Vectors as Arrays and Other Data Structures

THE OBJECTIVES OF THIS CHAPTER ARE:

- To enable you to solve problems which involve working carefully with array subscripts
- To introduce you to:
- Structures
- Cells and cell arrays

In MATLAB an array is just another name for a vector. So why have a large part of a chapter on arrays, when we have already been using vectors for most of the book? It is helpful to talk about arrays (as opposed to vectors) when we are concerned with handling individual elements by means of their subscripts, rather than with the vector as a whole. Thus, in the first three sections of this chapter we look at a number of problems which are best solved by treating vectors as arrays, usually with the help of for loops.

In the last three sections of this chapter we deal with other, more advanced, data structures.

10.1 UPDATE PROCESSES

In Chapter 8 (**Loops**) we considered the problem of calculating the temperature of orange juice (OJ) in a can as it cools in a fridge. This is an example of an *update process*, where the main variable is repeatedly updated over a period of time. We now examine how to solve this problem more generally.

The OJ is initially at temperature 25 °C when it is placed in the fridge, where the ambient temperature F is 10°. The standard way to solve such an update process is to break the time period up into a number of small steps, each of

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length dt. If T_i is the temperature at the beginning of step i, we can get T_{i+1} from T_i as follows:

$$T_{i+1} = T_i - K dt(T_i - F),$$
 (10.1)

where *K* is a physical constant, and units are chosen so that time is in minutes.

10.1.1 Unit time steps

We first solve the problem using a unit time step, i.e., dt=1. The simplest way is to use scalars for the time and the temperature, as we saw in Chapter 8 (although we didn't use a unit time step there):

Note the use of rem to display the results every 5 min: When time is an integer multiple of 5 its remainder when divided by 5 will be zero.

While this is undoubtedly the easiest way of writing the script, we cannot easily plot the graph of temperature against time. In order to do that, time and T must be vectors. The index of the for loop must be used as the subscript of each element of T. Here's the script (update1.m):

See Figure 10.1 for typical graphs.

Note:

- The statement time = 0:100 sets up a (row) vector for time where time(1) has the value 0 min, and time(101) has the value 100 min. This is necessary because the first subscript of a MATLAB vector must be 1.
- The statement T=zeros(1,101) sets up a corresponding (row) vector for the temperature, with every element initialized to zero (again there must be 101 elements, because the first element is for the temperature at time zero). This process is called *pre-allocation*. It serves two important purposes.
 - a. Firstly, it clears a vector of the same name left over from a previous run. This could cause a conflict when attempting to display or plot T against time, if the vectors have different sizes. To see this, run update1 as it stands. It should work perfectly. Now suppose you decide to do the calculations over a shorter time period, say 50 min. Remove the zeros statement, and make the following additional two changes and re-run the script (but don't clear the workspace):

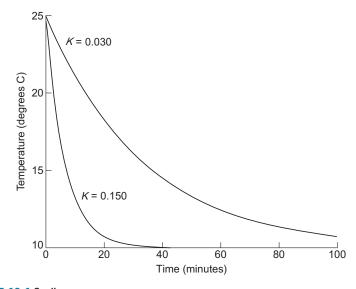


FIGURE 10.1 Cooling curves.

This time you get an error message:

```
??? Error using = \Rightarrow plot
Vectors must be the same lengths.
```

whos will reveal that time is correctly 51-by-1, but T is still 101-by-1. The plot command naturally requires these vectors to have the same lengths.

The problem is that, while the operation 0:50 redefines time correctly, the for loop does not have the same effect on T. Since T is being updated element by element, the unused elements from 51 to 101 from the previous run are left untouched in the workspace. Pre-allocation of the correct number of elements to T with zeros avoids this problem.

b. Secondly, although the script will work without the zeros statement, as we have seen, it will be much *slower*, since then T has to be *redimensioned* during each repeat of the for loop, in order to make space for a new element each time.

It's extremely instructive to do an experiment using a vector size of 10,000 elements, say, i.e.,

(and comment out the disp and plot statements, since these will obscure the issue). My Pentium II takes 0.99 s to run the script with pre-allocation of T, but 13.19 s to run without pre-allocation—more than 10 times longer. This could be a critical consideration in a script which does a lot of such element-by-element processing.

- The first element of T is set to the initial temperature of the OJ. This is the temperature at time zero.
- The for loop computes the values for T(2), ..., T(101).
 This arrangement ensures that temperature T(i) corresponds to time(i).
- The colon operator is used to display the results at 5-min intervals.

10.1.2 Non-unit time steps

It's not always appropriate and/or accurate enough to take dt=1 in Equation (10.1). There is a standard way of generating the solution vector in MATLAB, given (almost) any value of dt. We introduce a more general notation to do this.

Call the initial time a, and the final time b. If we want time steps of length dt, the number m of such steps will be:

$$m = (b - a)/dt$$
.

The time at the *end* of step *i* will therefore be a+i dt.

The following script, update 2.m, implements this scheme. It has some additional features. It prompts you for a value of dt, and checks that this value gives an integer number of steps m. It also asks you for the output interval opint (the intervals in minutes at which the results are displayed in table form) and checks that this interval is an integer multiple of dt. Try the script with same sample values, e.g., dt = 0.4 and opint = 4.

```
K = 0.05;
F = 10;
a = 0;
                      % initial time
                      % final time
b = 100;
load train
dt = input( 'dt: ');
opint = input( 'output interval (minutes): ');
if opint/dt ~= fix(opint/dt)
  sound(y, Fs)
  disp( 'output interval is not a multiple of dt!')
  break
end;
m = (b - a) / dt; % m steps of length dt
if fix(m) = m
                     % make sure m is integer
  sound(y, Fs)
  disp( 'm is not an integer - try again!');
  break
end:
T = zeros(1,m+1);
                     % pre-allocate (m+1) elements
time = a:dt:b;
T(1) = 25;
                      % initial temperature
for i = 1:m
  T(i+1) = T(i) - K * dt * (T(i) - F);
end;
disp( [time(1:opint/dt:m+1)', T(1:opint/dt:m+1)'] )
plot(time, T), grid
```

Note:

- The vectors T and time must each have *m* + 1 elements, because there are *m* time steps, and we need an extra element for the initial value of each vector.
- The expression opint/dt gives the index increment for displaying the results, e.g., dt = 0.1 and opint = 0.5 displays every (0.5/0.1)th element, i.e., every 5th element.

10.1.3 Using a function

The really cool way of solving this problem is to write a function to do it. This makes it a lot easier to generate a table of results for different values of dt, say, using the function from the command line. Here is update2.m rewritten as a function cooler.m:

Suppose you want to display a table of temperatures against time at 5 min intervals, using dt=1 and dt=0.1. Here is how to do it (in the Command Window):

```
dt = 1;
[t T m] = cooler(0, 100, 0.05, 10, dt, 25);
```

```
table(:,1) = t(1:5/dt:m+1)';
table(:,2) = T(1:5/dt:m+1)';
dt = 0.1;
[t T m] = cooler(0, 100, 0.05, 10, dt, 25);
table(:,3) = T(1:5/dt:m+1)';
format bank
disp(table)
```

Output:

0	25.00	25.00
5.00	21.61	21.67
10.00	18.98	19.09
100.00	10.09	10.10

Note:

- The advantage of using a function which generates a vector output variable is that even if you forget to pre-allocate the vector inside the function (with zeros) MATLAB automatically clears any previous versions of the output vector before returning from the function.
- The variable table is a two-dimensional array (or matrix). Recall that the colon operator may be used to indicate all elements of a matrix in a row or column. So table(:,1) means the elements in every row and column 1, i.e., the entire first column. The vectors t and T are row vectors, so they must be transposed before being inserted into columns 1 and 2 of table. The third column of table is inserted in a similar way.
- The results in the third column (for dt = 0.1) will be more accurate.

10.1.4 Exact solution

This cooling problem has an exact mathematical solution. The temperature T(t) at time t is given by the formula:

$$T(t) = F + (T_0 - F)e^{-Kt}, (10.2)$$

where T_0 is the initial temperature. You can insert values for this exact solution into a fourth column of table, by vectorizing the formula, as follows:

```
tab(:,4) = 10 + (T(1)-10)*exp(-0.05 * t(1:5/dt:m+1)');
```

The enlarged table should look something like this:

0	25.00	25.00	25.00
5.00	21.61	21.67	21.68
10.00	18.98	19.09	19.10

Note that the numerical solution generated by Equation (10.1) gets more accurate as dt gets smaller. That is because Equation (10.2) (the exact solution) is derived from Equation (10.1) in the limit as $dt \rightarrow 0$.

Exercise

To complete the analysis of the cooling problem, you may like to superimpose graphs for different values of K and draw them in different colours. It would also be useful to label each graph at some convenient place, e.g., with the label ${}^{L}K = 0.08$. The command:

```
gtext( 'text' )
```

causes an arrow to appear on the graphics screen. You can position the arrow with a mouse or the arrow keys. When you click (or press Enter), the specified text appears at that point.

The function gtext takes a string argument. If you want to label it with a variable numeric value, use sprintf to convert a number into a string. It works just like fprintf, except that the output goes into a string variable, e.g.,

gtext(sprintf('
$$K = \%5.3f', K$$
)

■ Draw some graphs, on the same set of axes, for different values of *K*, in different colours, and label them. Figure 10.1 shows what to aim for. Also superimpose the exact solution, given by Equation (10.2).

Plots can also be labelled interactively in plot editing mode or with the Properties Editor.

10.2 FREQUENCIES, BAR CHARTS, AND HISTOGRAMS

10.2.1 A random walk

Imagine an ant walking along a straight line, e.g., the x-axis. She starts at x = 40. She moves in steps of one unit along the line. Each step is to the left or the right with equal probability. We would like a visual representation of how much time she spends at each position.

Start by running the following script, ant.m:

```
f = zeros(1,100);
x = 40;

for i = 1:1000
    r = rand;
    if r >= 0.5
        x = x + 1;
    else
        x = x - 1;
    end
    if x ~= 0 | x ~= 100
```

Now enter the statement bar(f) in the Command Window. You should get a graph which is similar to the one in Figure 10.2.

Note:

- The function rand returns a random number in the range 0–1. If it's greater than 0.5, the ant moves right (x=x+1), otherwise she moves left (x=x-1).
- The vector f has 100 elements, initially all zero. We define f(x) as the number of times the ant lands at position x. Suppose her first step is to the right, so x has the value 41. The statement:

$$f(x) = f(x) + 1$$

then increases the value of f(41) to 1, meaning that she has been there once. When she next wanders past this value of x, f(41) will be increased to 2, meaning she's been there twice.

When I ran this script, the final value of f (41) was 33—the number of times the ant was there.

f(x) is called a frequency distribution, and the graph obtained from bar(f) is called a bar graph. Each element of f is represented by a vertical bar of proportional height. See also the help bar.

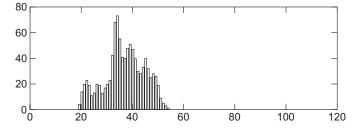


FIGURE 10.2 Random walk frequencies.

The script ant.m simulates the random movement of the ant. If you re-run it, you will get a different bar graph, because rand will generate a different sequence of random numbers. Simulation is discussed more fully in Chapter 13.

10.2.2 Histograms

Another helpful way of representing data is with a histogram.

As an example, suppose 12 students write a test and obtain the following marks (expressed as percentages), which are assigned to the vector m:

```
0 25 29 35 50 55 55 59 72 75 95 100
```

The statement hist(m) will draw a histogram. Try it. The histogram shows the distribution of marks in 10 "bins" (categories) which are equally spaced between the minimum (0) and maximum (100) marks in the set of data. The number of bins (10 by default) can be specified by a second argument, e.g., hist(m, 25).

To generate the frequences plotted by hist use the following form (which does not actually draw the histogram):

$$[nx] = hist(m)$$

n is a vector containing the frequences, viz.,

1 0 2 1 0 4 0 2 0 1 1

For example, there is one mark in the first bin (0-9), none in the second (10-19), two in the third, and so on.

The second output vector x contains the midpoints of the bins, such that bar(x, n) plots the histogram. See Help for more details.

Note the subtle difference between a histogram and a bar graph. The values plotted by hist are computed from the distribution of values in a vector, whereas bar generates a bar graph directly from the values themselves.

10.3 SORTING

One of the standard applications of arrays is sorting a list of numbers into, say, ascending order. Although MATLAB has its own sorting function (sort), you may be interested in how sorting algorithms actually work.

10.3.1 Bubble sort

The basic idea is that the unsorted list is assigned to a vector. The numbers are then ordered by a process which essentially passes through the vector many times, swopping consecutive elements that are in the wrong order, until all the elements are in the right order. Such a process is called a Bubble Sort, because the smaller numbers rise to the top of the list, like bubbles of air in water. (In fact, in the version shown below, the largest number will "sink" to the bottom of the list after the first pass, which really makes it a "Lead Ball" sort.) There are many other methods of sorting, such as the Quick Sort, which may be found in most textbooks on computer science. These are generally more efficient than the Bubble Sort, but its advantage is that it is by far the easiest method to program. A structure plan for the Bubble Sort is as follows:

- 1. Input the list X
- 2. Set N to the length of X
- 3. Repeat N-1 times with counter K: Repeat N-K times with counter J: If $X_j > X_{j+1}$ then Swop the contents of X_j and X_{j+1}
- 4. Stop since the list X is now sorted.

As an example, consider a list of five numbers: 27, 13, 9, 5, and 3. They are initially input into the vector X. Part of MATLAB's memory for this problem is sketched in Table 10.1. Each column shows the list during each pass. A stroke in a row indicates a change in that variable during the pass as the script works down the list. The number of tests $(X_j > X_{j+1}?)$ made on each pass is also shown in the table. Work through the table by hand with the structure plan until you understand how the algorithm works.

Sorting algorithms are compared by calculating the number of tests (comparisons) they carry out, since this takes up most of the execution time during the sort. On the Kth pass of the Bubble Sort there are exactly N-K tests, so the total number of tests is:

$$1+2+3+\cdots+(N-1)=N(N-1)/2$$

(approximately $N^2/2$ for large N). For a list of five numbers there are therefore 10 tests, but for 10 numbers there are 45 tests. The computer time needed goes up as the square of the length of the list.

Table 10.1 Memory During a Bubble Sort					
	1st pass	2nd pass	3rd pass	4th pass	
X₁:	27/13	13/9	9/5	5/3	
X ₂ : X ₃ : X ₄ : X ₅ :	13/27/9	9/13/5	5/9/3	3/5	
X ₃ :	9/27/5	5/13/3	3/9	9	
X ₄ :	5/27/3	3/13	13	13	
<i>X</i> ₅ :	3/27	27	27	27	
-	4 tests	3 tests	2 tests	1 test	

The function M-file bubble.m below departs slightly from the structure plan above, which will make N-1 passes, even if the list is sorted before the last pass. Since most real lists are partially sorted, it makes sense to check after each pass if any swops were made. If none were, the list must be sorted, so unnecessary (and therefore time-wasting) tests can be eliminated. In the function, the variable sorted is used to detect when the list is sorted, and the outer loop is coded instead as a non-deterministic while loop. Here it is:

```
function y = bubble( x )
n = length(x);
sorted = 0;
                         % flag to detect when sorted
                         % count the passes
k = 0;
while ~sorted
  sorted = 1;
                         % they could be sorted
  k = k + 1;
                         % another pass
  for j = 1:n-k % fewer tests on each pass if x(j) > x(j+1) % are they in order?
                         % no ...
      temp = x(j);
      x(j) = x(j+1);
      x(j+1) = temp;
      sorted = 0;
                       % a swop was made
    end
    end
  end;
  y = x;
```

You can test it on the command line to sort say 20 random numbers as follows:

```
r = rand(1,20);

r = bubble(r);
```

Note how bubble changes its input vector.

On my PC bubble takes 1.81 s to sort 200 random numbers, but 7.31 s to sort 400 numbers. This is consistent with the theoretical result obtained above.

10.3.2 MATLAB's sort

The built-in MATLAB function sort returns two output arguments: The sorted list (in ascending order) and a vector containing the *indexes* used in the sort, i.e., the positions of the sorted numbers in the original list. If the random numbers:

```
r = 0.4175 \quad 0.6868 \quad 0.5890 \quad 0.9304 \quad 0.8462
```

are sorted with the command:

$$[y, i] = sort(r)$$

the output variables are:

$$y = 0.4175$$
 0.5890 0.6868 0.8462 0.9304
 $i = 1$ 3 2 5 4

For example, the index of the second largest number (0.5890) is 3, which is its subscript in the original unsorted list r.

As a matter of fact, the built-in functions max and min also return second output variables giving indexes.

MATLAB's sort is *very* fast. My PC takes 2.36 s to sort a list of one million random numbers! This is because (a) a Quick Sort is used, and (b) the script has been compiled as a built-in function; this makes for faster code.

10.4 STRUCTURES

Up to now we have seen arrays with only one type of element—all numeric, or all character. A MATLAB *structure* allows you to put different kinds of data in its various *fields*. For example, we can create a structure called student with one field for a student's name,

```
student.name = 'Thandi Mangwane';
a second field for her student ID number,
student.id = 'MNGTHA003';
and a third field for all her marks to date,
student.marks = [36 49 74];
```

To see the whole structure, enter its name:

```
student
student =
    name: 'Thandi Mangwane'
    id: 'MNGTHA003'
    marks: [36 49 74]
```

Here's how to access her second mark:

```
student.marks(2)
ans =
    49
```

Note the use of the dot to separate the structure name from its fields when creating it and when accessing its fields.

To add further elements to the structure, use subscripts after the structure name:

```
student(2).name = 'Charles Wilson'
student(2).id = 'WLSCHA007'
student(2).marks = [49 98]
```

(the original student, Thandi Mangwane, is now accessed as student(1)). Note that field sizes do not have to conform across elements of a structure array: student(1).marks has three elements, while student(2).marks has only two elements.

The student structure now has size 1-by-2: It has two elements, each element being a student with three fields. Once a structure has more than one element MATLAB does not display the contents of each field when you type the structure name at the command line. Instead, it gives the following summary:

```
student =
1x2 struct array with fields:
    name
    id
    marks
```

You can also use fieldnames (student) to get this information.

A structure array can be preallocated with the struct function. See Help.

A structure field can contain any kind of data, even another structure—why ever not? Thus we can create a structure course of courses taken by students, where one field is the name of the course, and another field is a student structure with information on all the students taking that particular course:

```
course.name = 'MTH101';
course.class = student;
course

course =
    name: 'MTH101'
    class: [1x2 struct]
```

We can set up a second element of course for a different class of students:

```
course(2).name = 'PHY102';
course(2).class = ...

To see all the courses:
    course(1:2).name
    ans =
    MTH101
    ans =
    PHY102
```

To see all the students in a particular course:

```
course(1).class(1:2).name
ans =
Thandi Mangwane
ans =
Charles Wilson
```

There is a curious function called deal which "deals inputs to outputs." You can use it to generate "comma-separated variable lists" from structure fields:

```
[name1, name2] = deal(course(1).class(1:2).name);
```

(but you don't actually need the commas here ...).

You can use the rmfield function to remove fields from a structure.

10.5 CELL ARRAYS

A *cell* is the most general data object in MATLAB. A cell may be thought of as a "data container," which can contain any type of data: numeric arrays, strings, structures, or cells. An array of cells (and they almost always occur in arrays) is called a *cell array*. While you might think a cell sounds the same as a structure, a cell is more general, and there are also notational differences (which are confusing).

10.5.1 Assigning data to cell arrays

There are a number of ways of assigning data to cell arrays.

Cell indexing:

```
c(1,1) = {rand(3)};
c(1,2) = {char('Bongani', 'Thandeka')};
c(2,1) = {13};
c(2,2) = {student};

c =
   [3x3 double] [2x8 char]
   [ 13] [1x2 struct]
```

(assuming that the structure student created above still exists). Here the round brackets on the lefthand side of the assignments refer in the normal way to the elements of the cell array. What is different are the curly braces on the right. Curly braces indicate the *contents* of a cell; on the righthand side of an assignment they are technically cell array *constructors* (remember that each element of this array is a cell).

So read the first statement as:

"Construct a cell containing rand(3) and assign the cell to element 1,1 of the cell array c."

Content indexing:

```
c{1,1} = rand(3);
c{1,2} = char('Bongani', 'Thandeka');
c{2,1} = 13;
c{2,2} = student;
```

Here the curly braces on the left indicate the *contents* of the cell element at that particular location. So read the first statement as:

"The contents of the cell at location 1,1 becomes rand(3)."

You can use the curly braces to construct an entire cell array in one statement:

```
b = {[1:5], rand(2); student, char('Jason', 'Amy')}
b =
    [1x5 double]    [2x2 double]
    [1x2 struct]    [2x5 char ]
```

A cell may contain another cell array; nested curly braces may be used to create nested cell arrays.

■ The cell function enables you to preallocate empty cell arrays, e.g.,

You can then use assignment statements to fill the cells, e.g.,

```
a(2,2) = \{magic(3)\}
```

Note: if you already have a numeric array with a certain name, don't try to create a cell array of the same name by assignment without first clearing the numeric array. If you don't clear the numeric array, MATLAB will generate an error (it will think you are trying to mix cell and numeric syntaxes).

10.5.2 Accessing data in cell arrays

You can access cell contents using content indexing (curly braces):

```
r = c{1,1}
r =
0.4447 0.9218 0.4057
0.6154 0.7382 0.9355
0.7919 0.1763 0.9169
```

To access a subset of a cell's contents you can concatenate curly braces and round brackets if necessary:

```
rnum = c{1,1}(2,3)
rnum =
    0.9355
```

Here the curly braces (content indexing) indicate the contents of the cell array element c(1,1), which is a 3-by-3 numeric matrix. The subscripts (2,3) then indicate the appropriate element within the matrix.

Curly braces may be concatenated to access nested cell arrays.

10.5.3 Using cell arrays

Cell arrays come into their own when you need to access (different types of) data as "comma-separated variable lists," as the next example demonstrates.

The functions varargin and varargout, which allow a function to have any number of input or output arguments, are none other than cell arrays. The function testvar has a variable number of input arguments, which are doubled into the variable number of output arguments (assuming that the number of output arguments does not exceed the number of input arguments):

Command line:

When a function is called with the input argument varargin MATLAB automatically packs the list of corresponding input arguments into a cell array. You then simply unpack the cell array inside the function using the curly brace content indexing. You similarly pack up the output arguments into the cell array varargout.

Note that, should your function have some compulsory input and output arguments, varargin and varargout must appear at the end of their respective argument lists.

MATLAB has a discussion on when to use cell arrays in the section **Organizing Data in Cell Arrays**, to be found in MATLAB Help: **Programming and Data Types: Structures and Cell Arrays**.

10.5.4 Displaying and visualizing cell arrays

The function celldisp recursively displays the contents of a cell array.

The function cellplot draws a visualization of a cell array. Figure 10.3 depicts the contents of the cell array c, created above. Non-empty array elements are shaded.

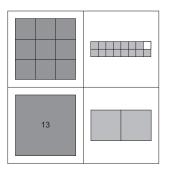


FIGURE 10.3 Result of cellplot(c).

10.6 CLASSES AND OBJECTS

You'd have to be living face down in a moon crater not to have heard about object-oriented programming. *Tom Swan*.

Object-oriented programming is an exceptionally bad idea which could only have originated in California. *Edsger Dijkstra*.

MATLAB, along with most other modern programming languages, espouses the cause of object-oriented programming, and has all the usual paraphernalia associated with this paradigm: classes, objects, encapsulation, inheritance, operator overloading, and so on.

Object-oriented programming is a subject which requires an entire book to do justice to it. If you want to learn about the concepts, there are many excellent books on the subject. If you want to see how MATLAB implements object-oriented programming, consult the online Help: MATLAB Help: Programming and Data Types: MATLAB Classes and Objects.

SUMMARY

- A MATLAB structure allows you to store different types of data in its various fields.
- Arrays of structures may be created.
- A cell is the most general data object in MATLAB, and can store any type of data.
- An array of cells is called a cell array—each cell in the array can store different types of data, including other cell arrays.
- A cell array is constructed with curly braces {}.
- The contents of cell in a cell array is accessed with content indexing (curly braces).
- Cell elements in a cell array are accessed in the usual way with round brackets (cell indexing).
- Arguments of functions using a variable number of arguments are packed into cell arrays.
- The function cellplot gives a visualization of a cell array.
- MATLAB implements object-oriented programming.