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Binomial Distribution

$n = p$
$q = 1 - p$

- x = no. of success
  $n - x$ = no. of failure

- $p(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

- No. of trials are finite
  " " are independent
  n = no. of trials × 1 each trial has only 2 outcomes i.e. success or failure
  constant
  the prob. of success for all is same in all trials.

- $E(X) = \Sigma x p(x)$

  mean = np
  varience = npq
  Std. dev. = $\sqrt{npq}$

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- The prob. of x no. of success's in n trials is given by

$$p(X=x) = {}^nC_x\, p^x q^{n-x}$$
$$x = 0, 1, 2, \ldots, n$$

$p+q=1$

- Mean $= E(X) = n.p.$
- Varience $= \cancel{E^2} = n.p.q$
- Std dev. : $\sqrt{npq}$

1. ~~Find~~ The mean & std. dev. of a binomial distribution are 5 & 2 resp. Determine the distribution.

$\longrightarrow$ Mean $= np = 5$

std dev $= \sqrt{npq} = 2$

$\therefore npq = 4$

$$\frac{npq}{np} = \frac{4}{5}$$

$$\therefore q = \frac{4}{5}$$

$$p = 1-q = 1-\frac{4}{5} = \frac{1}{5}$$

$np = 5$

$n\frac{1}{5} = 5$

$\therefore n = 25$

$\therefore$ The corresponding binomial distribution is

$$= 25C_x \left(\frac{1}{5}\right)^x \cdot \left(\frac{4}{5}\right)^{25-x}, \quad x = 0, 1, 2, \ldots, 25$$

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2. If 10% of bolts produced by machine are defective. Find the prob. that out of 5 bolts choosen at random.
1. None is defective
2. 1 is defective
3. atmost 2 are defective.

→ Let p = be prob. that bolt is defective.

$= \frac{10}{100} = 0.1$

$q = 1 - p = 1 - 0.1 = 0.9$

$n = 5$

∴ Prob of x no. of defective bolts

$P(X=x) = 5C_x (0.1)^x (0.9)^{5-x} \quad x = 0, 1, 2 \ldots 5$

1. $p(X=0) = 5C_0 (0.1)^0 (0.9)^{5-0}$
$= 0.5905$

2. $p(X=1) = 5C_1 (0.1)^1 (0.9)^{5-1}$
$= 5 \times 0.1 \times (0.9)^4$
$= 0.3281$

3. $p(X \leq 2) = p(X=0) + p(X=1) + p(X=2)$
$= 0.5905 + 0.3281 + {}^5C_2 (0.1)^2 (0.9)^3$
$= 0.5905 + 0.3281 + 0.0729$
$= 0.9915$

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3. 6 coins are tossed simultaneously. What is prob. of getting

1. 2 heads
2. atleast 2 heads
3. atmost 2 heads.

→ p = is prob. of getting head $= \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 6$

prob. of x no. of heads

$= {}^6C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{6-x}$ $x = 0, 1, 2 \ldots 6.$

1. $p(x=2)$

${}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4$

$= 15 \times 0.25 \times 0.0625$

$= 0.2343$

3. $p(x \leq 2)$

$= p(x=0) + p(x=1) + p(x=2)$

$= {}^6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^6 + {}^6C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^5 + {}^6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4$

$= 0.015625 + 0.09375 + 0.2343$

$= 0.3436$

atleast 2 means
2 or more than 2
instead we will do
$1 - (P(0) + P(1))$

2. $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - 0.1094$

$= 0.8906$

4. If $E(X) = 3\,var(X)$ & $p(X=0) = 1 - p(X=1)$
Find $p(X=0)$

$p(X=0) + p(X=1) = 1$

$\rightarrow E(X) = 3\,var(X)$

$\cancel{n}p = 3\cancel{n}pq$

$q = \frac{1}{3}$

$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

$n = 1$ as X can assume 2 values 0 & 1.

1. $p(X=0)$

$1 - 2C_0 \left(\frac{2}{3}\right)^{0} \left(\frac{1}{3}\right)^{0} = \cancel{1 \times 0.4489 \times 0.1089}$

$1 - 0.2178$

0.7822

$1 - {}^{1}C_1 p^1 q^{1-1}$

$= \boxed{1 - \left(1 \cdot \frac{2}{3}\right)}$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$

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1. The prob. that a man aged 60 will live up to 70 is 0.65. What is prob. that out of 10 such man now at 60 atleast 7 will live up to 70.

→ p = man who is 60 lives upto 70 $\quad p(x=x) = {}^{10}C_x (0.65)^x (0.35)^{10-x}$

$p = 0.65$ $\quad x = 0,1,2,\dots,10$

$q = 1-0.65 = 0.35$

$n = 10$

Ans 0.5138

1. $P(X \geq 7)$

$= P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_7 (0.65)^7 (0.35)^3 + {}^{10}C_8 (0.65)^8 (0.35)^2 + {}^{10}C_9 (0.65)^9 (0.35)^1 + {}^{10}C_{10} (0.65)^{10} (0.35)^0$

$= 120 \times 0.049 \times 0.0428 + 45 \times 0.0319 \times 0.1225 + 10 \times 0.020 \times 0.35 + 1 \times 0.0135$

$= 0.2522 + 0.1758 + 0.07 + 0.0135$

$= 0.5115$

Imp
Out of 800 families with 5 children how many would you expected to have
1. 3 boys
2. 5 girls
3. either 2 or 3 boys
4. atleast 1 boy
assuming equal probabilities for boys & girls.

→ p = single family with 5 children has a boy

$= \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

prob. distri. of a family having x boys out of 5 children

$p(x=x) = 5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$

$x = 0,1,2 \ldots 5$

$n=5$

i) $p(x=3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$= 10 \times 0.125 \times 0.25$

$= 0.3125$

No. of families out of 800 having 3 boys

$= 0.3125 \times 800 = 250$

ii) $p(x=5$ girls$)$ $p(x=0$ boys$)$

$5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5$

0.03125

No. of families out of 800 having 5 girls

$= 0.03125 \times 800 = 25$

iii) $p(x=2)+p(x=3)$

$${}^5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3+{}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$$

$= 10\times 0.25\times 0.125+10\times 0.125\times 0.25$

$= 0.3125+0.3125$

$= 0.625$

No. of families $= 800\times 0.625 = 500$

iv) $p(x\geq 1)$

$1-p(x\leq 0)$

$= 1-0.03125$

$= 0.96875$

No. of families $= 0.96875\times 800 = 775$

3. Assuming that half the population of town is vegetarian so that the chance of individual begin vegetarian is $\frac{1}{2}$ & assuming that 100 investigators ~~that~~ take a sample of 10 individuals to see whether they are vegetarian or not. How many investigators would you expect to report that 3 people or less in a sample were vegetarian?

→ p = individual is vegetarian

$p = \frac{1}{2}$ $\quad$ p = prob. of n persons were veg out of 10

$p = {}^{10}C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 10$

i) $P(X \leq 3)$

$P(X=0) + P(X=1) + P(X=2) + P(X=3)$

${}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10} + {}^{10}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^9 + {}^{10}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 + {}^{10}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7$

0.171

$= \cancel{1 \times 9.765 \times 10^{-4} + 10 \times 0.5 \times 1.95 \times 10^{-3} + 45 \times 0.25 \times 3.9 \times 10^{-3} + 120 \times 0.125 \times 7.81 \times 10^{-3}}$

9.75

$= \cancel{9.76 + 20.6 + 43.875 + 117.15}$

$= \cancel{196.5 \simeq 197}$

$9.765 \times 10^{-4} + 0.0975 +$

$9.765 + 9.75 + 0.0438 + 0.117$

$= 1 \times 1 \times 9.765 \times 10^{-4} + 10 \times 0.5 \times 1.953 \times 10^{-3} + 45 \times 0.25 \times 3.906 \times 10^{-3} + 120 \times 0.125 \times 7.8125 \times 10^{-3}$

$= 9.765 \times 10^{-4} + 9.765 \times 10^{-3} + 0.0439 + 0.117$

$= 0.171$

$= 100 \times 0.171 = 17$

$n \to \infty \cdot 0 \le p < 0.1$
$p \to 0 \quad 0 < p \le 0.5$

4. In sampling large no. of parts manufactured by a machine the mean no. of defectives in a sample of 20 is 2. Out of 1000 such samples how many would you expect exactly 2 defective parts.

$\longrightarrow n = 20$

Mean $= 2$

$np = 2$

$20p = 2$

$p = \frac{2}{20} = \frac{1}{10} = 0.1$

$q = 1 - \frac{1}{10} = \frac{9}{10} = 0.9$

p = prob. that the part manufactured to be defective $= 0.1$

prob. of x no. of defective parts in a sample of 20

$p(X = x) = {}^{20}C_x (0.1)^x (0.9)^{20-x} \quad x = 0, 1, 2, \ldots, 20$

1. $p(X = 2)$

${}^{20}C_2 (0.1)^2 (0.9)^{18}$

$190 \times 0.01 \times 0.15$

$= 0.285$

No. of samples with defective parts

$= 1000 \times 0.285 = 285$

Poisson Distribution (Rear events prob. so ans is too small)

(1847) No. of trials too large

$$p(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}, \quad x = 0, 1, 2 \ldots$$

$\lambda = np$ is called the parameter

- mean $= \lambda = np$ (n too large, p too small)

Varience $= \lambda$

Std dev. $= \sqrt{\lambda}$

- Law of improbable events:
  e.g. flood
  radiation emitted from radioactive substance.
  no. of printing mistake in a book.
  no. of accident on roads
  no. of defect in product of reputed company.

- $B(n, p)$
  λ is single parameter $n \to \infty$, $p \to 0$

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1. If mean of poisson distribution is 1.8. Find
i) $P(X>1)$
ii) $P(X=5)$
iii) $P(0<X\leq 5)$

$\longrightarrow$ Mean $=\lambda=np=1.8$

i) $P(X>1)$

$= 1-P(X\leq 1)$

$= 1-[P(X=0)+P(X=1)]$

$= 1-\left[\frac{e^{-1.8}(1.8)^0}{0!}+\frac{e^{-1.8}(1.8)^1}{1!}\right]$

$= 1-[0.1653+0.2975]$

$= 1-0.4628$

$= 0.5372$

ii) $P(X=5)$

$= \frac{e^{-1.8}(1.8)^5}{5!}$

$= \frac{(0.1653)(18.9)}{120}$

$= \frac{3.1242}{120}$

$= 0.026$

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x can b
$-$ve
but $>$

iii) $P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$$= \frac{e^{-1.8}(1.8)^1}{1!} + \frac{e^{-1.8}(1.8)^2}{2!} + \frac{e^{-1.8}(1.8)^3}{3!} + \frac{e^{-1.8}(1.8)^4}{4!}$$

$$= e^{-1.8}\left[1.8 + \frac{(1.8)^2}{2} + \frac{(1.8)^3}{6} + \frac{(1.8)^4}{24}\right]$$

$$= e^{-1.8}\left[1.8 + 1.62 + 0.972 + 0.4374\right]$$

$$= e^{-1.8}(4.8294)$$

$$= 0.1653(4.8294)$$

$$= 0.7983$$

~~2. For a poisson distribution~~

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2. If for a poisson distribution $\frac{3}{2} p(X=1) = p(X=3)$. find

i) $P(X \geq 1)$
ii) $P(X \leq 3)$

$\longrightarrow$ $\frac{3}{2} P(X=1) = P(X=3)$

$$\frac{3}{2}\frac{\cancel{e^{-\lambda}}\lambda^1}{1!} = \frac{\cancel{e^{-\lambda}}\lambda^3}{3!}$$

$$\frac{3\lambda}{2} = \frac{\lambda^3}{6}$$

$$18\lambda = 2\lambda^3$$

$$2\lambda^3 - 18\lambda = 0$$

$\cancel{2(\lambda^3 - 9\lambda) = 0}$ $2\lambda(\lambda^2 - 9) = 0$

$\cancel{\lambda^3 = 9\lambda}$ $\lambda = 0, \lambda = \pm 3$

$\cancel{\lambda = \sqrt{9\lambda}}$

$\therefore \boxed{\lambda = 3}$

i) $P(X \geq 1)$

$= 1 - P(X < 1)$
$= 1 - [P(X=0)]$
$= 1 - \left\{\frac{e^{-3}(3)^0}{0!}\right\}$

$= 1 - 0.04978$

$= 0.9502$

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$P(X \leq 3)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= \dfrac{e^{-3}(3)^0}{0!} + \dfrac{e^{-3}(3)^1}{1!} + \dfrac{e^{-3}(3)^2}{2!} + \dfrac{e^{-3}(3)^3}{3!}$

$= e^{-3}\left[1 + 3 + \dfrac{9}{2} + \dfrac{27}{6}\right]$

$= e^{-3}\left[1 + 3 + 4.5 + \overset{4.5}{\cancel{1.125}}\right]$

$= e^{-3}\left[\overset{13}{\cancel{7.375}}\right]$

$= 0.6472.$

3. Suppose on an avg. 1 house in 1000th in a certain district has a fire during the yr. If there are 2000 houses in that district what is prob. that exactly 5 houses will have a fire during a yr.?

house will catch fire

→ $n = 2000$, $p = 1/1000$ (= 0.001)

$\lambda = np$

$= 2000 \times \frac{1}{1000}$

$= 2$

$P(X=5)$

$= \frac{e^{-2}(2)^5}{5!}$ $x = 0, 1, 2, \ldots 2000$

$= \frac{(0.1353)(32)}{120}$

$= \frac{4.3296}{120}$

$= 0.0361$

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4. If 2% light bulbs are defective find the prob that

i) None defective
ii) atleast 1 defective in a sample of 100

→ $n=100$, $p=$ defective bulb $= \frac{2}{100}$

$\lambda = n.p$
$= \cancel{100} \times \frac{2}{\cancel{100}}$
$= 2$

According to poisson dis.
$p(X=x) = \frac{e^{-2} \cdot 2^{x}}{x!}$, $x = 0, 1, 2 \ldots 100$

i) $P(X=0)$
$\frac{e^{-2}(2)^{0}}{0!} = 0.1353$

ii) $P(X \geq 1)$
$= 1 - P(X=0)$
$= 1 - 0.1353$
$= 0.8647$

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5. Suppose a book has 585 pages contains 43 errors. What is prob. that 10 pages selected at random will be free from errors?

→ $n = 10$, $p = \frac{43}{585} = 0.0735$

$\lambda = 10 \times 0.0735$
$= 0.735$

According to poisson dis.

$P(X = x) = \frac{e^{-0.735} \, 0.735^{x}}{x!}$

$x = 0, 1, 2, \ldots 10$

n is not too large but we use poisson as prob. of success is too small.

$P(X = 0)$

$\frac{e^{-0.735} (0.735)^0}{0!}$

$= 0.4795$

6. In a certain company turning out blades there is a small chance of 1/500 for any blade to be defective. The blades are supplied in a packet of 10 use poisson distribution to calculate the approx. no. of packets containing no def. 1 def, 2 def. blades in a consignment of 10,000 packets.

→ p = blade is defective in a packet

$= \frac{1}{500}$

$n = 10$

$\lambda = np = 10 \times \frac{1}{500} = \frac{1}{50} = 0.02$

Accor. to poisson dis.

$$P(X=x) = \frac{e^{-0.02}(0.02)^x}{x!} \quad x = 0, 1, 2 \ldots 10{,}000$$

i) $P(X=0)$

No. of packets having '0' defective blades in 10000 packets

$$\frac{e^{-0.02}(0.02)^0}{0!} = 0.9802, \quad 0.9802 \times 10000 = 9802$$

ii) $P(X=1)$

0.9802×0.02

$$\frac{e^{-0.02}(0.02)^1}{1!}$$

$= 0.0196$

No. of packets having 1 defective blade in 10000 packets

$= 0.0196 \times 10000 = 196$

iii) $P(X=2)$

$$\frac{e^{-0.02}(0.02)^2}{2!}$$

0.00098

$$= \frac{(0.98)(4 \times 10^{-4})}{2}$$

$0.9802 \times (0.02)^2$

$= 1.96 \times 10^{-4}$

No. of packets having 2 def. blades in 10000 packets

$= 1.96 \times 10^{-4} \times 10000$

$= 1.96 \simeq 2$

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Exam

1. Fit a poisson distribution to the following data:

No. of deaths (x)	0	1	2	3	4
frequency (f)	122	60	15	2	1

$\Longrightarrow$ $\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{0+60+30+6+4}{122+60+15+2+1}$

$= \frac{100}{200}$

$= \frac{1}{2} = 0.5$

$P(X=0) = \frac{e^{-0.5}(0.5)^0}{0!} = 0.6065$

$P(X=1) = \frac{e^{-0.5}(0.5)^1}{1!} = 0.3033$

$P(X=2) = \frac{e^{-0.5}(0.5)^2}{2!} = 0.0758$

$P(X=3) = \frac{e^{-0.5}(0.5)^3}{3!} = 0.0126$

$P(X=4) = \frac{e^{-0.5}(0.5)^4}{4!} = 0.0016$

$200 \times P(X=0) = 200 \times 0.6065 = 121.3 \simeq 121$
$200 \times P(X=1) = 200 \times 0.3033 = 60.6 \simeq 61$
$200 \times P(X=2) = 200 \times 0.0758 = 15.16 \simeq 15$

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$200 \times P(x=3) = 200 \times 0.0126 = 2.52 \simeq 3$
$200 \times P(x=4) = 200 \times 0.0016 = 0.32 \simeq 0$

x	0	1	2	3	4
f	121	61	15	3	0

Poisson freq. distribution table.

2. Assuming that the typing mistakes committed by a typist follows a poisson distribution. Find the expected frequencies for following distribution of mistakes.

No. of mistakes (per page)	0	1	2	3	4	5
No. of pages	40	30	20	15	10	5

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{\cancel{0+30+40+45+40+25}\ 30+20+45+40+25}{40+30+20+15+10+5} = \frac{180}{120} = \frac{180}{40} = \frac{6}{8} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$P(x=x) = \frac{e^{-1.5}(1.5)^{x}}{x!}, \quad x = 0, 1, 2 \ldots 120$$

$$P(x=0) = \frac{e^{-1.5}(1.5)^{0}}{0!} = 0.2231$$

$$P(x=1) = \frac{e^{-1.5}(1.5)^{1}}{1!} = 0.3347$$

$$P(x=2) = \frac{e^{-1.5}(1.5)^{2}}{2!} = 0.2510$$

$$P(X=3) = \frac{e^{-1.5}(1.5)^3}{3!} = 0.1255$$

$$P(X=4) = \frac{e^{-1.5}(1.5)^4}{4!} = 0.0470$$

$$P(X=5) = \frac{e^{-1.5}(1.5)^5}{5!} = 0.0141$$

$f(0) = 0.0223 \times 120 = 26.7 \simeq 27$

$f(1) = 0.3347 \times 120 = 40$

$f(2) = 0.2510 \times 120 = 30$

$f(3) = 0.1255 \times 120 = 15$

$f(4) = 0.0471 \times 120 = 5$

$f(5) = 0.0141 \times 120 = 1.69 \simeq 2$

X	0	1	2	3	4	5
f(x)	27	40	30	15	5	2

3. The no. of defects per unit in a sample of 330 units was found as follows. Fit a poisson distribution to data & test the goodness of fit.

No. of defects	0	1	2	3	4
No. of units	214	92	20	3	1

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{0+92+40+9+4}{330} = \frac{145}{330} = 0.44$$

$$P(X=x) = \frac{e^{-0.44}(0.44)^x}{x!} \quad x = 0, 1, 2 \ldots 330$$

$$P(X=0) = \frac{e^{-0.44}(0.44)^0}{0!} = 0.6440$$

$$P(X=1) = \frac{e^{-0.44}(0.44)^1}{1!} = 0.2834$$

$$P(X=2) = \frac{e^{-0.44}(0.44)^2}{2!} = 0.0623$$

$$P(X=3) = \frac{e^{-0.44}(0.44)^3}{3!} = 0.0091$$

$$P(X=4) = \frac{e^{-0.44}(0.44)^4}{4!} = 0.0010$$

$$f(0) = 0.6440 \times 330 = 212.52 \simeq 212$$

$f(1) = 0.2834 \times 330 = 93.52 \simeq 94$

$f(2) = 0.0623 \times 330 = 20.559 \simeq 21$

$f(3) = 0.0091 \times 330 = 3$

$f(4) = 0.001 \times 330 = 0.33 \simeq 0$

x	0	1	2	3	4
$f(x)$	212	94	21	3	0

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Normal Probability Distribution

- A continuous random variable x is said to follow a normal probability distribution if

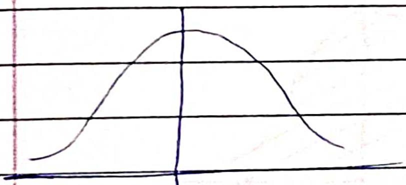
$$p(X=x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty$$

μ = mean
σ = std. dev.

Note:
- limiting case of both BD & PD
- Large N & P

- Remarks:

1. The curve of normal distribution is given by



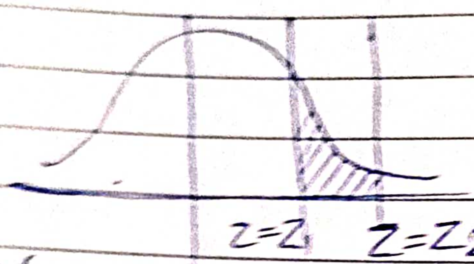
2. This curve is symmetric about the line $x=\mu$.
3. Area under this bell-shaped curve = 1.
4. $P(-\infty < X < \infty) = 1$
5. $P(-\infty < x < \mu) = P(\mu < x < \infty) = 1/2$
6. For this distribution: mean = median = mode.
7. $P(x_1 < X < x_2)$

$$= P\left(\frac{x_1-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{x_2-\mu}{\sigma}\right)$$

Where $P(z_1 < z < z_2)$

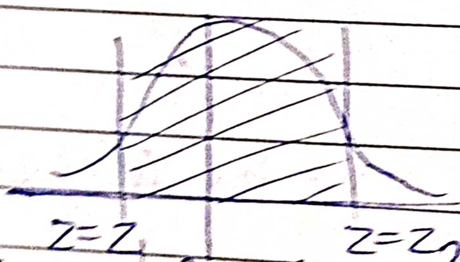
$z = \frac{x-\mu}{\sigma}$, $z_1 = \frac{x-\mu}{\sigma}$, $z_2 = \frac{x_2-\mu}{\sigma}$

- Case I $P(z_1 < z < z_2)$



$= P(0 < z < z_2) - P(0 < z < z_1)$

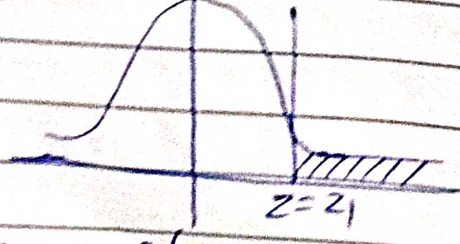
- Case II $P(z_1 < z < z_2)$



$= P(-z_1 < z < 0) + P(0 < z < z_2)$
$= P(0 < z < z_1) + P(0 < z < z_2)$

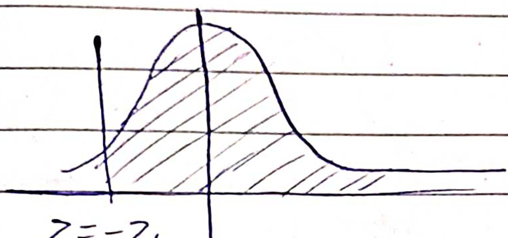
- Case III $P(z > z_1)$

$z_1 > 0$



$= 0.5 - P(0 < z < z_1)$

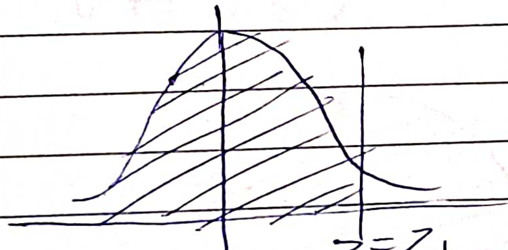
- Case IV $P(Z > -Z_1)$



$= 0.5 + P(-Z_1 < Z < 0)$

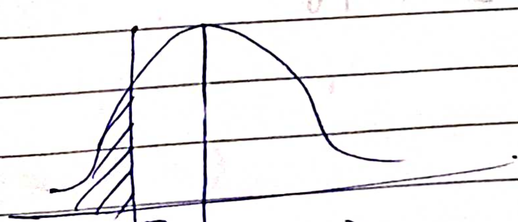
$= 0.5 + P(0 < Z < Z_1)$

- Case V $(Z < Z_1)$ $Z_1 > 0$



$\therefore = 0.5 + P(0 < Z < Z_1)$

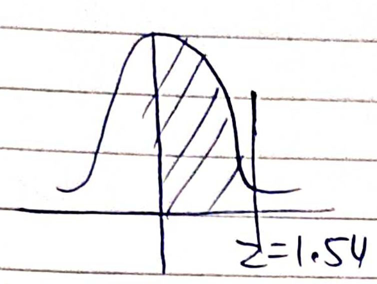
- Case VI $(Z < -Z_1)$



$= 0.5 - P(-Z_1 < Z < 0)$

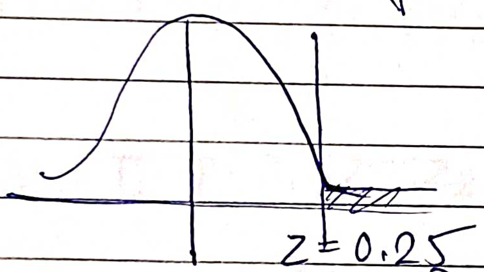
$= 0.5 - P(0 < Z < Z_1)$

1. Find the ar. under the normal curve for $z=0$ to $z=1.54$



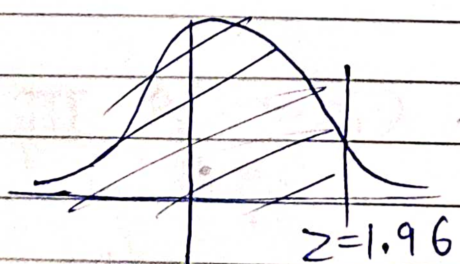
$= 0.4382$

2. Find the ar. right to $z=0.25$



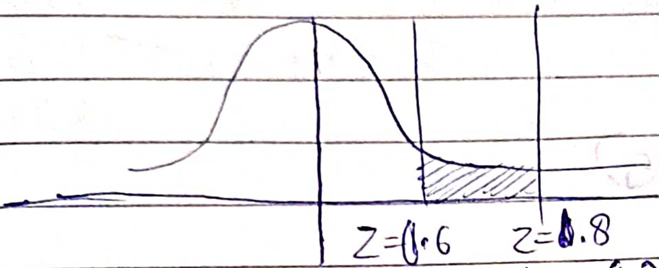
$= 0.5 - P(0<z<0.25)$
$= 0.5 - 0.0987$
$= 0.4013$

3. Find the ar. left of $z=1.96$



$= 0.5 + P(0<z<1.96)$
$= 0.5 + 0.4750$
$= 0.9750$

4. Find the prob. of z lies Betwn $P(0.6<z<1.8)$

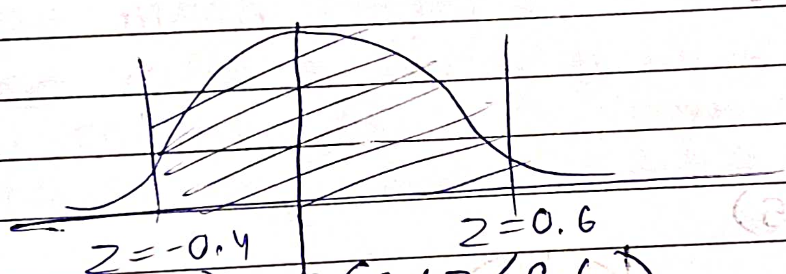


$= P(0<z<1.8) - P(0<z<0.6)$

$= 0.4641 - 0.2257$

$= 0.2384$

5. $P(-0.4 \text{ to } 0.6)$



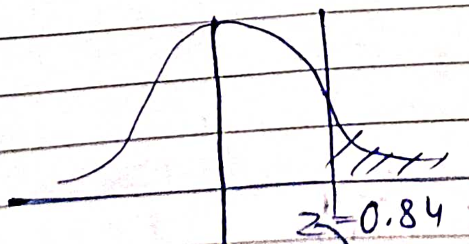
$= P(-0.4<z<0) + P(0<z<0.6)$

$= P(0<z<0.4) + P(0<z<0.6)$

$= 0.1554 + 0.2257$

$= 0.3811$

6. $P(z>0.84)$



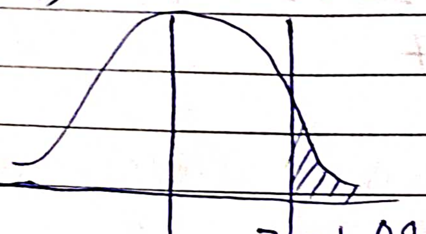
$= 0.5 - P(0<z<0.84)$

$= 0.5 - 0.2995 = 0.2005$

21/3/23

1. Find
1. $P(z > 1.09)$
2. $P(z < -1.65)$
3. $P(-1 < z < 1.96)$
4. $P(1.25 < z < 2.75)$

→ 1. $P(z > 1.09)$

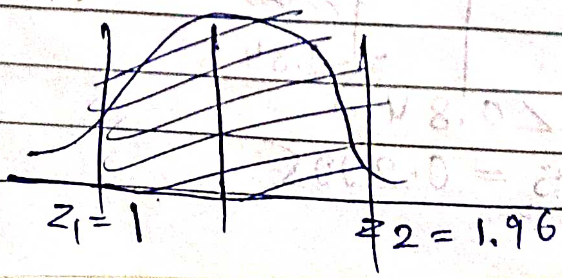


$= 0.5 - P(0 < z < 1.09)$
$= 0.5 - 0.3621$
$= 0.1379$

2. $P(z < -1.65)$

$= 0.5 - P(-z_1 < z < 0)$
$= 0.5 - P(0 < z < z_1)$
$= 0.5 - 0.4505$
$= 0.0495$

3. $P(-1 < z < 1.96)$



$= P(-z_1 < z < 0) + P(0 < z < z_2)$

$= P(0 < z < 1) + P(0 < z < 1.96)$

$= 0.3413 + 0.4750$

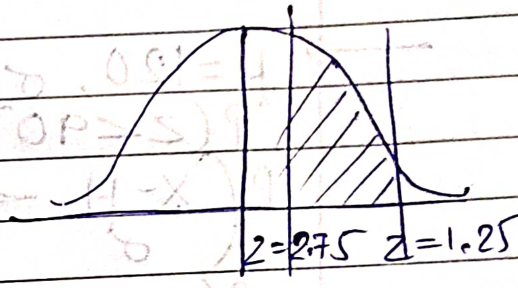
$= 0.8163$

4. $P(1.25 < z < 2.75)$ 1.25

$= P(0 < z < 2.75) - P(0 < z < 1.25)$

$= 0.4970 - 0.3944$

$= 0.1026$



8. Assume mean height of soliders to be 68.22 inches with a varience of 10.8 inches. How many soliders in a regiment of 1000 would you expect to be over 6 feet tall?

6 feet = 72 inches

$\mu = 10.8, \quad \sigma = \sqrt{10.8} = 3.29$

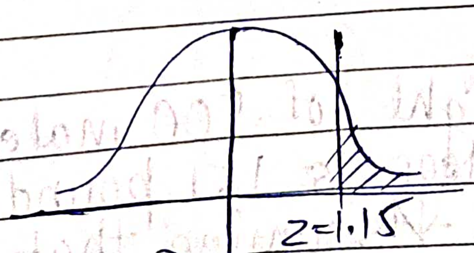
$P(X > 72) = ?$

$P\left(\frac{X-\mu}{\sigma} > \frac{72 - 68.22}{3.29}\right)$

upto 2 decimal places approx.

$P(z > 1.15)$

We should draw for confirm



$= 0.5 - P(0 < z < 1.15)$

$= 0.5 - 0.3749$

$= 0.1251$

∴ No. of soliders having height ~~greater~~ more than 72 inches $= 1000 \times 0.1251$

$= 125.1 \simeq 125$

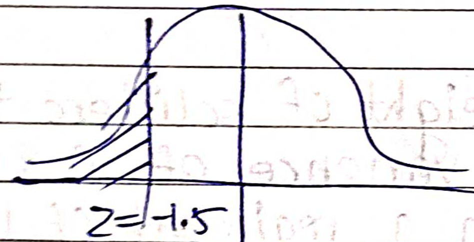
3. 1000 light bulbs with a mean life 120 days are installed in a new factory, their length of life is normally distributed with std. dev. of 20 days. How many bulbs will expire in less than 90 days?

→ $\mu = 120,\ \sigma = 20$

$P(z < 90) = ?$

$P\left(\frac{x-\mu}{\sigma} < \frac{90-120}{20}\right)$

$P(z < -1.5)$



$= 0.5 - P(-1.5 < z < 0)$

$= 0.5 - P(0 < z < 1.5)$

$= 0.5 - 0.4332$

$= 0.0668$

No. of bulbs will expire in less than 90 days

$= 1000 \times 0.0668$

$= 66.8 \simeq 67$

4. The mean weight of 500 males students in a certain college is 151 pounds (lbs) & std. dev. is 15 pounds. Assuming that weights are normally distributed, find how many students weigh between

1. 120 & 155 pounds
2. more than 185 pounds.

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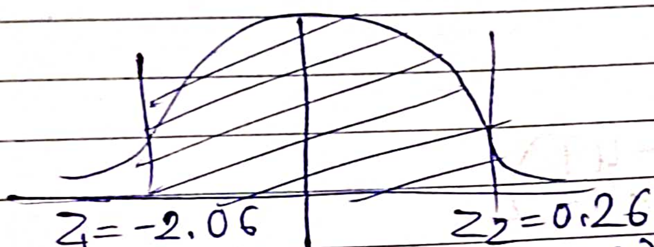
→ $\mu = 151, \sigma = 15$

$P(120 < z < 155)$

$P\left(\frac{120-151}{15} < \frac{x-151}{15} < \frac{155-151}{15}\right)$

1. $P(-2.06 < z < 0.26)$



col(a) if is same

$= P(-2.06 < z < 0) + P(0 < z < 0.26)$

$= P(0 < z < 2.06) + P(0 < z < 0.26)$

$= 0.4803 + 0.1026$

$= 0.5829$

∴ No. of students having weight betwⁿ 120 & 155 pounds

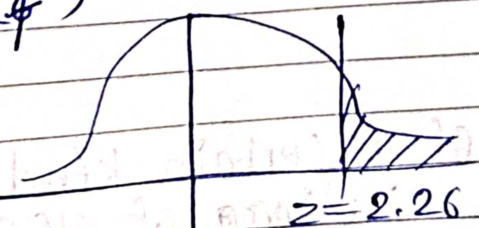
$= 0.5829 \times 500$

$= 291.45 \simeq 291$

2. $P(x > 185)$

$P\left(\frac{x-\mu}{\sigma} > \frac{185-151}{15}\right)$

$P(z > 2.27)$



$= 0.5 - P(0 < z < 2.26)$

$= 0.5 - 0.4884$

$= 0.0116$

∴ No of students having weight more than 185 pounds

$= 0.0116 \times 500 = 5.8 \simeq 6$

20M
Q1-MCQ, FIB-5M, Objective, Define
Q2-OR, Q3-OR, Q4-NO option
5M 5M

Whole ~~Qset~~ of 5M or distributed
Poisson, Binomial, Normal ~~dist~~:
tables are given.

Date: / /

22/3/23
Poisson - given
Binomial - decide on own

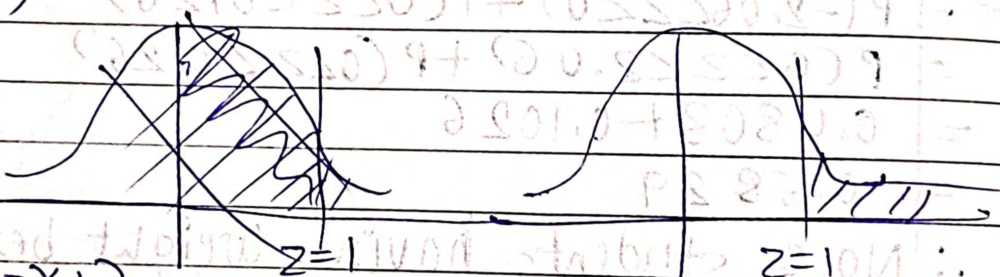
1. 15000 students appeared for an exam. The mean marks were 49 & std. dev. of marks was 6. Assuming that marks are normally distributed What proportion of students scored more than 55 marks?

$\rightarrow$ $\mu = 49,\ \sigma = 6$

$P(X > 55)$

$P\left(\frac{X-\mu}{\sigma} > \frac{55-49}{6}\right)$

$P(Z > 1)$



$= 0.5 - P(0 \le Z \le 1)$
$= 0.5 - 0.3413$
$= 0.1587$

$\therefore$ ~~Prop of students who scored more than 55 marks~~ proportion
$=$ ~~0.1587 × 15000~~
$=$ ~~2380.5~~
$=$ 15.87%.

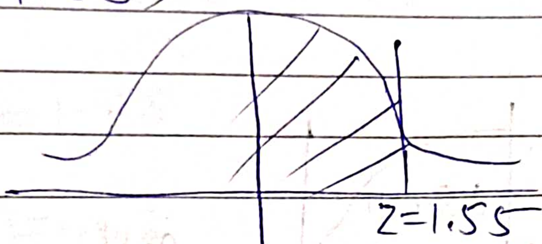
2. The lifetime of a certain kind of batteries has a mean lifetime of 400 hrs & std. dev. as 45 hrs. Assuming distribution of lifetime to be normal, find the % of batteries with a lifetime of i) atleast 470 hrs.
ii) the proportion of batteries with lifetime betw^n^ 385 & 415 hrs.

$\rightarrow \mu = 400, \sigma = 45$

i) $P(X \geq 470)$

$$P\left(\frac{X-\mu}{\sigma} > \frac{470-400}{45}\right)$$

$P(Z > 1.55)$



$= 0.5 - P(0 < z < 1.55)$

$= 0.5 - 0.4394$

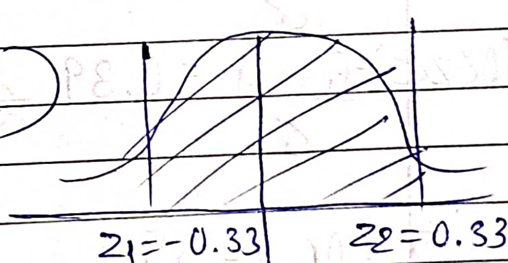
$= 0.0606$

$= 6.06\%$

ii) $P(385 < X < 415)$

$$P\left(\frac{385-400}{45} < \frac{X-\mu}{\sigma} < \frac{415-400}{45}\right)$$

$P(-0.33 < Z < 0.33)$



$2P(0.33)$

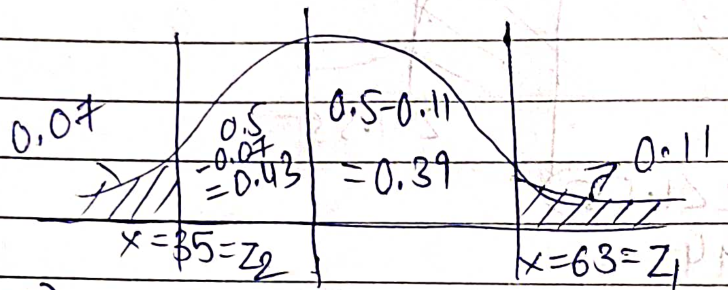
$= 2 \times 0.1293$

$= 0.2586$

$= 25.86\%$

3. Find the mean & std. dev. of a normal distributio in which 7% of items are under 35 & 89% of items are under 63.

→ $P(X<35) = 0.07$
$P(X<63) = 0.89$



$P(X<63) = 0.89$
$P(X \geq 63) = 1-0.89 = 0.11$
⇒ For $x = 63$
$$\frac{X-\mu}{\sigma} = Z_1$$
$$P(0<Z<Z_1) = \frac{63-\mu}{\sigma} = 0.39 \quad \text{--- (1)}$$

⇒ For $x = 35$
$$\frac{X-\mu}{\sigma} = Z_2$$
$$P(0<Z<Z_2) = \frac{35-\mu}{\sigma} = 0.43 \quad \text{--- (2)}$$

Solving (1) & (2)
$\cancel{2205 - 35\mu = 13.65}$
$\cancel{2205 - 63\mu =}$

$P(0<Z<Z_1) = 0.39$
$Z_1 = 1.23$

$P(0<Z<Z_2) = 0.43$
$Z_2 = 1.48$

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$$\frac{x_1-\mu}{\sigma}=1.23$$

$$\frac{63-\mu}{\sigma}=1.23$$

$$63-\mu=1.23\sigma \text{ —— } (1)$$

$$\frac{x_2-\mu}{\sigma}=1.48$$

$$-\left(\frac{35-\mu}{\sigma}\right)=1.48$$

$$35-\mu=1.48\sigma \text{ —— } (2)$$

Solving (1) & (2)

$$63-\mu=1.23\sigma$$
$$-35-\mu=1.48\sigma$$
$$28=-0.25\sigma$$

$$\sigma=\frac{28}{0.25}=112$$

$$63-112=1.23\sigma$$
$$-49=1.23\sigma$$
$$\sigma$$

$$63-\mu=1.23\times112$$
$$63-\mu=137.76$$
$$63-137.76=\mu$$
$$\mu=74.76$$

$$35-\mu=-1.48\sigma$$
$$-63-\mu=1.23\sigma$$
$$-28=2.71\sigma$$

$$\boxed{\sigma=\frac{28}{2.71}=10.33}$$

$$63-\mu=1.23\times10.33$$
$$63-\mu=12.7$$
$$63-12.7=\mu \Rightarrow \boxed{\mu=50.3}$$