modified Brink's and Sharma/Zhang equations should be recommended for use as taper equations for Chinese fir because of their high accuracy and variable exponent. The relationships between some parameters of the three selected equations and stand planting densities can be built by adopting some simple mathematical functions to examine the effects of stand planting density on tree taper. The modelling and prediction precision of the three taper equations were compared with or without incorporation of the stand density variable. The predictive accuracy of the model was improved by including the stand density variable and the mean absolute bias of the modified Brink's and Sharma/Zhang equations with a stand density variable were all below 1.0 cm in the study area. The modelling results showed that the trees have larger butt diameters and more taper when stand density was lower than at higher stand density.

#### References

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