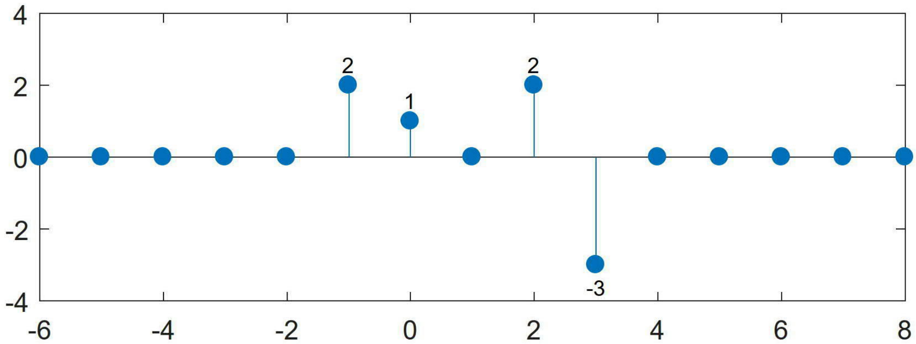
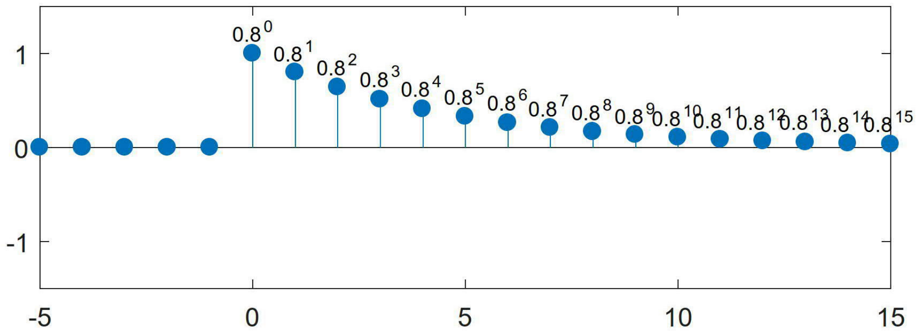
SGN-11007 Introduction to Signal Processing,
Exercise 4, Fall 2020

Task 1. *(Pen & paper)*

(a) Calculate the z-transform expression of the signal in the figure below.



(b) Calculate the z-transform expression of the signal in the figure below.



(c) Calculate the DTFT expressions of the signals in (a) and (b).

Task 2. *(Pen & paper)* The z-transform of the impulse response of a filter is

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 + z^{-1} + z^{-2}}.$$

Draw the pole-zero plot of the system. Is the filter stable?

Task 3. *(Pen & paper)* The filter

$$y(n) = \frac{1}{4}x(n) - \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$$

is implemented in hardware with the sampling rate 16000 Hz. What is the amplitude response (i.e. amplification/attenuation) of the filter at the frequency 4000 Hz? *Hint:* Calculate $H(z)$ and $H(e^{i\omega})$, substitute normalized angular frequency ω into the formula you obtained and take the absolute value. The normalized angular frequency corresponding to the frequency f is $\omega = 2\pi f/F_s$, where F_s is the sampling rate.

Task 4. *(Pen & paper)* Assume that the input $x(n)$ and the output $y(n)$ of a causal LTI system satisfy the following difference equation:

$$y(n) = x(n) - 2x(n-1) + \frac{5}{4}x(n-2) + y(n-1) - \frac{5}{16}y(n-2).$$

(a) Determine the transfer function $H(z)$ of the system.

(b) Draw the pole-zero plot of the system.

(c) Is the system stable?

Task 5. *(Pen & paper)* The signal $x(n) = u(n)\sin(\frac{1}{8} \cdot 2\pi n)$ is filtered by a system having the impulse response

$$h(n) = \begin{cases} \frac{1}{2}, & \text{when } n = 0 \text{ or } n = 2, \\ 1, & \text{when } n = 1, \\ 0, & \text{otherwise.} \end{cases}$$

The output is of the form $y(n) = Au(n)\sin(\frac{1}{8} \cdot 2\pi n + \phi)$. Determine the values of the real numbers A and ϕ.

Task 6. *(Matlab)* Consider points $n = 0 : 70$. Filter the signal $x(n) = u(n)\sin(0.05 \cdot 2\pi n)$ with the system in the example on pp. 72-73 in the lecture handout:

$$y(n) = 0.0349x(n) + 0.4302x(n-1) - 0.5698x(n-2) + 0.4302x(n-3) + 0.0349x(n-4).$$

Compare the result with the estimated response $y(n) = 0.3050u(n)\sin(0.05 \cdot 2\pi n - 0.6283)$. Plot the original signal, the estimated response and the true response in the same figure.

Task 7. *(Matlab)* An LTI system is implemented by the difference equation

$$\begin{aligned} y(n) &= 1.143y(n-1) - 0.4128y(n-2) + 0.0675x(n) + 0.1349x(n-1) \\ &+ 0.0675x(n-2) \end{aligned}$$

in a system having sampling frequency 20000 Hz. How large is the attenuation (in dB) for an input signal oscillating at the frequency 5000 Hz? *Hint:* Calculate first the normalized angular frequency $\omega = 2\pi f/F_s$, then the corresponding point at complex plane $z = e^{i\omega}$, and finally the value of the transfer function $H(z)$ at this point. The transfer function $H(z)$ must be calculated manually.

Task 8. *(Matlab)* The transfer function of an LTI system is

$$H(z) = \frac{0.0122 + 0.0226z^{-1} + 0.0298z^{-2} + 0.0204z^{-3} + 0.0099z^{-4}}{1 - 0.9170z^{-1} + 0.0540z^{-2} - 0.2410z^{-3} + 0.1990z^{-4}}.$$

Assign the coefficients to the vectors `a` and `b` and plot the pole-zero plot (`help zplane`), the amplitude and phase responses (`help freqz`) and the impulse response (`help impz`) of the system. Compare these to the corresponding ones of the inverse system

$$H^{-1}(z) = \frac{1 - 0.9170z^{-1} + 0.0540z^{-2} - 0.2410z^{-3} + 0.1990z^{-4}}{0.0122 + 0.0226z^{-1} + 0.0298z^{-2} + 0.0204z^{-3} + 0.0099z^{-4}}.$$

Note that you can utilize the original system when studying the inverse system. So do not rewrite the coefficients.

Load Matlab's test signal `handel` to variable `y` with the command `load handel`. Filter it with filter $H(z)$ and plot the spectrogram. Filter the obtained result with the inverse filter $H^{-1}(z)$ and plot the spectrogram of the result. This spectrogram should be similar to the spectrogram of the original signal. Is it?

Task 9. *(Matlab)* Create a signal `y` with steadily increasing frequency with the commands

```
t=0:1/8192:4;
y=chirp(t,0,1,1000);
```

Listen to the result (if possible) with the command `soundsc(y)`. Alternatively, you can study the signal with the command `spectrogram`. Filter the signal with the filter in Task 7. Listen to the result and/or study its spectrogram. Compare the result to the amplitude response of the filter.

Task 10. *(Matlab)* Download the file `number.mat` from the course Moodle (`Ex_4.zip`). The file contains dialing tone (seven digits) for the push-button phone. Your task is to find out what phone number it is.

Load the signal to Matlab using the command `load number.mat`. The signal is then in the vector called `secret`. You can listen to it with the command `sound(secret)`.

Dialing tones consist of the sum of two components having different frequency (Table 1). Identify these components using Matlab command `spectrogram`. If the identification is difficult, you can zoom in the image by using the magnifying glass tool of Matlab.

Table 1: Dual tone multiple frequencies (DTMF) for push-button phone when the sampling rate is 8192 Hz. For example, the signal corresponding to the button '5' is $x(n) = \sin(0.1880\pi n) + \sin(0.3262\pi n)$.

	0.2952	0.3262	0.3606
0.1702	1	2	3
0.1880	4	5	6
0.2080	7	8	9
0.2297	*	0	#