SOLVED EXAMPLES

Example 1 Calculate the frequency and wavelength of a photon whose energy is 75 eV.

Solution Given energy $E = 75\text{ eV} = 75 \times 1.6 \times 10^{-19}$ J.

Formula used is

$$E = h\nu = \frac{hc}{\lambda}$$

$$\text{Frequency } (\nu) = \frac{E}{h} = \frac{75 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 18.13 \times 10^{15}\text{ Hz}$$

and wavelength

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8}}{18.13 \times 10^{15}} = 1.655 \times 10^{-8}\text{ m} = 165.5 \times 10^{-10}\text{ m}$$

or $\lambda = \mathbf{165.5\ Å}$

Example 2 Find the number of quanta of energy emitted per second if a radio station operates at a frequency of 98 MHz and radiates power of 2×10^5 W.

Solution Given $\nu = 98 \times 10^6$ cycles / sec and Power (P) $= 2 \times 10^5$ W $= 2 \times 10^5$ J/sec.

Energy of each quanta is

$$E = h\nu$$

$$\therefore E = 6.62 \times 10^{-34} \times 98 \times 10^6$$

$$= 6.4876 \times 10^{-26}\text{ J/quanta}$$

$$= 6.5 \times 10^{-26}\text{ J/quanta}$$

Number of quanta emitted per second

$$= \frac{\text{Power}}{\text{quantum energy}}$$

$$= \frac{2 \times 10^5\,(\text{J/sec})}{6.5 \times 10^{-26}\,(\text{J/quanta})}$$

$$= \mathbf{3.08 \times 10^{30}\ quanta/sec}$$

Example 3 A certain spectral line has wavelength 4000 Å. Calculate the energy of the photon.

Solution Given $\lambda = 4.0 \times 10^{-7}$ m.

Formula used is

$$E_k = h\nu = \frac{hc}{\lambda}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7}\text{ m}}$$

$$= \mathbf{4.965\ \ 10^{-19}\ J}$$

Example 4 Calculate the number of photons of green light of wavelength 5000 Å require to make one erg of energy.

Solution Given $\lambda = 5 \times 10^{-7}$ m.

Formula used is

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{5 \times 10^{-7}}$$

$$= 3.972 \times 10^{-19}\text{ J}$$

$$= 3.972 \times 10^{-12}\text{ erg}$$

Number of photons of green light emitted (per energy)

$$= \frac{1.0}{3.972 \times 10^{-12}}$$

$$= \mathbf{252 \times 10^{9}}$$

Example 5 Calculate the wavelength of a photon of energy 5×10^{-19} J.

Solution Given $E = 5 \times 10^{-19}$ J

Formula used is

$$E = \frac{hc}{\lambda}$$

or

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{5 \times 10^{-19}}$$

$$= 3.972 \times 10^{-7}\text{ m}$$

$$= \mathbf{4000\ Å}$$

Example 6 Calculate the energy of an electron of wavelength 4.35×10^{-7} m.

Solution Given $E = 4.35 \times 10^{-7}$ m.

Formula used is

$$E = h\nu = \frac{hc}{\lambda}.$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{4.35 \times 10^{-7}}$$

$$= \mathbf{4.566 \times 10^{-19}\ J}$$

Example 7 How many watts of power at the threshold is received by the eye, if it receives 120 photons per second of the visible light of wavelength = 5600 Å.

Solution Given $\lambda = 5.6 \times 10^{-7}$ m and number of photons = 120.

Energy of a photon $E = h\nu = \frac{hc}{\lambda}$

or $E = \frac{6.62\times10^{-34}\times3\times10^{8}}{5.6\times10^{-7}} = 3.55\text{J}$

The energy received by the eye per second = 3.55 × 120 J/sec

$= 425.57$ **W**

Example 8 How many photons of yellow light of wavelength 5500 Å constitute 1.5 J of energy.

Solution Given $\lambda = 5.5 \times 10^{-7}$ m and energy of n photons = 1.5 J

Formula used is $E = h\nu = \frac{hc}{\lambda}$

Energy of a photon of yellow light, i.e.

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{5.5\times10^{-7}}$$
$$= 3.61\times10^{-19}\text{J}$$

Given

$n\times$ energy of one photon = 1.5 J

or $n = \frac{1.5}{3.61\times10^{-19}}$

$= \mathbf{4.155\times10^{18}}$

Example 9 Calculate the work function, stopping potential and maximum velocity of photoelectrons for a light of wavelength 4350 Å when it incidents on sodium surface. Consider the threshold wavelength of photoelectrons to be 5420 Å.

Solution Given $\lambda_0 = 5.42 \times 10^{-7}$ m and $\lambda = 4.35 \times 10^{-7}$ m.

ormulae used are

$$\phi_0 = \frac{hc}{\lambda_0} = h\nu_0$$

$$\frac{1}{2}mv_{max}^2 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \quad \text{and}$$

$$eV = h\nu - h\nu_0 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$

or $eV = \frac{1}{2}mv_{max}^2 = (E_k)_{max}$

$$\phi_0 = \frac{hc}{\lambda_0} = \frac{6.62\times10^{-34}\times3\times10^{8}}{5.42\times10^{-7}} = 3.664\times10^{-19}\text{J}$$

$$\phi_0 = \frac{hc}{\lambda_0} = \frac{6.62\times10^{-34}\times3\times10^{8}}{5.42\times10^{-7}} = 3.664\times10^{-19}\text{J}$$

$$\frac{1}{2}mv_{max}^2 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$

$$v_{max}^2 = \frac{2hc}{m}\left[\frac{\lambda_0 - \lambda}{\lambda_0\lambda}\right]$$

$$= \frac{2\times6.62\times10^{-34}\times3\times10^{8}}{9.1\times10^{-31}}\left[\frac{(5.42-4.35)\times10^{-7}}{5.42\times4.35\times10^{-14}}\right] = 0.1981\times10^{1}$$

$$\therefore v_{max} = 0.445\times10^{6}\text{m/sec}$$
$$= 4.45\times10^{5}\text{m/sec}$$

$$eV = \frac{1}{2}mv_{max}^2$$

The stopping potential

$$V = \frac{mv_{max}^2}{2e} = \frac{9.1\times10^{-31}\times(4.45\times10^{5})^2}{2\times1.6\times10^{-19}}$$
$$= \mathbf{0.56\ volts}$$

Example 10 The threshold frequency for photoelectric emission in copper is 1.1×10^{15} Hz. Find the maximum energy in eV when light of frequency 1.2×10^{15} Hz is directed on the copper surface.

Solution Given $\nu_0 = 1.1 \times 10^{15}$ Hz and $\nu = 1.2 \times 10^{15}$ Hz.

Formula used is

$$\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0 = h(\nu - \nu_0)$$
$$= 6.62\times10^{-34}[1.2-1.1]\times10^{15}$$
$$= 0.662\times10^{-19}\text{J}$$
$$= \mathbf{0.414eV}$$

Example 11 Calculate the work function in electron volts of a metal, given that photoelectric threshold is (i) 6200 Å (ii) 5000 Å.

Solution Given (i) $\lambda_0 = 6.2 \times 10^{-7}$ m (ii) $\lambda_0 = 5.0 \times 10^{-7}$ m.

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

(i) $\phi_0 = \frac{6.62\times10^{-34}\times3\times10^{8}}{6.2\times10^{-7}\times1.6\times10^{-19}}$ eV

$= \mathbf{2.0\ eV}$

(ii) $\lambda_0 = 5.0\times10^{-7}$m

$$\phi_0 = \frac{6.62\times10^{-34}\times3\times10^{8}}{5\times10^{-7}\times1.6\times10^{-19}}$$
$$= 2.483\text{ eV}$$
$$= \mathbf{2.48\ eV}$$

Example 12 Find out the maximum energy of the photoelectron, work function and threshold frequency when a light of wavelength 3132 Å is incident on a surface of cesium and the stopping potential for the photo electron is 1.98 volt.

Solution Given V= 1.98 volts and λ=3.132x10^{-7}m.

Formulae used are

$$E_k = \frac{1}{2}mv_{max}^2 = eV_0, \quad V_0 = \text{stopping potential}$$

$$\text{and } E_k = h(\nu - \nu_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Then maximum energy of the photoelectron (E_{max})

$$= eV_0 = 1.6\times10^{-19}\times1.98 \text{ J}$$

$$E_k = 3.168\times10^{-19} \text{ J}$$

$$E_k = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$3.168\times10^{-19} = 6.62\times10^{-34}\times3\times10^8\left[\frac{1}{3.132\times10^{-7}} - \frac{1}{\lambda_0}\right]$$

$$\frac{1}{6.2689\times10^{-7}} = \frac{1}{3.132\times10^{-7}} - \frac{1}{\lambda_0}$$

$$\text{or } \frac{1}{\lambda_0} = 3.193\times10^6 - 1.595\times10^6 = 1.598\times10^6$$

$$\lambda_0 = \frac{1}{1.598\times10^6} = 6258 \text{ Å}$$

$$\text{Work function}(\phi_0) = \frac{hc}{\lambda_0} = \frac{6.62\times10^{-34}\times3\times10^8}{6.258\times10^{-7}}$$

$$= \mathbf{3.174\times10^{-19}\ J}$$

Example 13 Is it possible to liberate an electron from a metal surface having work function 4.8 eV with an incident radiation of wavelength (i) 5000 Å and (ii) 2000 Å.

Solution Given $\phi_0 = 4.8$ eV.

Formula used is $E_k = \frac{hc}{\lambda}$.

$$\text{(i) Energy}(E_k) = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^8}{5\times10^{-7}} \text{ J}$$

$$= \frac{6.62\times10^{-34}\times3\times10^8}{5\times10^{-7}\times1.6\times10^{-19}} \text{ eV}$$

$$= \mathbf{2.48\ eV}$$

From the above it is clear that the energy corresponding to wavelength 5000 Å is found to be less than the work function i.e. 4.8 eV. So it will not be able to liberate an electron.

$$\text{(ii) } E_k = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^8}{2.0\times10^{-7}} = 9.93\times10^{-19} \text{ J}$$

$$\text{or } E_k = \frac{9.93\times10^{-19}}{1.6\times10^{-19}} \text{eV} = 6.206 \text{ eV}$$

$$E_k = \mathbf{6.21\ eV}$$

As the energy corresponding to wavelength 2000 Å is greater than the work function. So it is sufficient to liberate electrons.

Example 14 Find the maximum energy of the photoelectron, the work function and threshold frequency, if the potassium surface is illuminated by a light of wavelength 5893 Å. The stopping potential for the emitted electron be 0.36V.

Solution Given stopping potential V_0 = 0.36V and λ = 5893 Å.

Formula used is

$$E_k = eV = h\nu - \phi_0.$$

$$E_k = eV = 0.36\text{eV}$$

Work function

$$(\phi_0) = h\nu - eV = \frac{hc}{\lambda} - eV$$

$$= \frac{6.62\times10^{-34}\times3\times10^8}{5.893\times10^{-7}\times1.6\times10^{-19}} - 0.36 \text{ eV}$$

$$= 2.11 - 0.36 = 1.75\text{eV}$$

Thus the work function is 1.75 eV.

Threshold frequency

$$\nu_0 = \frac{\phi}{h}$$

$$= \frac{1.75\times1.6\times10^{-19}}{6.62\times10^{-34}}$$

$$= \mathbf{4.23\times10^{14}\ cycles/sec}$$

Example 15 Find the maximum kinetic energy of the emitted electrons and the stopping potential if the light of wavelength 5890 Å is incident on the surface for which threshold frequency is 7320 Å.

Solution Given $\lambda = 5.89\times10^{-7}$ m and $\lambda_0 = 7.32\times10^{-7}$ m.

Formula used is

$$E_k = h\nu - h\nu_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$= \frac{6.62\times10^{-34}\times3\times10^8}{10^{-7}}\left[\frac{1}{5.89} - \frac{1}{7.32}\right]$$

$$= 19.86\times10^{-19}\left[\frac{7.32-5.89}{5.89\times7.32}\right]$$

$$= \mathbf{6.587\times10^{-20}\ J}$$

$$Ve = E_k \quad \text{or} \quad V = \frac{E_k}{e}$$

$$\text{Stopping potential}(V) = \frac{6.587\times10^{-20}}{1.6\times10^{-19}}$$
$$= \mathbf{0.412\ V}$$

Example 16 The threshold wavelength for photoelectric emission in tungsten is 2300 Å. What wavelength of light must be used in order for electrons with a maximum energy of 1.5 eV to be ejected?

Solution Given $\lambda_0 = 2.3 \times 10^{-7}$ m and $E_k = 1.5$ eV.

Formula used is

$$E = h\nu - h\nu_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$\frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{E}{hc}$$

$$\text{or} \quad \frac{1}{\lambda} = \frac{1.5\times1.6\times10^{-19}}{6.62\times10^{-34}\times3\times10^{8}} + \frac{1}{2.3\times10^{-7}}$$
$$= 1.2085\times10^{6} + 4.3478\times10^{6}$$
$$\frac{1}{\lambda} = 5.556\times10^{6}$$
$$\text{or} \quad \lambda = 1.7998\times10^{-7}\ \text{m}$$
$$\lambda = \mathbf{1799.8\ Å}$$

Example 17 The work function of tungsten is 4.53 eV. If ultraviolet light of wavelength 1500 Å is incident on the surface, does it cause photoelectron emission? If so, what is the kinetic energy of the emitted electron?

Solution Given work function $\phi_0 = 4.53$ eV and $\lambda = 1.5 \times 10^{-7}$ m.

Formula used is $E_k = \dfrac{hc}{\lambda}$

Energy corresponding to incident photon of wavelength 1.5×10^{7} m

$$E_k = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{1.5\times10^{-7}}\ \text{J}$$
$$= 13.24\times10^{-19}\ \text{J}$$
$$E_k = 8.28\ \text{eV}$$

The kinetic energy of the electron

$$E_k = \frac{1}{2}mv_{max}^2 = h\nu - h\nu_0 = h\nu - \phi_0$$
$$= 8.28 - 4.53$$
$$= \mathbf{3.75\ eV}$$

Example 18 The work function of sodium metal is 2.3 eV. What is the longest wavelength of light that cause photoelectric emission from sodium?

Solution Given $\phi_0 = 2.3\ \text{eV} = 2.3 \times 1.6 \times 10^{-19}$ J.

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

Longest wavelength = Threshold wavelength

$$\lambda_0 = \frac{hc}{\phi_0} = \frac{6.62\times10^{-34}\times3\times10^{8}}{2.3\times1.6\times10^{-19}}$$
$$= \mathbf{5396.74\ Å}$$

Example 19 Evaluate the threshold wavelength of photoelectric material whose work function is 2.0 eV.

Solution Given $\phi_0 = 2.0\ \text{eV} = 2 \times 1.6 \times 10^{-19}$ J.

Formula used is

$$\lambda = \frac{hc}{\phi_0}$$

$$\text{or} \quad \lambda = \frac{6.62\times10^{-34}\times3\times10^{8}}{2\times1.6\times10^{-19}}$$
$$= \mathbf{6206\ Å}$$

Example 20 Calculate the threshold wavelength and the wavelength of incident electromagnetic radiation so that the photoelectrons emitted from potassium have a maximum kinetic energy of 4eV. Take the work function of potassium as 2.2 eV.

Solution Given $E_{max} = 4.0 \times 1.6 \times 10^{-19}$ J and $\phi_0 = 2.2 \times 1.6 \times 10^{-19}$ J.

Formulae used are

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0} \quad \text{and} \quad E_k = h\nu - \phi_0 = h\nu - h\nu_0$$

$$\lambda_0 = \frac{hc}{\phi_0} = \frac{6.62\times10^{-34}\times3\times10^{8}}{2.2\times1.6\times10^{-19}}$$
$$\lambda_0 = 5642\ \text{Å} \quad \text{(Threshold wavelength)}$$
$$E_k = 4\times1.6\times10^{-19} = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$
$$\text{or} \quad \frac{1}{\lambda} = \frac{4\times1.6\times10^{-19}}{6.62\times10^{-34}\times3\times10^{8}} + \frac{1}{\lambda_0}$$
$$= 3.223\times10^{6} + 1.772\times10^{6}$$
$$\frac{1}{\lambda} = 4.995\times10^{6}$$
$$\lambda = \frac{10}{4.995\times10^{7}}$$
$$= \mathbf{2002\ Å}$$

Example 21 Ultraviolet light of wavelength 350 nm and intensity 1.0 watt/m² is directed at a potassium surface. (i) Find the maximum kinetic energy of photoelectron (ii) 0.5% of incident photons produce photoelectrons, how many photoelectrons are emitted per second if the surface of potassium is 1.0 cm². Work function of potassium is 2.1 eV.

Solution Given $\lambda = 3.5 \times 10^{-7}$ m and $\phi_0 = 2.1$ eV.

(i) Formula used is

$$E_k = \frac{1}{2} m v_{max}^2 = h\nu - \phi_0.$$

$$= \frac{hc}{\lambda} - \phi_0 = \frac{6.62\times10^{-34}\times3\times10^8}{3.5\times10^{-7}\times1.6\times10^{-19}} - 2.1\text{ eV}$$

$$E_k = (3.546 - 2.1)\text{eV} = 1.45\text{ eV}$$

$$= 2.3136\times10^{-19}\text{ J}$$

$$= \mathbf{2.314\times10^{-19}\ J}$$

(ii) Energy incident per second on 1.0 cm² surface of potassium = 10^{-4} Joule

The energy which produces photoelectron per second = 0.5%.

Effective energy which will be used to produce photoelectrons = $\frac{0.5}{100} \times 10^{-4}\text{ J} = 5\times10^{-7}\text{ J}$

Minimum energy required to eject one electron from the surface

$$= 2.314 \times 10^{-19}\text{ J}$$

So the number of electrons emitted per second from 1.0 cm² area of the surface of potassium will be $= \frac{5\times10^{-7}}{2.314\times10^{-19}}$.

$$= \mathbf{2.16\times10^{12}}$$

Example 22 Calculate the value of Planck's constant from the following data, assuming that the electronic charge *e* has value of 1.6×10^{-19} Coulomb. A surface when irradiated with light of wavelength 5896 Å emits electrons for which the stopping potential is 0.12 volts. When the same surface is irradiated with light of wavelength 2830 Å, it emits electrons for which the stopping potential is 2.2 volts.

Solution If the radiation of wavelength is incident on the surface of the metal having work function ϕ_0 and stopping potential V_0 for the emitted electrons, then ϕ_0 and V_0 satisfy the following relation.

$$eV_0 = \frac{hc}{\lambda} - \phi_0 \qquad \text{(i)}$$

(i) Given $\lambda = 5.896 \times 10^{-7}$m and $V_0 = 0.12$ volts

$$\frac{hc}{\lambda} = eV_0 + \phi_0$$

$$\frac{h\times3\times10^8}{5.896\times10^{-7}} = 1.6\times10^{-19}\times0.12 + \phi_0 \qquad \text{(ii)}$$

(ii) Given $\lambda = 2.83 \times 10^{-7}$ m and $V_0 = 2.2$ volts, then

$$\frac{h\times3\times10^8}{2.83\times10^{-7}} = 1.6\times10^{-19}\times2.2 + \phi_0 \qquad \text{(iii)}$$

On subtracting Eq. (ii) from Eq. (iii), we get

$$h\left[\frac{3\times10^8}{2.83\times10^{-7}} - \frac{3\times10^8}{5.896\times10^{-7}}\right] = \left[1.6\times10^{-19}\times2.2 - 1.6\times10^{-19}\times0.12\right]$$

$$h\times\frac{3\times10^{15}[5.896-2.83]}{2.83\times5.896} = 1.6\times10^{-19}\times2.08$$

$$h = \mathbf{6.04\times10^{-34}\ Jsec}$$

Example 23 Calculate Compton shift if X-rays of wavelength 1.0 Å are scattered from a carbon block. The scattered radiation is viewed at 90° to the incident beam.

Solution Given $\lambda = 1.0$ Å $= 10^{-10}$m and $\phi = 90°$.

Formula used is

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi).$$

$$= \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^8}(1 - \cos 90°)$$

$$= 0.242\times10^{-11}\text{m}$$

$$= 0.024\times10^{-10}\text{m}$$

$$= \mathbf{0.0242\ Å}$$

Example 24 An X-ray photon is found to have doubled its wavelength on being scattered by 90°. Find the energy and wavelength of incident photon.

Solution Given $\phi = 90°$.

Formula used is

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\phi). \qquad \text{(i)}$$

$$= \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^8}(1 - \cos 90°)$$

$$= \mathbf{0.242\times10^{-11}\,m = 0.024\ Å}$$

As $\Delta\lambda = \lambda' - \lambda$, where λ is the wavelength of incident photon and λ' is the wavelength of scattered photon, then

$$\lambda' = \lambda + \Delta\lambda \qquad \text{(ii)}$$

Given

$$\lambda' = 2\lambda \qquad \text{(iii)}$$

From Eqs. (2) and (3), we get

$$2\lambda = \lambda + \Delta\lambda$$

or $\lambda = \Delta\lambda = 0.0242 \times 10^{-10}$ m $= 0.0242$ Å

Energy of the incident photon (E) $= h\nu = \frac{hc}{\lambda}$

For neutron

$$E = \frac{(6.62\times10^{-34})^2}{2\times1.7\times10^{-27}\times(10^{-10})^2} = \frac{43.8244\times10^{-68}}{3.4\times10^{-47}}$$
$$= 12.89\times10^{-21}\,\text{J}$$
$$= \mathbf{0.081\ eV}$$

Example 38 Calculate deBroglie wavelength of an electron whose kinetic energy is (i) 500 eV, (ii) 50 eV and (iii) 1.0 eV.

Solution Formula used is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

(i) $E = 500\,\text{eV} = 500\times1.6\times10^{-19} = 8.0\times10^{-17}\,\text{J}$

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times8.0\times10^{-17}}} = 5.486\times10^{-11}\,\text{m}$$
$$= \mathbf{0.5486\text{Å}}$$

(ii) $E = 50\,\text{eV} = 50\times1.6\times10^{-19} = 8.0\times10^{-18}$

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times8\times10^{-18}}}$$
$$\lambda = 1.735\times10^{-10}\,\text{m}$$

or $\lambda = \mathbf{1.735\text{Å}}$

(iii) $E = 1.0\,\text{eV} = 1.6\times10^{-19}\,\text{J}$

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}}}$$
$$\lambda = \mathbf{12.267\text{Å}}$$

Example 39 Calculate the ratio of deBroglie wavelengths associated with the neutrons with kinetic energies of 1.0 eV and 510 eV.

Solution Formula used is

$$\lambda = \frac{h}{\sqrt{2mE}}.$$

r $E = 1.0\,\text{eV} = 1.6\times10^{-19}\,\text{J}$ and $m_n = 1.7\times10^{-27}\,\text{kg}$

$$\lambda_1 = \frac{6.62\times10^{-34}}{\sqrt{2\times1.7\times10^{-27}\times1.6\times10^{-19}}}$$
$$= 2.838\times10^{-11}$$
$$\lambda_1 = \mathbf{0.284\text{Å}}$$

For $E = 510\,\text{eV} = 510\times1.6\times10^{-19} = 816\times10^{-19}\,\text{J}$

$$\lambda_2 = \frac{6.62\times10^{-34}}{\sqrt{2\times1.7\times10^{-27}\times816\times10^{-19}}}$$
$$\lambda_2 = 0.01257\text{Å}$$
$$= \mathbf{0.0126\,\text{Å}}$$

and ratio of deBroglie wavelength is

$$\frac{\lambda_1}{\lambda_2} = \frac{0.284}{0.0126} = \mathbf{22.54:1}$$

Example 40 Calculate the ratio of deBroglie waves associated with a proton and an electron each having the kinetic energy as 20 M eV [$m_p = 1.67\times10^{-27}$ kg and $m_e = 9.1\times10^{-31}$ kg].

Solution Given energy of each proton and electron is $20\times10^6\times10^{-19}\,\text{J} = 3.2\times10^{-12}\,\text{J}$.

Formula used is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

For proton

$$\lambda_p = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times3.2\times10^{-12}}}$$
$$= 6.4\times10^{-15}\,\text{m}$$

For electron

$$\lambda_e = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.2\times10^{-12}}}$$
$$= 2.74\times10^{-13}\,\text{m}$$

The ratio of λ_p to λ_e is

$$\lambda_p : \lambda_e = \mathbf{1:43}$$

Example 41 Calculate the deBroglie wavelength of 1.0 M eV proton. Do we require relativistic calculation?

Solution Given Energy $E = 1.0\times10^6\times1.6\times10^{-19}\,\text{J} = 1.6\times10^{-13}\,\text{J}$

Formula used for velocity of Proton

$$E = \frac{1}{2}mv^2 \quad \text{or} \quad v^2 = \frac{2E}{m}$$

$$\text{or} \quad v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times1.6\times10^{-13}}{1.67\times10^{-27}}}$$
$$= \mathbf{1.38\times10^7\,m/sec}$$

From the above result it is clear that the velocity of proton is nearly one twentieth of the velocity of light. So the relativistic calculations are not required.

Example 42 Calculate the deBroglie wavelength associated with a proton moving with a velocity equal to $\frac{1}{20}$ of velocity of light.

Solution Given $v = \frac{c}{20} = \frac{3\times10^8}{20} = 1.5\times10^7$ m/sec and $m = 1.67\times10^{-27}$ kg

Formula used is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.62\times10^{-34}}{1.67\times10^{-27}\times1.5\times10^7}$$
$$= \mathbf{2.643\times10^{-14}\,m}$$

Example 43 Calculate the kinetic energy of a proton and an electron so that the deBroglie wavelengths associated with them is the same and equal to 5000 Å.

Solution Given wavelength of proton and electron = 5.0×10^{-7} m.

Formula used in

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{or} \quad E = \frac{h^2}{2m\lambda^2}$$

For proton $m = m_p = 1.67 \times 10^{-27}$ kg and $\lambda = 5.0 \times 10^{-7}$ m

$$E = \frac{(6.62\times10^{-34})^2}{2\times1.67\times10^{-27}\times(5.0\times10^{-7})^2}$$
$$= \frac{43.8244\times10^{-68}}{83.5\times10^{-41}} = 0.5248\times10^{-27}\,J$$
$$= \mathbf{5.248\times10^{-28}\,J}$$

For electron $m = m_e = 9.1 \times 10^{-31}$

$$E = \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times(5\times10^{-7})^2} = \frac{43.8244\times10^{-68}}{4.55\times10^{-43}}$$
$$E = \mathbf{9.63\times10^{-25}\,J}$$

Example 44 Find deBroglie wavelength of an electron in the first Bohr's orbit of hydrogen atom.

Solution Energy of an electron in the first Bohr's orbit of hydrogen atom can be obtained by using the relation

$$E_n = \frac{-13.6}{n^2}$$

$$E_1 = \frac{-13.6}{1^2} = -13.6\,eV$$
$$E_1 = -13.6\times1.6\times10^{-19}\,J = -2.176\times10^{-18}\,J$$

Magnitude of energy is $= 2.176\times10^{-18}$ J

$$\text{Wavelength } \lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times21.76\times10^{-19}}}$$
$$= 3.3\times10^{-10}\,m$$
$$= \mathbf{3.3\,Å}$$

Example 45 Calculate the ratio of deBroglie wavelengths of a hydrogen atom and helium atom at room temperature, when they move with thermal velocities. Given mass of hydrogen atom

$m_H = 1.67\times10^{-27}$ kg and mass of helium atom $m_{He} = 4\times m_p = 4\times1.67\times10^{-27}$ kg at room temperature T = 27°C = 300 K and Boltzmann's constant k = 1.376×10^{-23} J/K.

Solution deBroglie wavelength can be calculated by the relation

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

ydrogen atom

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{3\times1.67\times10^{-27}\times1.376\times10^{-23}\times300}}$$
$$= 1.456\times10^{-10}\,m$$
$$\lambda = \mathbf{1.456\,Å}$$

Helium atom

$$\lambda_{He} = \frac{6.62\times10^{-34}}{\sqrt{3\times4\times1.67\times10^{-27}\times1.376\times10^{-23}\times300}}$$
$$= 0.728\times10^{-10}\,m$$
$$= \mathbf{0.728\,Å}$$

ratio of wavelengths i.e

$$\frac{\lambda_H}{\lambda_{He}} = \frac{1.456}{0.728} = \frac{2}{1}$$
$$\mathbf{\lambda_H : \lambda_{He} = 2:1}$$

ample 46 A proton and a deuteron have the same kinetic energy. Which has a longer wavelength?

Solution m_p = mass of porton, $m_d = 2m_p$ and v_p and v_d are the velocities of proton and deuteron.

tic energy of proton is given by

$$E_p = \frac{1}{2}m_p v_p^2$$

l kinetic energy of deuteron is

$$E_d = \frac{1}{2}m_d v_d^2 = \frac{1}{2}(2m_p)v_d^2$$
$$E_d = m_p v_d^2$$

But $E_p = E_d$, then

$$m_p v_d^2 = \frac{1}{2}m_p v_p^2$$
$$\text{or} \quad v_d = \frac{v_p}{\sqrt{2}}$$

oglie wavelength corresponding to moving proton and deuteron are

$$\lambda_p = \frac{h}{m_p v_p} \quad \text{and}$$

$$\lambda_d = \frac{h}{m_d v_d} = \frac{h}{2m_p v_p / \sqrt{2}} = \frac{h}{\sqrt{2} m_p v_p}$$

$$\frac{\lambda_d}{\lambda_p} = \frac{h}{\sqrt{2}\, m_p v_p} \times \frac{m_p v_p}{h} = \frac{1}{\sqrt{2}}$$

$$\lambda_p = \sqrt{2}\, \lambda_d$$