MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1. (a) Governments issue bonds rather than printing money because excessive money creation leads to inflation, while bonds allow governments to finance spending without immediately eroding the currency's value [1].

   (b) The long-term part of the yield curve might flatten if investors expect slower long-term economic growth and inflation, increasing demand for long-term bonds and pushing their yields down toward short-term rates [4, 3].

   (c) Quantitative easing is a monetary policy in which the Fed purchases large quantities of government and mortgage-backed securities to lower long-term interest rates, which it has done extensively since COVID-19 to support liquidity, stabilize financial markets, and stimulate economic activity[2, 5].

2. The ten selected bonds are: CAN 0.25 Mar 26, CAN 1.00 Sep 26, CAN 1.25 Mar 27, CAN 2.75 Sep 27, CAN 3.50 Mar 28, CAN 3.25 Sep 28, CAN 4.00 Mar 29, CAN 3.50 Sep 29, CAN 2.75 Mar 30, and CAN 2.75 Sep 30. These bonds were chosen because they provide two bonds per calendar year from 2026 to 2030 with maturities spaced approximately six months apart. This spacing is ideal for bootstrapping because it minimizes interpolation error when deriving spot rates at intermediate time points [6].

3. When we have stochastic processes representing points along a curve, the covariance matrix quantifies how movements at different points co-vary over time. The eigenvalues represent the variance captured by independent factors (principal components) ranked by importance, with their sum equaling the total variance in the system. The eigenvectors describe the directional loading pattern of each factor across the curve points, indicating whether that factor causes parallel shifts (all points moving together), tilts (opposite movements at different ends), or more complex deformations. This decomposition allows us to reduce complex multi-dimensional dynamics into a few uncorrelated, interpretable components, with the largest eigenvalues identifying the dominant drivers of variation [7, 8, 9].

Empirical Questions - 75 points

4. (a) Cubic spline interpolation with natural boundary conditions was employed to construct smooth yield curves between the 10 bond maturities. This method ensures continuous first and second derivatives, avoiding arbitrage-implying discontinuities while preventing the oscillation issues common to high-degree polynomial interpolation. The yield curve is shown in Figure 1.[1]

[1] All calculations use dirty prices (closing price + accrued interest) to accurately reflect settlement values.

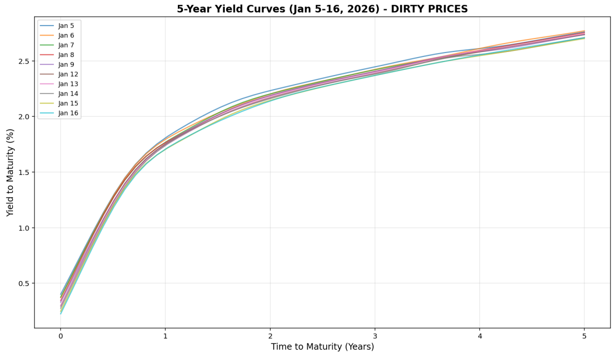

Figure 1: Yield curve across maturities. Boundary effects from natural cubic spline interpolation outside the observed maturity range induce artificial flattening near $T = 0$.

(b) Spot rates are bootstrapped iteratively by discounting known coupon payments using previously computed spot rates and solving for the remaining unknown spot rate under continuous compounding. The following high-level algorithm was utilized to find the spot rate curve.

Algorithm 1 Finding Spot Rates

Initialize spot[0] with YTM of first bond
for $i = 1, 2, \ldots, N-1$ **do**
  Compute T_i, c_i, and number of periods n_i
  prevSum $= \sum_{j=1}^{n_i-1} c_i e^{-s(t_j)t_j}$
  remainingPV $= P_i -$ prevSum
  finalPayment $= c_i + 100$
  spot[i] $= \dfrac{\ln(\text{finalPayment}) - \ln(\text{remainingPV})}{T_i}$
end for

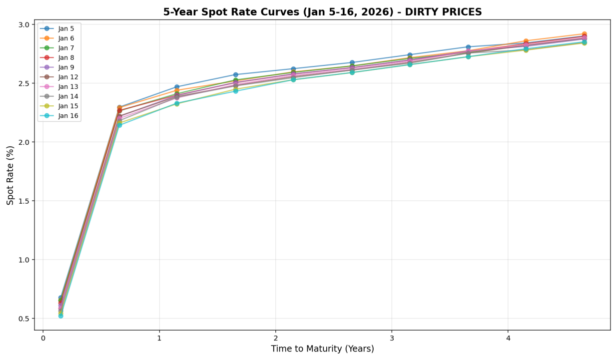

Figure 2: Spot rate curve derived from bond prices, representing zero-coupon yields for each maturity

(c) One-year forward rates are computed from continuously compounded spot rates by interpolating the required spot maturities and applying the forward–spot rate relationship:

$$f_{t_1,t_2} = \frac{s(t_2)\,t_2 - s(t_1)\,t_1}{t_2 - t_1}, \qquad t_1 = 1,\ t_2 \in \{2,3,4,5\}.$$

The following high-level algorithm was utilized to find the 1 year forward rates.

Algorithm 2 Finding 1-Year Forward Rates

Interpolate spot rate at $t_1 = 1$ year
for $t_2 = 2, 3, 4, 5$ **do**
    Obtain spot rate at t_2 by interpolation or extrapolation
    forward $= \dfrac{s(t_2)t_2 - s(t_1)t_1}{t_2 - t_1}$
    Store forward rate
end for

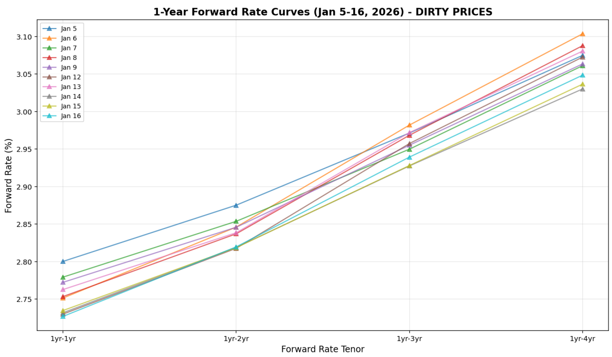

Figure 3: Implied forward rate curve, showing market expectations of future short-term interest rates.

5. The covariance matrix of yield log-returns and forward rates is:

$$\Sigma_{\text{yield}} = \begin{bmatrix} 4.0060 & 0.4886 & 0.4332 & 0.8680 & 0.4581 \\ 0.4886 & 2.9994 & 2.2890 & -0.1824 & -1.4697 \\ 0.4332 & 2.2890 & 1.8147 & 0.1461 & -0.7549 \\ 0.8680 & -0.1824 & 0.1461 & 3.3279 & 4.2430 \\ 0.4581 & -1.4697 & -0.7549 & 4.2430 & 5.9943 \end{bmatrix} \qquad \Sigma_{\text{fwd}} = \begin{bmatrix} 12.2828 & 6.7703 & -1.0930 & -3.7695 \\ 6.7703 & 3.9086 & -0.0071 & -1.5160 \\ -1.0930 & -0.0071 & 5.5240 & 6.7594 \\ -3.7695 & -1.5160 & 6.7594 & 8.9343 \end{bmatrix}.$$

All matrix entries are scaled by a factor of 10^{-5}.

6. For the yield covariance matrix, the eigenvalues and corresponding eigenvectors are

$$\lambda_{\text{yield}} = \begin{bmatrix} 9.59 \\ 5.24 \\ 3.27 \\ 0.00287 \\ 0.000734 \end{bmatrix} \times 10^{-5}, \qquad V_{\text{yield}} = \begin{bmatrix} -0.1207 & -0.5776 & 0.8043 & 0.0250 & 0.0659 \\ 0.2246 & -0.6086 & -0.4153 & -0.5341 & 0.3484 \\ 0.1252 & -0.4902 & -0.3442 & 0.7747 & -0.1597 \\ -0.5516 & -0.2361 & -0.1841 & -0.2909 & -0.7221 \\ -0.7842 & 0.0024 & -0.1682 & 0.1714 & 0.5721 \end{bmatrix}.$$

For the forward-rate covariance matrix, the eigenvalues and eigenvectors are:

$$\lambda_{\text{fwd}} = \begin{bmatrix} 1.9132 \times 10^{-4} \\ 1.1353 \times 10^{-4} \\ 1.2166 \times 10^{-6} \\ 4.2829 \times 10^{-7} \end{bmatrix}, \qquad V_{\text{fwd}} = \begin{bmatrix} -0.7003 & -0.5030 & 0.4715 & -0.1851 \\ -0.3651 & -0.3367 & -0.6999 & 0.5133 \\ 0.3185 & -0.5589 & -0.3833 & -0.6628 \\ 0.5243 & -0.5668 & 0.3753 & 0.5129 \end{bmatrix}.$$

The largest eigenvalue in each case represents a dominant common factor, and its associated eigenvector indicates that most variation in yield and forward-rate log-returns is driven by parallel (level) movements across maturities.

References

[1] Mishkin, F. S. (2018). *The Economics of Money, Banking, and Financial Markets* (12th ed.). Pearson.

[2] Bernanke, B. S. (2020). The New Tools of Monetary Policy. *American Economic Review*, 110(4), 943–983.

[3] Fabozzi, F. J., & Mann, S. V. (2021). *Bond Markets, Analysis, and Strategies* (10th ed.). MIT Press.

[4] Gürkaynak, R. S., Sack, B., & Wright, J. H. (2007). The U.S. Treasury Yield Curve: 1961 to the Present. *Journal of Monetary Economics*, 54(8), 2291–2304.

[5] Board of Governors of the Federal Reserve System. (2024). *Federal Reserve Statistical Release H.4.1: Factors Affecting Reserve Balances*. Retrieved from `https://www.federalreserve.gov/releases/h41/`

[6] Hull, J. C. (2022). *Options, Futures, and Other Derivatives* (11th ed.). Pearson.

[7] Litterman, R., & Scheinkman, J. (1991). Common Factors Affecting Bond Returns. *The Journal of Fixed Income*, 1(1), 54–61.

[8] Jolliffe, I. T., & Cadima, J. (2016). Principal Component Analysis: A Review and Recent Developments. *Philosophical Transactions of the Royal Society A*, 374(2065), 20150202.

[9] Alexander, C. (2008). *Market Risk Analysis, Volume I: Quantitative Methods in Finance*. John Wiley & Sons.

Github Link to Code

`https://github.com/Nupri-03/APM466-Assignment-1.git`