Basics of Electrical & Electronics

Unit-II

AC Fundamentals

INTRODUCTION

As discussed earlier, the flow of current in the circuits was steady and in only one direction, that is, direct current (DC). The use of DC is limited to few applications, for example electro-plating, charging of batteries, electric traction, electronic circuits, etc. However, for large-scale power generation, transmission, distribution, and utilization, an alternative current (AC) system is invariably adopted. In AC system, voltage acting in the circuit changes polarity and magnitude at regular interval of time, and hence, the flow of current in the circuits will reverse the direction periodically.

The most important advantages of AC system over DC system are the following:

1. An alternating voltage can be stepped up and stepped down efficiently by means of transformer. To transmit huge power over a long distance, the voltages are stepped up (up to 400 kV) for economical reasons at the generating stations, whereas they are stepped down to a very low level (400/230 V) for the utilization of electrical energy from safety point of view.
2. The AC motors (i.e., induction motors) are cheaper in cost, simple in construction, more efficient and robust as compared to DC motors.
3. The switchgear (e.g., switches, circuit breakers, etc.) for AC system is simpler than DC system.

ALTERNATING VOLTAGE AND CURRENT

A voltage that changes its polarity and magnitude at regular intervals of time is called an 'alternating voltage'.

When an alternating voltage source is connected across a load resistor R as shown in Figure 6.1, the current flows through it in one direction and then in opposite direction when the polarity is reversed.

Figure 6.1(c) shows the wave shape of the source voltage (representing the variation of voltage with respect to time) and current flowing through the circuit (i.e., load resistor R).

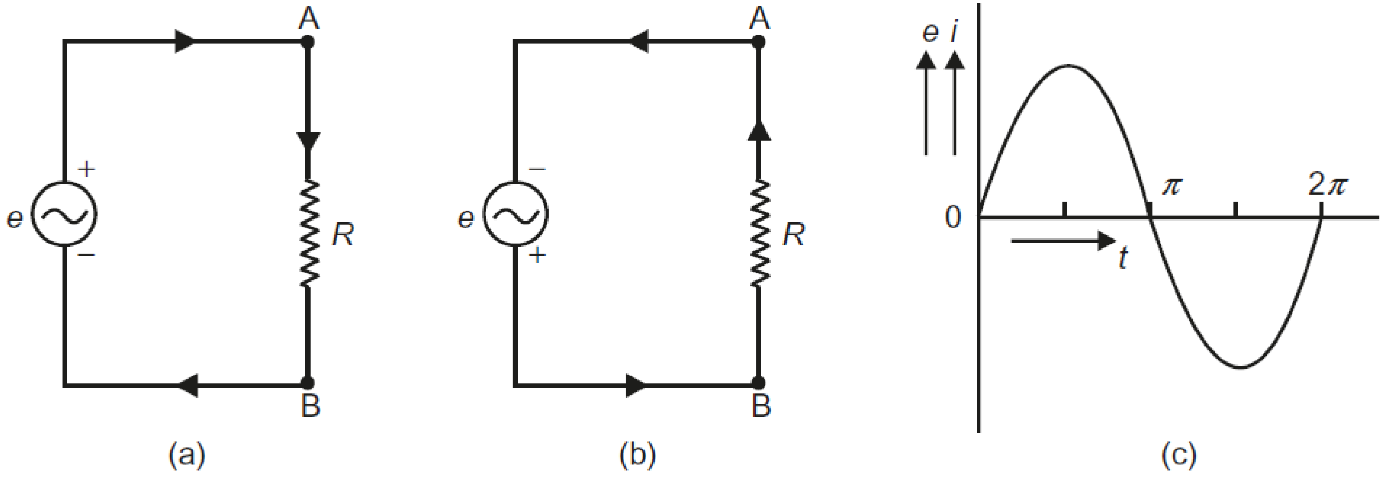


Fig. 6.1 **Alternating voltage and current (a) AC voltage applied across resistor, flow of current during first half cycle (b) AC voltage applied across resistor, flow of current during next half cycle**

Wave Form

As shown in graph (Fig. 6.2), an alternating voltage or current changes with respect to time is known as 'wave form or wave shape'. While plotting a graph, usually the instantaneous values of the alternating quantities are taken along Y-axis and time along X-axis. The alternating voltage or current may vary in different manner, as shown in Figure 6.2, and accordingly, their wave shapes are named in different ways such as irregular wave, triangular wave, square wave, periodic wave, sawtooth wave, sine wave, etc.

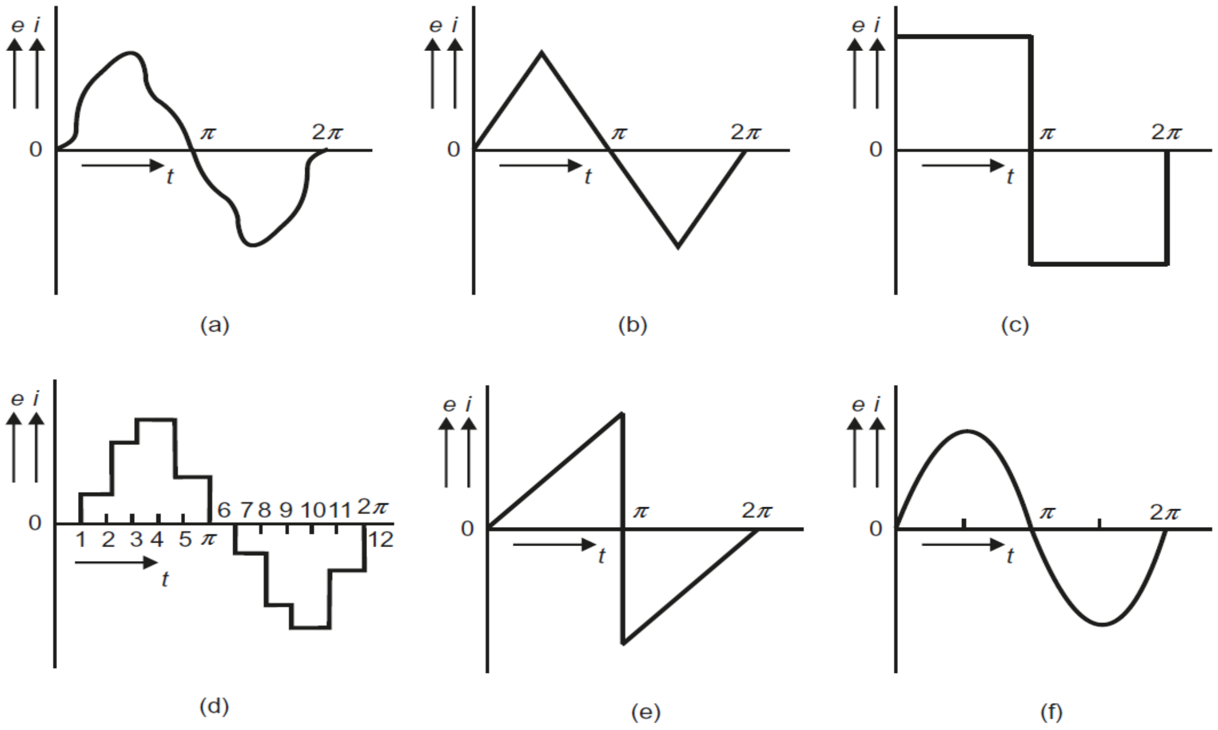


Fig. 6.2 **AC wave shapes (a) General ac wave (b) Triangular wave (c) Square wave (d) Periodic wave (e) Triangular/saw tooth wave (f) Sinusoidal wave**

SINUSOIDAL ALTERNATING QUANTITY

An alternating quantity (i.e., voltage or current) that varies according to sine of angle $\theta (\theta = \omega t)$ is known as 'sinusoidal alternating quantity'. Its wave shape is shown in Figure 6.2(f). For the generation of electric power, sinusoidal voltages and currents are selected all over the world due to the following reasons:

1. The sinusoidal voltages and currents cause low iron and copper losses in AC rotating machines and transformers. This improves the efficiency of AC machines.
2. The sinusoidal voltages and currents offer less interference to nearby communication system (e.g., telephone lines)
3. They produce least disturbance in the electrical circuits.

Whenever the word 'alternating voltage or current' is used in this text, it means sinusoidal alternating voltage or current unless stated otherwise.

GENERATION OF ALTERNATING VOLTAGE AND CURRENT

An alternating voltage can be generated either by rotating a coil in a uniform magnetic field at constant speed as shown in Figure 6.3 or by rotating a uniform magnetic field within a stationary coil at a constant speed as shown in Figure 6.4.

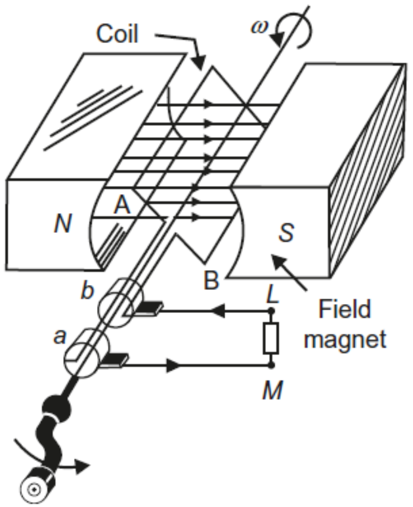


Fig. 6.3 Production of ac voltage (rotating coil, field stationary)

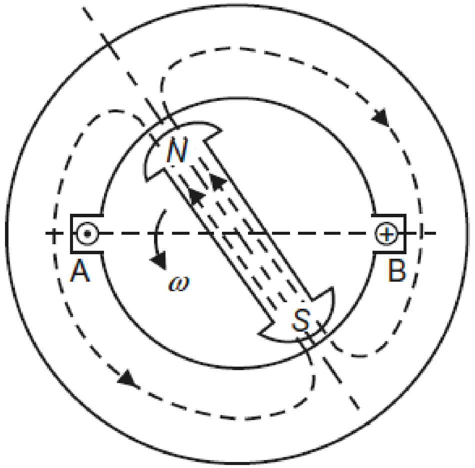


Fig. 6.4 Production of ac voltage (rotating field, coil stationary)

The first method is generally applied in small AC generators, whereas second method is applied in large AC generators due to economical considerations. In both cases, magnetic field is cut by the conductors (or coil sides) and an emf is induced in them. The direction and magnitude of the induced emf in the conductors depend upon the position of the conductors explained as follows:

For simplicity, consider a coil placed in a uniform magnetic field to which a load (*LM*) is connected through brushes and slip rings as shown in Figure 6.5. When it is rotated in anticlockwise direction at a constant angular velocity of ω radians per second, an emf is induced in the coil sides. The cross-sectional view of the coil and its different positions at different instants are shown in Figure 6.5.

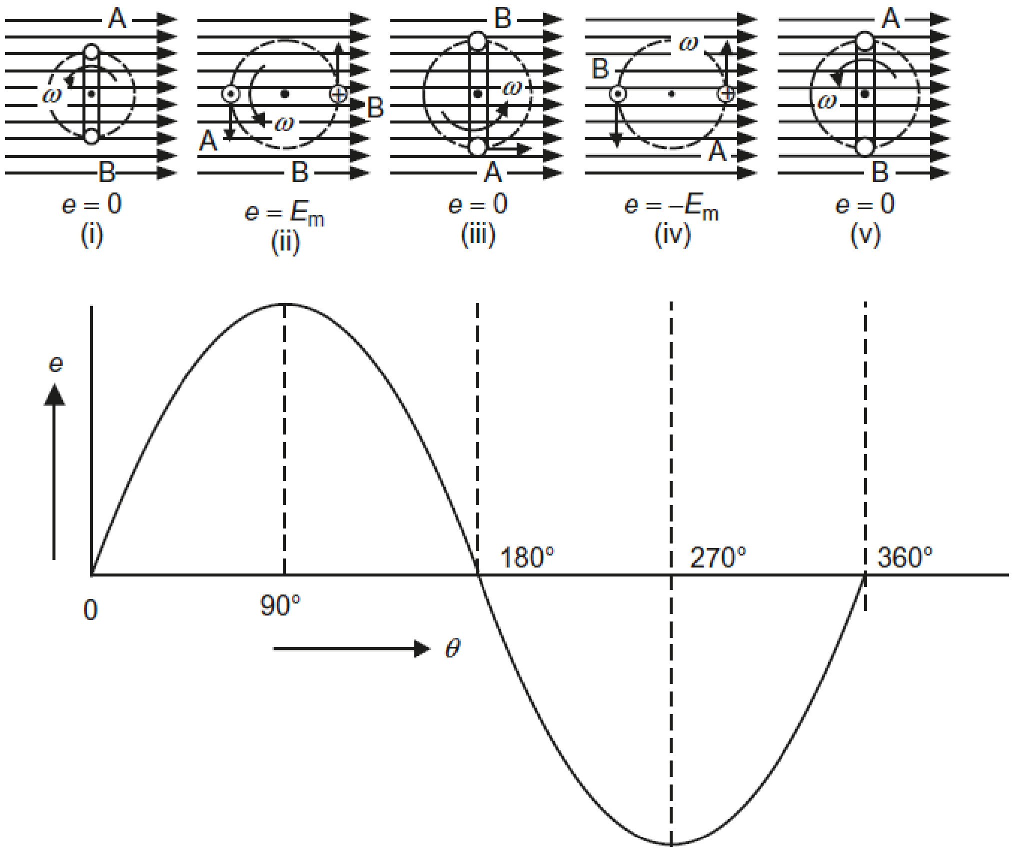


Fig. 6.5 **Position of coil at various instants. Wave shape of generated voltage**

The magnitude of induced emf depends upon the rate at which the flux is cut by the conductors. At (i), (iii), and (v) instants, induced emf in the conductors A and B is zero as they are moving parallel to the magnetic lines of force and the rate of flux cut is zero, whereas the magnitude of emf induced in the conductors A and B is maximum at instant (ii) and (iv) as the conductors are moving perpendicular to the magnetic lines of force and the rate of flux cut is maximum.

The direction of emf induced in the conductors is determined by applying Fleming's right-hand rule. At instant (ii), the direction of emf induced in conductor A is outward, whereas at instant (iv), the direction of induced emf in the conductor A is inward (i.e., the direction of induced emf at this instant is opposite to that of the direction of induced emf at instant (ii).

The wave shape of the emf induced in the coil is also shown in Figure 6.5.

EQUATION OF ALTERNATING EMF AND CURRENT

Consider a coil having 'N' turns rotating in a uniform magnetic field of density B Wb/m^2 in the counterclockwise direction at an angular velocity of ω radians per second as shown in Figure 6.6.

At the instant, as shown in Figure 6.6(b), maximum flux ϕ_m is linking with the coil. After t seconds, the coil is rotated through an angle $\theta = \omega t$ radians. The component of flux linking with

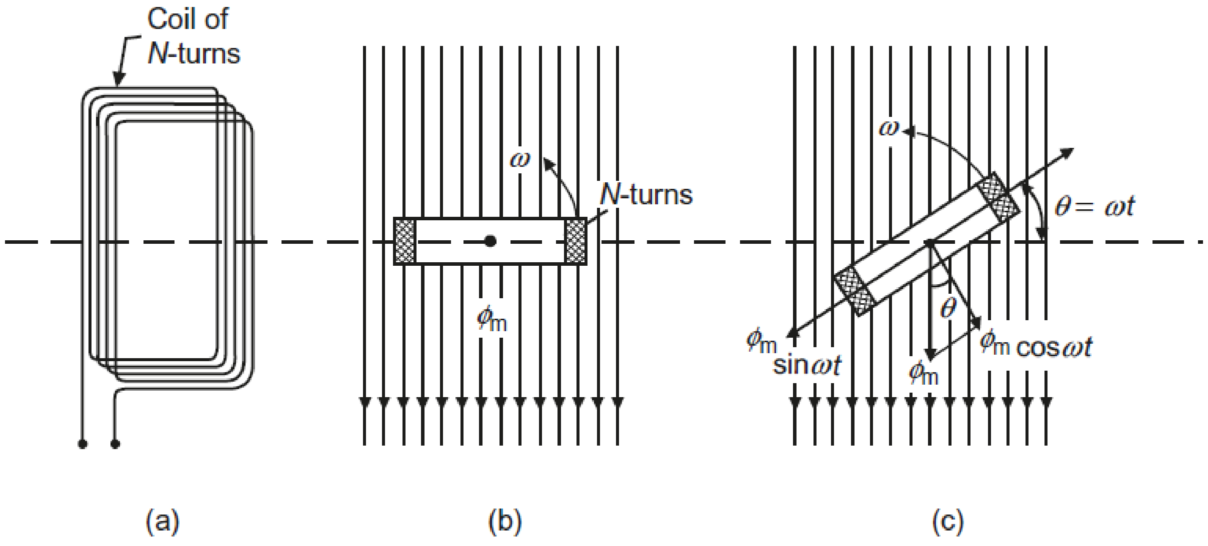


Fig. 6.6 Multi-turn coil rotating in a constant magnetic field (a) Multi-turn coil (b) Position of coil at an instant (c) Position of coil after t second

the coil at this instant is $\phi_m \cos\omega t$, whereas the other component $\phi_m \sin\omega t$ is parallel to the plane of the coil.

According to Faraday's laws of electromagnetic induction, the magnitude of emf induced in the coil at this instant, that is,

Instantaneous value of emf induced in the coil,

$e = -N \frac{df}{dt}$ (negative sign indicates that an induced emf is opposite to the very cause which produces it)

$$\text{or} \qquad e = -N \frac{d}{dt} \phi_m \cos t\ \omega\, t (\phi = \phi_m \cos \omega t)$$

$$\text{or} \qquad e = -N \phi_m (-\omega \sin \omega t)$$

$$\text{or} \qquad e = \omega N \phi_m \sin \omega t \tag{6.1}$$

The value of an induced emf will be maximum when angle q

$$\text{or} \qquad \omega t = 90^\circ \text{ (i.e., } \sin \omega t = 1)$$

$$\therefore \qquad E_m = \omega N \phi_m \tag{6.2}$$

Putting this value in equation (6.1), we get,

$$e = E_m \sin \omega t = E_m \sin \theta$$

From the above equation, it is clear that the magnitude of the induced emf varies according to sine of angle θ. The wave shape of an induced emf is shown in Figure 6.7. This wave form is called sinusoidal wave.

If this voltage is applied across resistor, an alternating current will flow through it varying sinusoidally, that is, following a sine law and its wave shape will be same as shown in Figure 6.7.

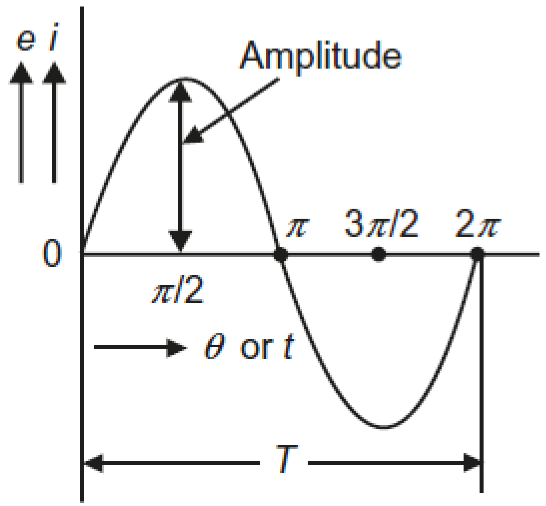


Fig. 6.7 Wave shape of a sinusoidal voltage or current

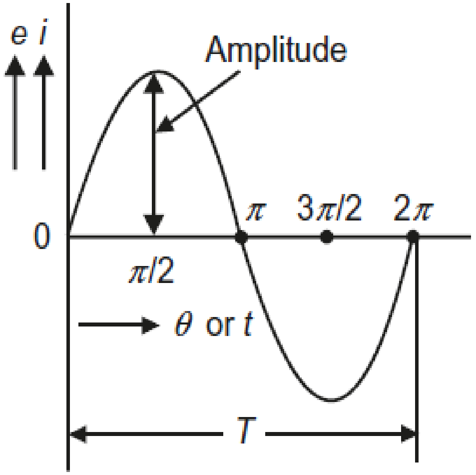
This alternating current is given by the following equation:

$$i = I_m \sin \omega t = I_m \sin \theta$$

IMPORTANT TERMS

An alternating voltage or current changes its magnitude and direction at regular intervals of time. A sinusoidal voltage or current varies as a sine function of time t or angle $\theta (= \omega t)$. The following important terms are generally used in alternating quantities:

1. **Wave form:** The shape of the curve obtained by plotting the instantaneous values of alternating quantity (voltage or current) along Y-axis and time or angle ($\theta = \omega t$) along X-axis is called 'wave form or wave shape'. Figure 6.7 shows the waveform of an alternating quantity varying sinusoidally.
2. **Instantaneous value:** The value of an alternating quantity, that is, voltage or current at any instant is called its instantaneous value and is represented by 'e' or 'i', respectively.
3. **Cycle:** When an alternating quantity goes through a complete set of positive and negative values or goes through 360 electrical degrees, it is said to have completed one cycle.
4. **Alternation:** One half-cycle is called 'alternation'. An alternation spans 180 electrical degrees.
5. **Time period:** The time taken in seconds to complete one cycle by an alternating quantity is called time period. It is generally denoted by 'T'.
6. **Frequency:** The number of cycles made per second by an alternating quantity is called 'frequency'. It is measured in cycles per second (c/s) or hertz (Hz) and is denoted by 'f'.
7. **Amplitude:** The maximum value (positive or negative) attained by an alternating quantity in one cycle is called its 'amplitude or peak value or maximum value'. The maximum value of voltage and current is generally denoted by E_m (or V_m) and I_m, respectively.



IMPORTANT RELATIONS

Some of the terms used in AC terminology have definite relations among themselves as given below:

1. **Relation between frequency and time period:** Consider an alternating quantity having a frequency of f c/s. Then, time taken to complete f cycle = 1 s
   Time taken to complete 1 cycle = $1/f$ s
   Hence, time period, $T = 1/f$ s or $f = 1/T$ c/s
2. **Relation between frequency and angular velocity:** Consider an alternating quantity having a frequency of f c/s.
   Angular distance covered in one cycle = 2π radian
   $\therefore$ Angular distance covered per second in f cycles = 2π radian

   Hence, $\omega = 2\pi f$ radian/s

DIFFERENT FORMS OF ALTERNATING VOLTAGE EQUATION

The alternating voltage is given by the following standard equation:

$$e = E_m \sin \theta \text{ or } e = E_m \sin \omega t$$

or $$e = E_m \sin 2\pi ft \; (\sin \omega = 2\pi f) = E_m \sin 2\pi t/T \; (\text{since } f = 1/T)$$

Which form of the above equation is to be applied will depend upon the data given?

To determine the various values, for example, maximum value, frequency, time period, angular velocity, etc., the given equation is compared with the standard equation of any one of the form given above. For instant,

Maximum value of voltage, E_m = Coefficient of sine of time angle

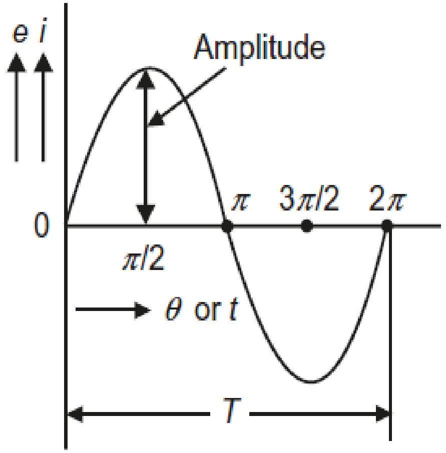
Frequency, $$f = \frac{\text{Coefficient of time in the angle}}{2\pi}$$

VALUES OF ALTERNATING VOLTAGE AND CURRENT

The voltage and current in DC system are constant so that there is no problem of specifying their magnitudes, whereas in AC system, the alternating voltage and current vary from time to time. Hence, it is necessary to explain the ways to express the magnitude of alternating voltage and current. The following three ways are adopted to express the magnitude of these quantities:

1. Peak value
2. Average value or Mean value
3. Effective value or rms value

The rms value of an alternating quantity (voltage or current) represents the real magnitude, whereas the peak and average values are important in some of the engineering applications.



PEAK VALUE

The maximum value attained by an alternating quantity during one cycle is called 'peak value'. This is also called 'maximum value or crest value or amplitude'. A sinusoidal alternating quantity obtains its maximum value at 90° as shown in Figure 6.7. The peak of an alternating voltage and current is represented by E_m and I_m. The knowledge of peak value is important in case of testing dielectric strength of insulating materials.

AVERAGE VALUE

The arithmetic average of all the instantaneous values considered an alternating quantity (current or voltage) over one cycle is called average value.

In case of symmetrical waves (such as sinusoidal current or voltage wave), the positive half is exactly equal to the negative half; therefore, the average value over a complete cycle is zero. Since work is being done by the current in both the positive and the negative half cycle, average value is determined regardless of signs. Hence, to determine average value of alternating quantities having symmetrical waves, only (positive half) cycle is considered.

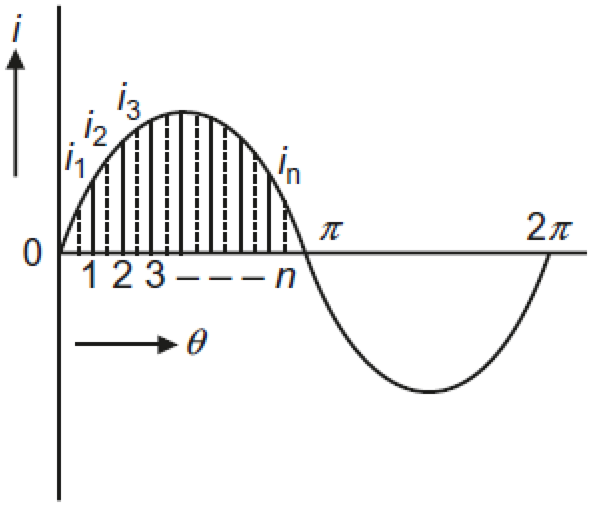


Fig. 6.8 Positive half cycle divided into n equal parts

Divide the positive half cycle into 'n' number of equal parts as shown in Figure 6.8. Let $i_1, i_2, i_3, ..., i_n$ be the mid-ordinates.

Average value of current, I_{av} = mean of mid-ordinates.

$$= \frac{i_1 + i_2 + i_3 + \ldots + i_n}{n}$$

$$= \frac{\text{Area of alternation}}{\text{Base}}$$

AVERAGE VALUE OF SINUSOIDAL CURRENT

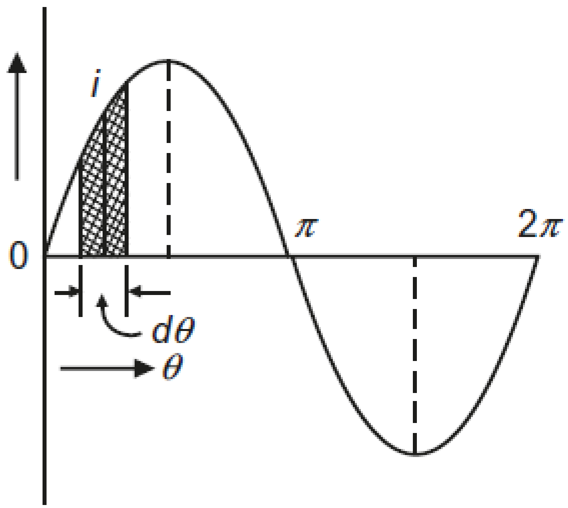


Fig. 6.9 Current varying sinusoidally

The alternating current varying sinusoidally, as shown in Figure 6.9, is given by the equation:

$$i = I_m \sin\theta$$

Consider an elementary strip of thickness $d\theta$ in the positive half cycle, i be its mid-ordinate. Then,

Area of strip $= id\theta$

$$\text{Area of half cycle } \int_0^{\pi} i\, d\theta = \int_0^{\pi} I_m \sin\theta d\theta$$

$$= I_m(-\cos\theta)_0^{\pi} = I_m(-(\cos\pi - \cos 0))$$

$$= I_m[-1(-1-1)] = 2I_m$$

$$\text{Base} = 0 \text{ to } \pi = \pi - 0 = \pi$$

∴ Average value, $$I_{av} = \frac{\text{Area of alternation}}{\text{base}} = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = 0.637\, I_m$$

EFFECTIVE OR RMS VALUE

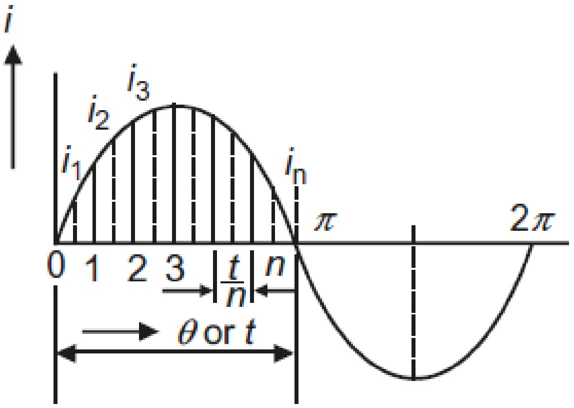


Fig. 6.10 Positive half cycle divided into n equal parts

The steady current when flows through a resistor of known resistance for a given time produces the same amount of heat as produced by an alternating current when flows through the same resistor for the same time is called effective or rms value of an alternating current.

Let i be an alternating current flowing through a resistor of resistance R for time t seconds which produces the same amount of heat as produced by I_{eff} (direct current). The base of one alternation is divided into n equal parts, as shown in Figure 6.10, so that interval is of $\frac{t}{n}$ second. Let $i_1, i_2, i_3, ..., i_n$ be the mid-ordinate.

Then, heat produced in the

First interval $= i_1^2 \; Rt/Jn$ calorie

Second interval $= i_2^2 \; Rt/Jn$ calorie

Third interval $= i_3^2 \; Rt/Jn$ calorie

nth interval $= i_n^2 \; Rt/Jn$ calorie

$$\text{Total heat produced} = \frac{Rt}{J}\left(\frac{i_1^2 + i_2^2 + i_3^2 + \ldots\ldots i_n^2}{n}\right)\text{calorie} \qquad (6.3)$$

Since I_{eff} is considered as the effective value of this current.

Then, total heat produced by this current = calorie (6.4)

Equating equation (6.3) and (6.4), we get,

$$\frac{I_{eff}^2 Rt}{J} = \frac{Rt}{J}\left(\frac{i_1^2 + i_2^2 + i_3^2 + \ldots\ldots i_n^2}{n}\right)$$

or $$I_{eff} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2}{n}} = \sqrt{\text{mean of squares of instantaneous values}}$$

or I_{eff} = Square root of mean of squares of instantaneous values

= root-mean-square (rms) value

It is the actual value of an alternating quantity which tells us the energy transfer capability of an AC source. For example, if we say that 5 A AC is flowing through a circuit, it means the rms value of an AC which flows through the circuit is 5 A. It transfers the same amount of energy as is transferred by 5 A DC.

The ammeters and voltmeters record the rms values of alternating currents and voltages, respectively. The domestic single-phase AC supply is 230 V, 50 Hz. Where 230 V is the rms value of an alternating voltage.

RMS VALUE OF SINUSOIDAL CURRENT

An alternating current varying sinusoidally is given by the following equation:

$$i = I_m \sin\theta$$

To determine the rms value, the squared wave of the alternating current is drawn as shown in Figure 6.11.

Considering an elementary strip of thickness $d\theta$ in the first half-cycle of the squared wave, let i^2 be its mid-ordinate. Then,

$$\text{Area of strip} = i^2 d\theta$$

Area of first half cycle of squared wave

$$= \int_0^{\pi} i^2 d\theta = \int_0^{\pi} (I_m \sin\theta)^2 d\theta$$

$$= I_m^2 \int_0^{\pi} \sin^2\theta d\theta = I_m^2 \int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{2} \int_0^{\pi} (1-\cos 2\theta)\, d\theta = \frac{I_m^2}{2} \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

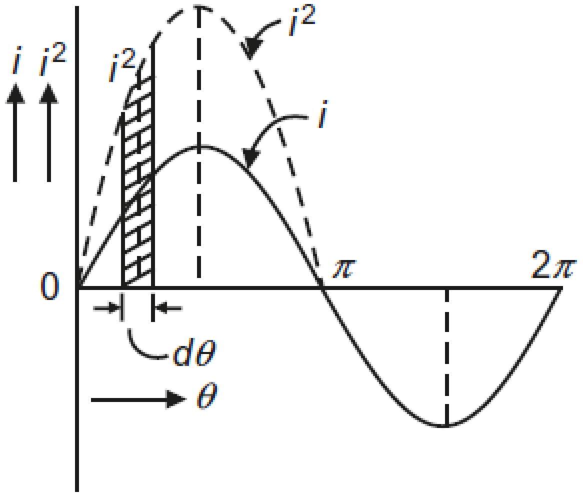


Fig. 6.11 Squared-wave shape of a sine wave

$$= \frac{I_m^2}{2}\left((\pi - 0) - \frac{\sin 2\pi - \sin 0}{2}\right)$$

$$= \frac{I_m^2}{2}[(\pi - 0) - (0 - 0) = \frac{\pi I_m^2}{2}$$

Base = 0 to $\pi = \pi - 0 = \pi$

Effective or rms value,

$$I_{rms} = \sqrt{\frac{\text{Area of first half of squared wave}}{\text{Base}}} = \sqrt{\frac{\pi I_m^2}{2\pi}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Usually, rms value of an AC is simply represented by I instead of I_{rms}. Similarly, rms value of an alternating voltage is represented by E or V.

FORM FACTOR AND PEAK FACTOR

There exists a definite relation among the average value, rms value, and peak value of an alternating quantity. The relationship is expressed by the two factors, namely form factor and peak factor.

1. **Form factor:** The ratio of rms value to average value of an alternating quantity is called form factor.

   Mathematically, form factor $= \dfrac{I_{\text{rms}}}{I_{\text{av}}}$ or $\dfrac{E_{\text{rms}}}{E_{\text{av}}}$

   For the current varying sinusoidally

   $$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{I_{\text{m}}/\sqrt{2}}{2I_{\text{m}}/\pi} = \frac{\pi I_{\text{m}}}{2\sqrt{2}\, I_{\text{m}}} = 1.11$$

2. **Peak factor:** The ratio of maximum value to rms value of an alternating quantity is called peak factor.

   Mathematically, peak factor $= \dfrac{I_{\text{m}}}{I_{\text{rms}}}$ or $\dfrac{E_{\text{m}}}{E_{\text{rms}}}$

   For current varying sinusoidally

   $$\text{Peak factor} = \frac{I_{\text{m}}}{I_{\text{rms}}} = \frac{I_{\text{m}}}{I_{\text{m}}/\sqrt{2}} = \sqrt{2} = 1.4142$$

Example

The equation of an alternating current is $i = 42.42 \sin 628\, t$

Determine (i) its maximum value; (ii) Frequency; (iii) rms value; (iv) Average value; and (v) Form factor.

Solution:

Given equation is

$$i = 42.42 \sin 628\, t$$

Comparing above equation with the standard equation, $i = I_m \sin \omega t$

$$i_m = 42.42 \text{ A and } \omega = 628 \text{ rad/sec}$$

(i) Maximum value, $I_m = 42.42$ A (Ans.)

(ii) Here, $\omega = 628$ rad/sec or $2\pi f = 628$

$$\text{Frequency, } f = \frac{628}{2\pi} = 100 \text{ Hz (Ans.)}$$

(iii) RMS value, $I_{rms} = \dfrac{I_m}{\sqrt{2}} = \dfrac{42.42}{\sqrt{2}} = 30$ A (Ans.)

(iv) Average value, $I_{av} = \dfrac{2\, I_m}{\pi} = \dfrac{2 \times 42.42}{\pi} = 27$ A (Ans.)

(v) Form factor $= \dfrac{I_{rms}}{I_{av}} = \dfrac{30}{27} = 1.11$ (Ans.)

Example

An alternating voltage is given by $v = 141.1 \sin 314t$. Find the following:

(i) Frequency
(ii) RMS value
(iii) Average value
(iv) The instantaneous value of voltage when 't' is 3 msec
(v) The time taken for voltage to reach 100 V for the first time after passing through zero value.

Solution:

(i) Given that, $v = 141.14 \sin 314\, t$

Comparing above equation with

$v = V_{in} \sin \omega t$

$\omega = 314$ rad/sec.

∴ Frequency, $$f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} = 50 \text{ Hz (Ans.)}$$

(ii) RMS value of the voltage, $$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{141.4}{1.414} = 100 \text{ V (Ans.)}$$

(iii) Average value, $$V_{av} = \frac{2V_m}{\pi} = \frac{2 \times 141.4}{\pi} = 90 \text{ V (Ans.)}$$

(iv) $v = 141.1 \sin 314t.$

At $t = 3$ ms $= 3 \times 10^{-3}$ s

$v = 141.4 \sin 314 \times 3 \times 10^{-3} = 114.4$ V (Ans.)

(v) $v = 141.4 \sin 314t$

or $$t = \frac{1}{314}\sin^{-1}\left(\frac{100}{141.4}\right) = 2.5 \times 10^{-3} \text{ s} = 2.5 \text{ m s (Ans.)}$$

PHASOR REPRESENTATION OF SINUSOIDAL QUANTITY

It has been seen that an alternating quantity (varying sinusoidally) can be represented in the form of wave and equation. The wave form shows the graphical representation, whereas the equation represents the mathematical expression of the instantaneous value of an alternating quantity.

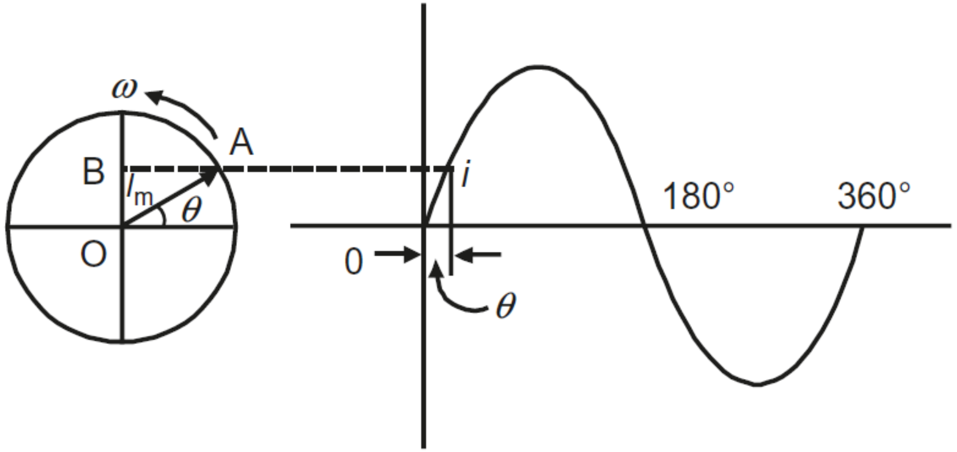


Fig. 6.22 Phasor representation and wave diagram

The same alternating quantity can also be represented by a line of definite length (representing its maximum value) rotating in counterclockwise direction at a constant velocity (ω radians/second). Such a rotating line is called a 'phasor'.

Thus, an alternating quantity can be represented by a phasor that shows its magnitude and direction at that instant.

For instant, consider an alternating quantity (current) represented by the equation $i = I_m \sin \omega \times t$. Take a line OA to represent the maximum value of current I_m to scale. Imagine this line is rotating in counterclockwise direction at an angular velocity of ω radian/s about point O. After t s, the line is rotated through an angle θ ($\theta = \omega \times t$), from its horizontal position as shown in Figure 6.22. The projection of line OA on the Y-axis is OB.

$$\text{OB} = \text{OA} \sin \theta = I_m \sin \omega t$$
$$= i \text{ (the value of current at that instant)}$$

Hence, the projection of the phasor OA on the Y-axis (i.e., OB) at any instant gives the value of current at that instant.

Thus, a sinusoidal alternating quantity is represented by a phasor (vector) of length to scale equal to its maximum value rotated through an angle θ with the axis of reference (i.e., X-axis).

The phasor representation of an alternating quantity enables us to understand its magnitude and position on the axis. The alternating quantities can be added and subtracted with a fair degree of ease by representing them vectorially (phasor diagram).

PHASE AND PHASE DIFFERENCE

The phase of an alternating quantity (current or voltage) at an instant is defined as the fractional part of a cycle through which the quantity has advanced from a selected origin (Fig. 6.23). In actual practice, we are more concerned with the phase difference between the two alternating quantities rather than their absolute phase.

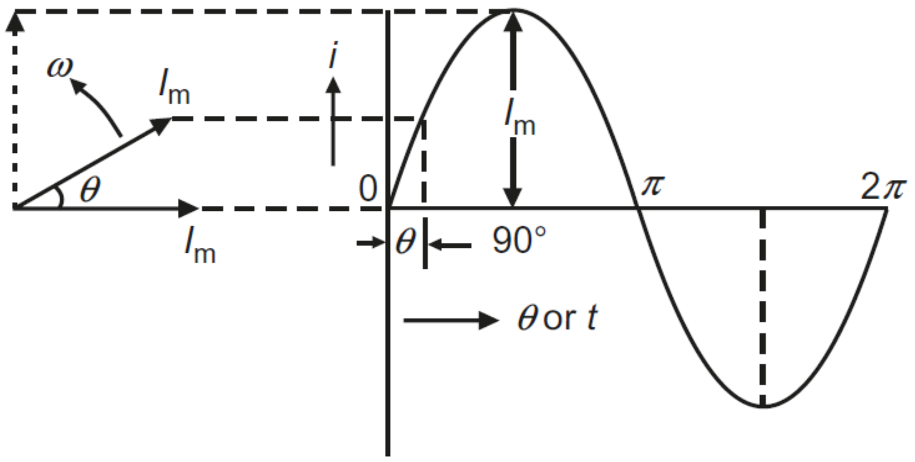


Fig. 6.23 Phasor representation and its instantaneous value on the wave diagram

The two alternating quantities having same frequency, when attain their zero value at different instants, the quantities are said to have a phase difference. This angle between zero points (and are becoming positive) of two alternating quantities is called angle of phase difference.

In Figure 6.24, two alternating currents of magnitude I_{m1} and I_{m2} are shown vectorially. Both the vectors are rotating at same angular velocity of ω radian per second. The zero values are obtained by the two currents at different instants. Therefore, they are said to have a phase difference of angle ϕ.

In other words, the phase difference may be defined as the angular displacement between the maximum positive value of the two alternating quantities having the same frequency.

The quantity that attains its positive maximum value prior to the other is called a 'leading quantity', whereas the quantity that attains its positive maximum value after the other is called a 'lagging quantity'. In this case, current I_{m1} is leading current with respect to I_{m2} or is other words current I_{m2} is the lagging current with respect to I_{m1}.

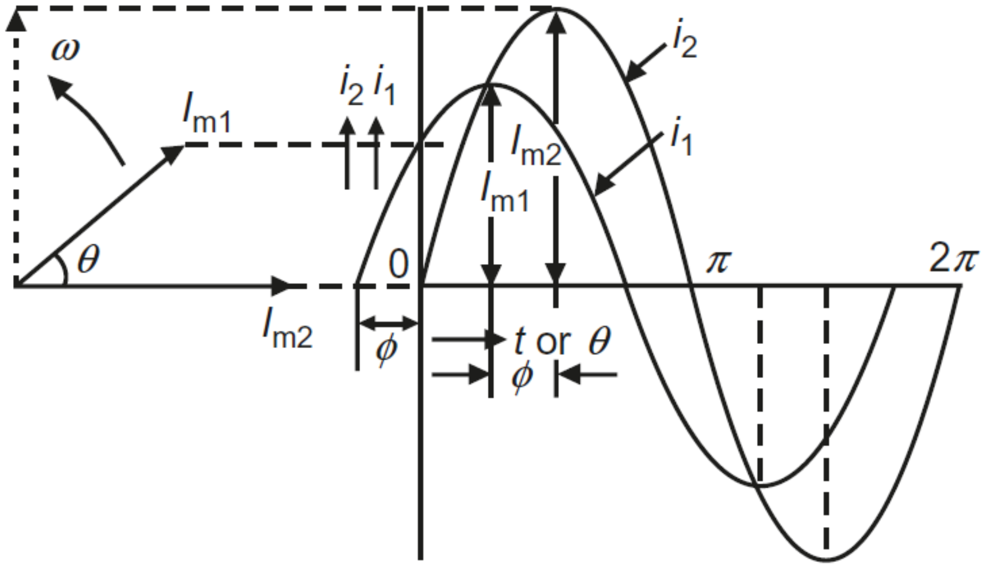


Fig. 6.24 Phasor and wave diagram of two ac quantities with phase difference

Poly-phase System

INTRODUCTION

Although single-phase system is employed for the operation of almost all the domestic and commercial appliances, for example, lamps, fans, electric irons, refrigerators, TV sets, washing machines, exhaust fans, computers, etc. However, it has its own limitations in the field of generation, transmission, distribution, and industrial applications. Due to this, it has been replaced by polyphase system. Polyphase (three-phase) system is universally adopted for generation, transmission, and distribution of electric power because of its unchangeable superiority. In this chapter, we shall confine our attention to three-phase system and its practical utility in the field of engineering.

POLYPHASE SYSTEM

Poly means many (more than one) and *phase* means windings or circuits, each of them having a single alternating voltage of the same magnitude and frequency. Hence, a polyphase system is essentially a combination of two or more voltages having same magnitude and frequency, but displaced from one another by equal electrical angle. This angular displacement between the adjacent voltages is called phase difference and depends upon the number of phases.

$$\text{Phase difference} = \frac{360 \text{ electrical degrees}}{\text{Number of phases}}$$

However, the abovementioned relation does not hold good for two-phase system, where the voltages are displaced by 90° electrical. Thus, an AC system having a group of (two or more than two) equal voltages of same frequency arranged to have equal phase difference between them is called a polyphase system.

The polyphase system may be two-phase system, three-phase system, or six-phase system. However, for all practical purposes, three-phase system is invariably employed. Therefore, whenever a polyphase system is mentioned, we mean by that a three-phase system unless stated otherwise.

ADVANTAGES OF THREE-PHASE SYSTEM OVER SINGLE-PHASE SYSTEM

The following are the main advantages of three-phase system over single-phase system:

1. **Constant power:** In single-phase circuits, the power delivered is pulsating. Even when the voltage and current are in phase, the power is zero twice in each cycle. While in polyphase system, power delivered is almost constant when the loads are balanced.
2. **High rating:** The rating (output) of a three-phase machine is nearly 1.5 times the rating (output) of a single-phase machine of the same size.
3. **Power transmission economics:** To transmit the same amount of power over a fixed distance at a given voltage, three-phase system requires only 75% of the weight of conducting material as required by single-phase system.

4. **Superiority of three-phase induction motors:** The three-phase induction motors have widespread field of applications in the industries because of the following advantages:
   (a) Three-phase induction motors are self-starting, whereas single-phase induction motors have no starting torque without using auxiliary means.
   (b) Three-phase induction motors have higher power factor and efficiency than that of single-phase induction motors.

GENERATION OF THREE-PHASE EMFS

In a three-phase system, there are three equal voltages (or emfs) of the same frequency having a phase difference of 120°. These voltages can be produced by a three-phase, AC generator having three identical windings (or phases) displaced 120° electrical apart. When these windings are rotated in a stationary magnitude field (see Fig. 8.1(a)) or when these windings are kept stationary and the magnetic field is rotated (see Fig. 8.1(b)), an emfs is induced in each winding or phase. These emfs are of same magnitude and frequency, but are displaced from one another by 120° electrical.

Consider three identical coils a_1a_2, b_1b_2, and c_1c_2 mounted, as shown in Fig 8.1(a) and (b). Here, a_1, b_1, and c_1 are the start terminals, while a_2, b_2, and c_2 are the finish terminals of the three coils. It may be noted that a phase difference of 120° electrical is maintained between the corresponding start terminals a_1, b_1, and c_1. Let the three coils mounted on the same axis be rotated (or the magnetic field system be rotated keeping coils stationary) in anticlockwise direction at ω radians/s, as shown in Figure 8.1(a) and (b), respectively.

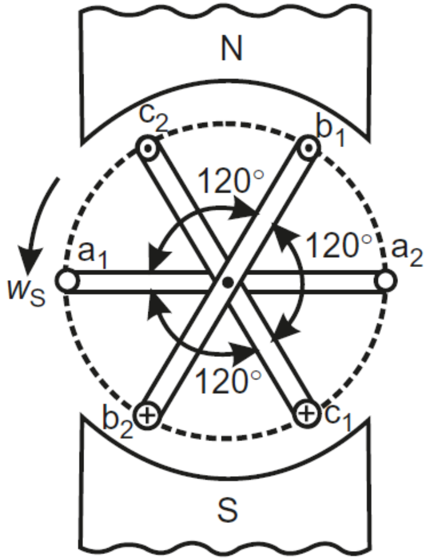
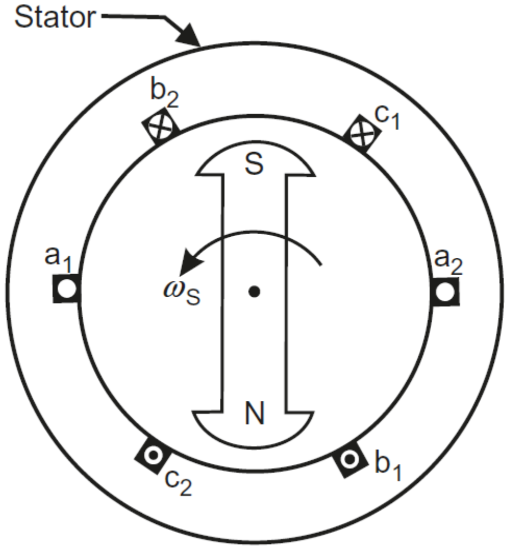
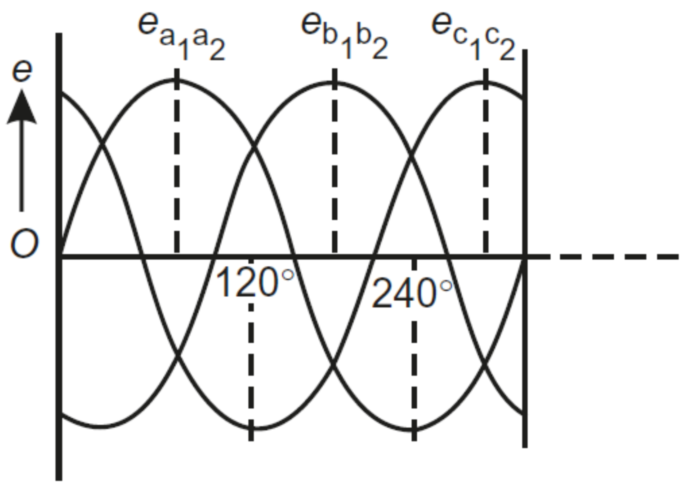


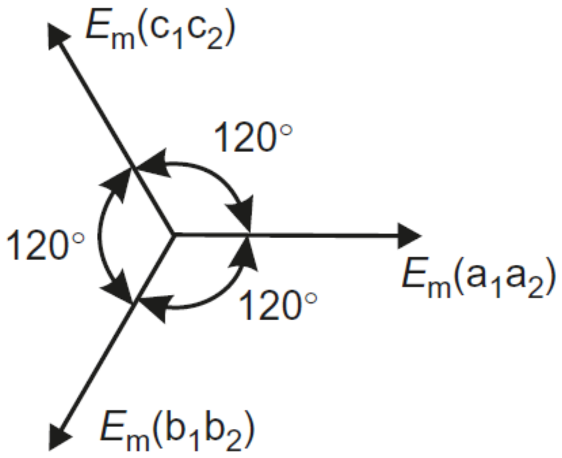
Fig. 8.1 (a) Coils rotating in stationary magnetic field



(b) Magnetic field rotating in stationary coils



(c) Wave diagram of induced emfs in three coils



(d) Phasor diagram of induced emfs in three coils

Three emfs are induced in the three coils, respectively. Their magnitudes and directions, at this instant, are as follows:

1. The emf induced in coil a_1a_2 is zero (consider start terminal a_1) and is increasing in the positive direction, as shown by wave e_{a1a2} in Figure 8.1(c).
2. Coil b_1b_2 is 120° (electrical) behind coil a_1a_2. The emf induced in this coil is negative and is becoming maximum negative (consider start terminal b_1) as shown by e_{b1b2} in Figure 8.1(c).
3. Coil c_1c_2 is 120° (electrical) behind b_1b_2 or 240° (electrical) behind a_1a_2. The emf induced in this coil is positive and is decreasing (consider start terminal c_1) as shown by wave e_{c1c2} in Figure 8.1(c).

Phasor Diagram

The emfs induced in three coils are of the same magnitude and frequency, but are displaced by 120° (electrical) from each other as shown by phasor diagram in Figure 8.1(d). These can be represented by the equations:

$$e_{a1a2} = E_m \sin \omega t; e_{b1b2} = E_m \sin(\omega t - 2\pi/3); \; e_{c1c2} = E_m \sin(\omega t - 4\pi/3) = E_m \sin(\omega t - 240°)$$

NAMING THE PHASES

The three phases may be represented by numbers (1, 2, and 3) or by letters (a, b, and c) or by colours (red, yellow, and blue, i.e., R, Y, and B). In India, they are named by R, Y, and B, that is, red, yellow, and blue.

PHASE SEQUENCE

In a three-phase system, there are three voltages having same magnitude and frequency displaced by an angle of 120° electrical. They are attaining their positive maximum value in a particular order. The order in which the voltages (or emfs) in the three phases attain their maximum positive value is called the phase sequence.

The emfs induced in the three coils or phases attain their positive maximum value in the order of a_1a_2, b_1b_2, and c_1c_2; therefore, the phase sequence is a, b, and c. However, if the coils or phases are being named out as R, Y, and B in place of a, b, and c, respectively, then the phase sequence will be RYB.

The sequence RYB (or YBR or BRY) is considered as positive phase sequence, while the RBY (or BYR or YRB) is considered as negative phase sequence. The sequence knowledge of phase sequence is essential in the following important applications:

1. The direction of rotation of three-phase induction motors depends upon the phase sequence of three-phase supply. To reverse the direction of rotation, the phase sequence of the supply given to the motor has to be changed.
2. The parallel operation of three-phase alternators and transformers is only possible if phase sequence is known.

DOUBLE-SUBSCRIPT NOTATION

An alternating quantity is generally represented by a double-subscript notation. In this notation, two letters are placed at the foot of the symbol for voltage or current, as shown in Figure 8.2. This conveys the following two scenarios:

1. The subscript of the symbol for voltage or current indicates the portion of the circuit where the quantity is located.
2. The order of the subscript indicates the positive direction of the quantity in which it acts.

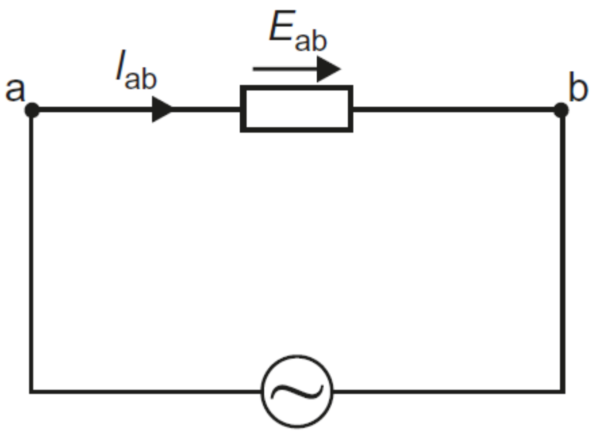


Fig. 8.2 Circuit to represent double subscript notation

For instant, the current is represented as I_{ab}. It means that

1. The portion ab of the circuit is considered.
2. The current flows from a to b.

The double-subscript notation is very useful in solving AC circuits having a number of voltages and currents.

INTERCONNECTION OF THREE PHASES

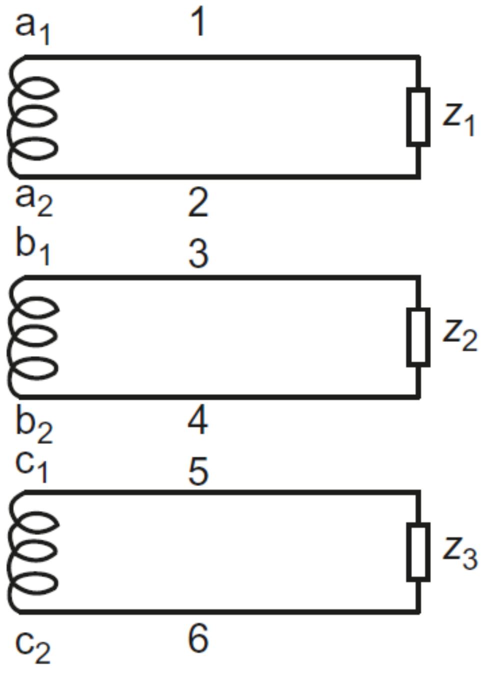


Fig. 8.3 **Three-phases supplying power independently**

In a three-phase AC generator, three are three windings. Each winding has two terminals (start and finish). If a separate load is connected across each phase winding as shown in Figure 8.3, then each phase supplies an independent load through a pair of leads (wires). Thus, six wires will be required in this case to connect the load to generator. This will make the whole system complicated and expensive.

In order to reduce the number of the conductors, the three-phase windings of the AC generator are suitably interconnected. The following are the two universally adopted methods of interconnecting the three phases:

1. Star or wye (Y) connection
2. Mesh or delta (Δ) connection.

STAR OR WYE (Y) CONNECTION

In star or wye (Y) connections, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point. The three line conductors are run from the remaining three free terminals called line conductors. Ordinarily, only three wires are carried to the external circuit giving three-phase, three-wire star-connected system. However, sometimes a fourth wire is carried from the star point to the external circuit, called neutral wire, giving three-phase, four-wire star-connected system.

As shown in Figure 8.4, the finish terminals a_2, b_2, and c_2 of the three windings are connected to form a star or neutral point. From the remaining three free terminals, three conductors are run, named R, Y, and B. The current flowing through each phase is called phase current I_{ph} and current

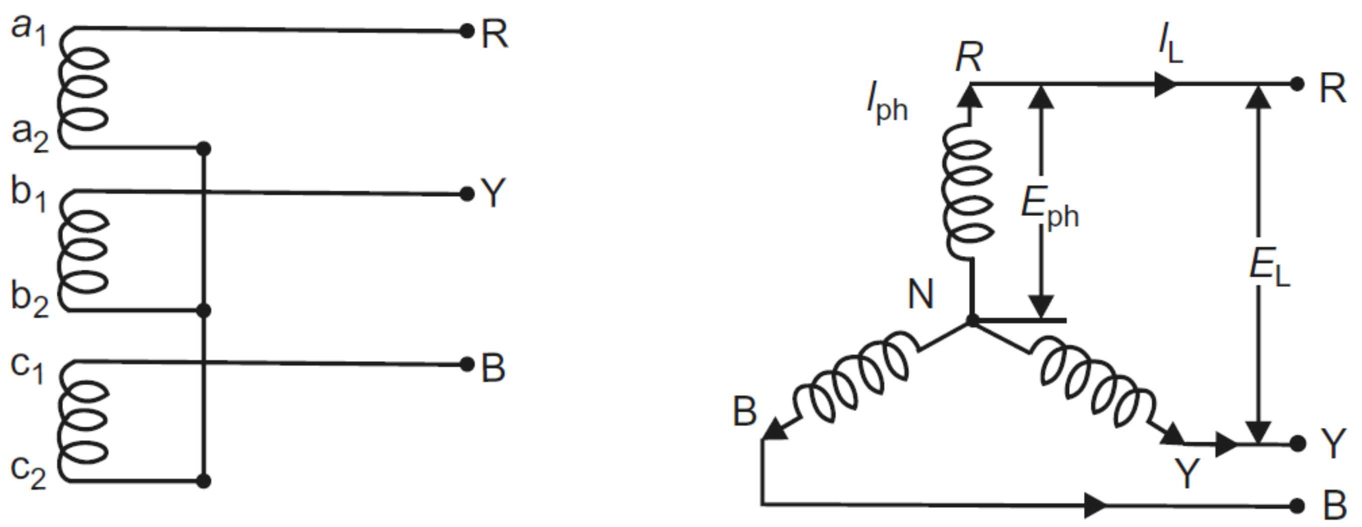


Fig. 8.4 (a) 3-phases connected in star (b) Star connected system

flowing through each line conductor is called line current I_L. Similarly, voltage across each phase is called phase voltage (E_{ph}) and voltage across two line conductors is called line voltage (E_L).

Relation between Phase Voltage and Line Voltage

The connections are shown in Figure 8.5(a). Since the system is balanced, three voltages E_{NR}, E_{NY}, and E_{NB} are equal in magnitude, but displaced from one another by 120° electrical. Their phasor diagrams are shown in Figure 8.5(b). The arrow heads on emfs and currents indicate the positive direction and not their actual direction at any instant.

Now, $E_{NR} = E_{NY} = E_{NB} = E_{ph}$ (in magnitude)

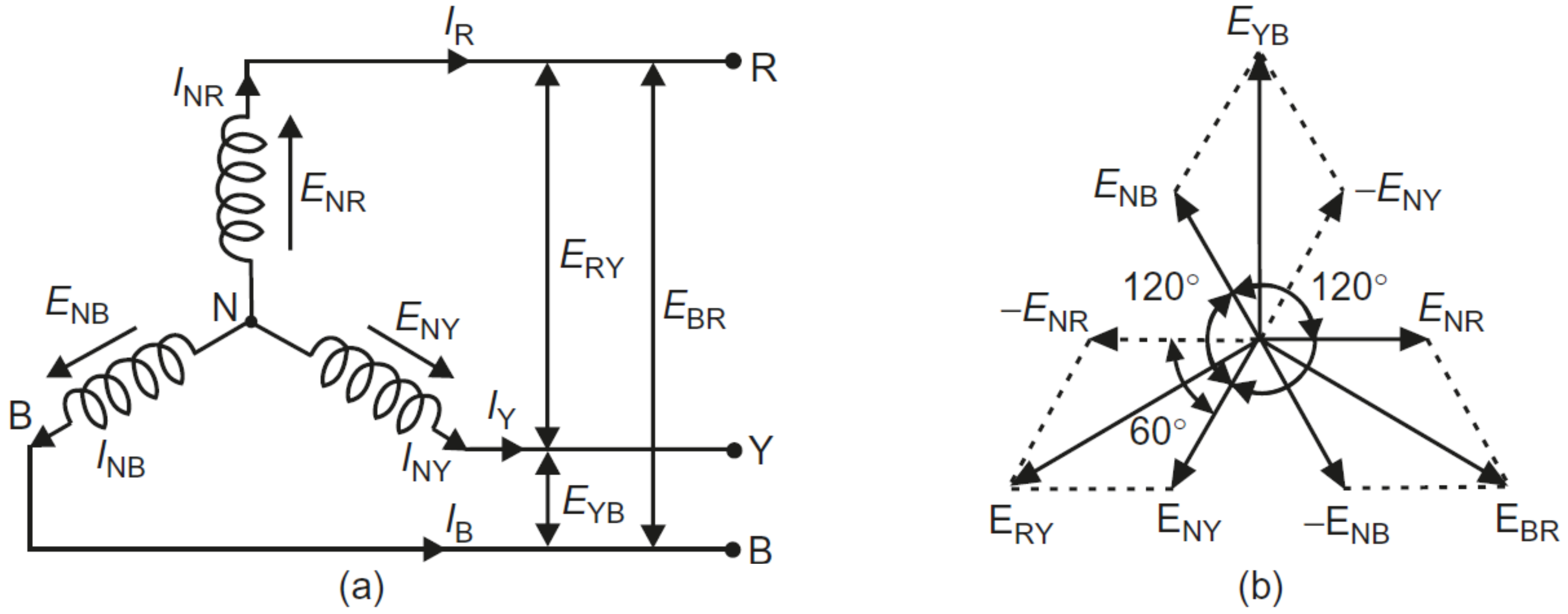


Fig. 8.5 (a) Circuit representing phase and line voltages and currents in star connections (b) Phasor diagram of phase and line voltages in star connections

It may be seen that between any two lines, there are two phase voltages.

Tracing the loop NRYN, we get $\overline{E}_{NR} + \overline{E}_{RY} - \overline{E}_{NY} = 0$

or $$\overline{E}_{RY} = \overline{E}_{NY} - \overline{E}_{NR} \quad \text{(vector difference)}$$

To find the vector sum of E_{NY} and $-E_{NR}$, reverse the vector E_{NR} and add it vectorially with E_{NY} as shown in Figure 8.5(b).

Therefore, $$E_{RY} = \sqrt{E_{NY}^2 + E_{NR}^2 + 2E_{NY}E_{NR}\cos 60°}$$

or $$E_L = \sqrt{E_{ph}^2 + E_{ph}^2 + 2E_{ph}E_{ph} \times 0.5} = \sqrt{3E_{ph}^2} = \sqrt{3}E_{ph} \qquad \text{(in magnitude)}$$

Similarly, $$\overline{E}_{YB} = \overline{E}_{NB} - \overline{E}_{NY} \quad \text{or} \quad E_L = \sqrt{3}E_{ph} = \overline{E}_{NR} - \overline{E}_{NB} \quad \text{or} \quad E_L = \sqrt{3}E_{ph}$$

Hence, in star connections, line voltage = $\sqrt{3}$ × phase voltage.

Relation between Phase Current and Line Current

From Figure 8.5(a), it is clear that same current flows through phase winding as well as the line conductor since line conductor is just connected in series with the phase winding.

$$I_R = I_{NR};\ I_Y = I_{NY} \text{ and } I_B = I_{NB}$$

where $I_{NR} = I_{NY} = I_{NB} = I_{ph}$ (phase current) and $I_R = I_Y = I_B = I_L$ (line current)

Hence, in star connections, line current = phase current.

MESH OR DELTA (Δ) CONNECTION

In delta (Δ) or mesh connections, the finish terminal of one winding is connected to start terminal of the other winding and so on, which forms a closed circuit. The three line conductors are run from three junctions of the mesh called line conductors, as shown in Figure 8.6.

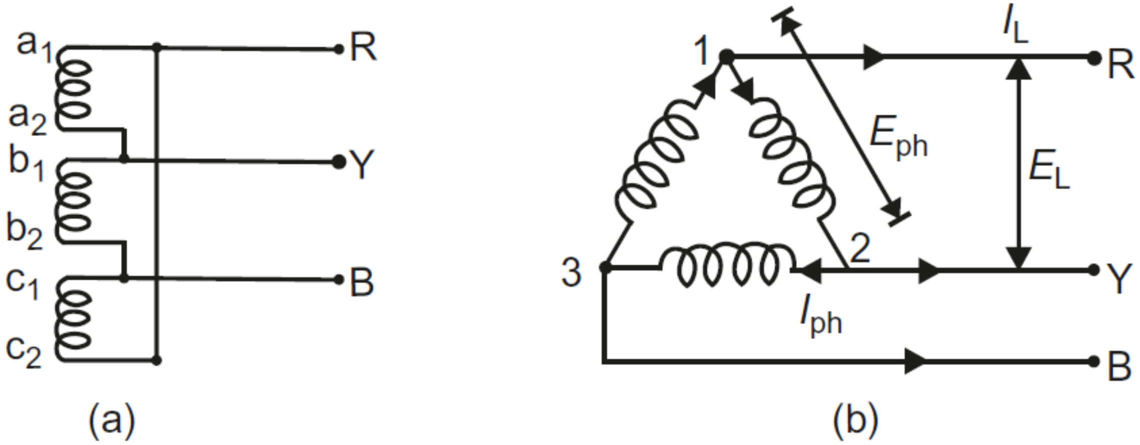


Fig. 8.6 (a) Three phases connected in delta (b) Delta connected system

To obtain delta connection, a_2 is connected with b_1, b_2 is connected with c_1, and c_2 is connected with a_1, as shown in Figure 8.6(a). Three conductors R, Y, and B are run from the three junctions called line conductors. The current flowing through each phase is called phase current (I_{ph}) and the current flowing through each line conductor is called line current (I_L), as shown in Figure 8.6(b). Similarly, voltage across each phase is called phase voltage (E_{ph}) and voltage across two line conductors is called line voltage (E_L).

Relation between Phase Voltage and Line Voltage

From Figure 8.7(a), it is clear that voltage across terminals 1 and 2 is the same as across terminals R and Y.

Therefore, $E_{12} = E_{RY}$; similarly, $E_{23} = E_{YB}$ and $E_{31} = E_{BR}$,

where $E_{12} = E_{23} = E_{31} = E_{ph}$ (phase voltage) and $E_{RY} = E_{YB} = E_{BR} = E_L$ (line voltage)

Hence, in delta connection, line voltage = phase voltage.

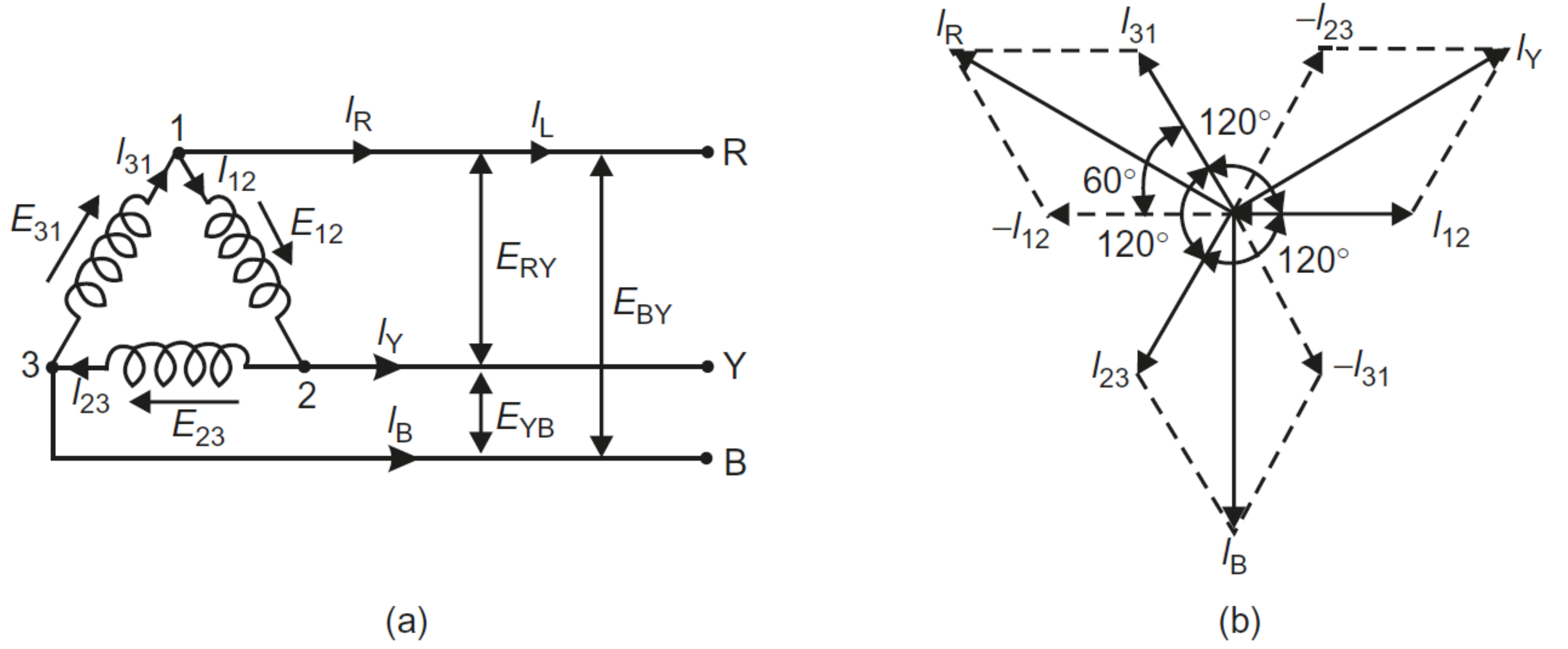


Fig. 8.7 **(a) Circuit representing phase and line voltages and currents in delta connections (b) Phasor diagram of phase and line currents in delta connections**

Relation between Phase Current and Line Current

Since the system is balanced, and therefore, three phase currents I_{12}, I_{23}, and I_{31} are equal in magnitude, but displaced from one another by 120° electrical. Their phasors are shown in Figure 8.7(b).

Thus, $$I_{12} = I_{23} = I_{31} = I_{ph} \text{ (in magnitude)}$$

In Figure 8.7(a), it may be seen that current is divided at every junction 1, 2, and 3.

By applying Kirchhoff's first law at junction 1,

incoming currents = outgoing currents

$$\overline{I_{31}} = \overline{I_R} + \overline{I_{12}} \quad \text{or} \quad \overline{I_R} = \overline{I_{31}} - \overline{I_{12}} \text{ (vector difference)}$$

To find the vector sum of I_{31} and $-I_{12}$, reverse the vector I_{12} and add it vectorially with I_{31}, as shown in Figure 8.7(b).

$$I_R = \sqrt{I_{12}^2 + I_{12}^2 + 2I_{31}I_{12}\cos 60^\circ} = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph} \times 0.5} \quad (-I_R = I_l)$$

or $$I_L = \sqrt{3I_{ph}^2} = \sqrt{3}I_{ph} \text{ (in magnitude)}$$

Similarly, $$\overline{I_Y} = \overline{I_{12}} - \overline{I_{23}} \quad \text{or} \quad I_L = \sqrt{3}I_{ph} \quad \text{and}$$

$$\overline{I_B} = \overline{I_{23}} - \overline{I_{31}} \quad \text{or} \quad I_L = \sqrt{3}I_{ph}$$

CONNECTIONS OF THREE-PHASE LOADS

Similar to three-phase supply, the three-phase loads may also be connected in star or delta.

The three-phase loads connected in star and delta are shown in Figure 8.8(a) and (b), respectively. The three-phase loads may be balanced or unbalanced. If the three loads (impedances) Z_1, Z_2, and Z_3 are having same magnitude and phase angle, then the three-phase load is said to be a balanced load. Under such connections, all the phase or line currents and all the phase or line voltages are equal in magnitude. Throughout this book, balanced three-phase system will be considered unless stated otherwise.

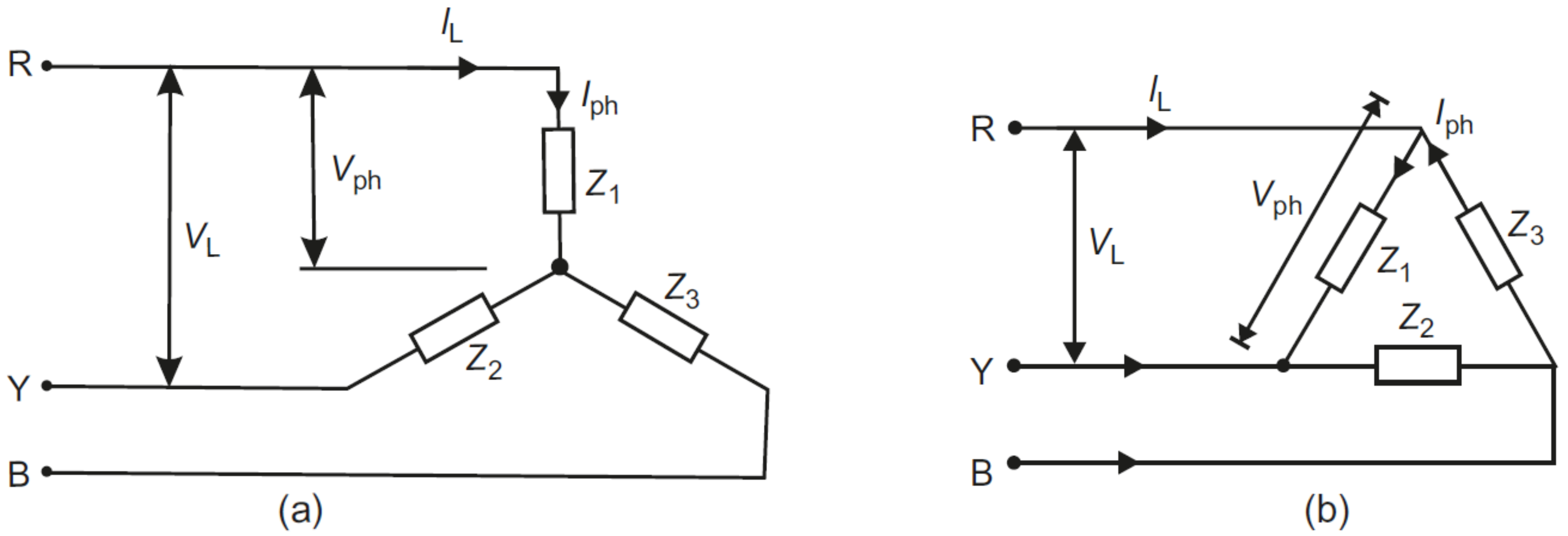


Fig. 8.8 (a) Three phase load connected in star (b) Three phase load connected in delta

POWER IN THREE-PHASE CIRCUITS

Power in single-phase system or circuit is given by the relation:

$$P = VI\cos\phi$$

where V = voltage of single phase (i.e., V_{ph});

I = current of single phase (i.e., I_{ph}); and

$\cos\phi$ = power factor of the circuit.

In three-phase circuits (balanced load), the power is just the sum of powers in three phases

$$P = 3V_{ph}I_{ph}\cos\phi$$

In star connections, $P = 3\frac{V_L}{\sqrt{3}}I_L\cos\phi$ (since $V_{ph} = V_L/\sqrt{3}$ and $I_{ph} = I_L$

or $P = \sqrt{3}\,V_L I_L\cos\phi$

In delta connections, $P = 3\,V_L\frac{I_L}{\sqrt{3}}\cos\phi$ (since $V_{ph} = V_L$ and $I_{ph} = I_L/\sqrt{3}$)

or $P = \sqrt{3}\,V_L I_L\cos\phi$

Thus, the total power in a three-phase balanced load, irrespective of connections (star or delta), is given by the relation $\sqrt{3}\,V_L I_L\cos\phi$. Its units are kW or W.

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or $P = \sqrt{3}\,V_L I_L\cos\phi$

Thus, the total power in a three-phase balanced load, irrespective of connections (star or delta), is given by the relation $\sqrt{3}\,V_L I_L\cos\phi$. Its units are kW or W.

Apparent power, $P_a = \sqrt{3}\,V_L\,I_L$ (kVA or VA)

Reactive power, $P_r = \sqrt{3}\,V_L I_L\sin\phi$ (kVAR or VAR)

Example

Three 100-Ω resistors are connected first in star and then in delta across 415 V, three-phase supply. Calculate the line and phase currents in each case and also the power taken from the source.

Solution:

The resistors are connected in star as shown in Figure 8.9.

Phase voltage, $V_{ph} = V_L/\sqrt{3} = 415/\sqrt{3} = 239.6\text{ V}$

Phase current, $I_{ph} = V_{ph}/Z_{ph} = 239.6/100 = 2.396\text{ A}$

Line current, $I_L = I_{ph} = 2.396\text{ A}$

Power drawn, $P = 3I_{ph}^2 R_{ph} = 3\times(2.396)^2\times 100 = 1,722\text{ W}$

The resistors are connected in delta as shown in Figure 8.10.

$V_{ph} = V_L = 415\text{ V}$

$I_{ph} = V_{ph}/Z_{ph} = 415/100 = 4.15\text{ A}$

$I_L = \sqrt{3}\times 4.15 = 7.188\text{ A}$

Power drawn, $P = 3I_{ph}^2 R_{ph} = 3\times(4.15)^2\times 100 = 5,166\text{ W}$

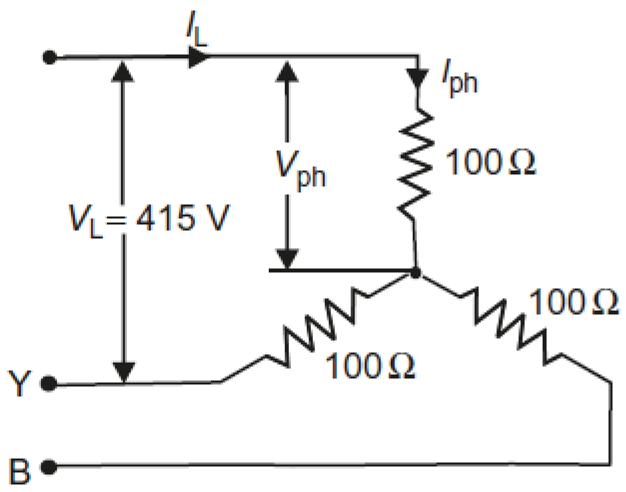

Fig. 8.9 Resistors connected in star

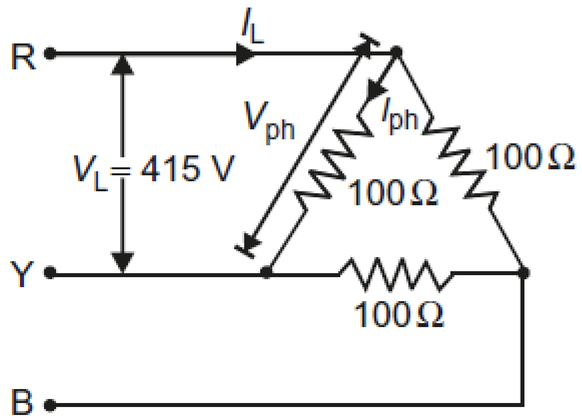

Fig. 8.10 Resistors connected in delta