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Probability of any event is denoted by P(A) & is defined as

$$P(A) = \frac{\text{no. of outcomes favourable to event A}}{\text{total no. outcomes}}$$

1. Find the probability of getting 1. exactly 2 heads
2. atleast one tail
3. at most 2 heads in tossing 3 coins together.

→ S = { HHH, HHT, HTH, THH, HTT, THT, TTH, TTT }

1. A = getting exactly 2 heads

$P(A) = \frac{3}{8}$

2. B = getting atleast 1 tail

$P(B) = \frac{7}{8}$

3. C = atmost 2 heads

$P(C) = \frac{7}{8}$

2. A card is drawn from well-shuffled pack of 52 cards. Find the probability of
1. getting a king card
2. getting a red card
3. getting a card between 2 & 7 both inclusive.

→ ~~E(A)~~ A = getting a king card

$P(A) = \frac{4}{52} = \frac{1}{13}$

B = getting a red card

$P(B) = \frac{26}{52} = \frac{1}{2}$

C = getting card betwⁿ 2 & 7 both inclusive

$P(C) = \frac{24}{52}$

- Permutation : Order imp.
- Combinatⁿ : order not imp. (selection)

$^nP_r = \frac{n!}{(n-r)!}$ $\quad$ $^nC_r = \frac{n!}{(n-r)!\,r!}$

Probability always has combinatⁿ //.

AND - (×)
OR - (+)

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1. A class consits of 6 girls & 10 boys. If a committee of 3 is choosen at random from the class find the probability that

1. 3 boys are selected.
2. Exactly 2 girls are selected.

~~$A = \frac{^{10}C_3}{^{16}C_3} = 10$~~ Total no. of students $= 6+10=16$.
A committee is formed taking 3 students
$\therefore$ total no. of such committees $= {}^{16}C_3$

1. A = 3 boys are selected

$$P(A) = \frac{^{10}C_3}{^{16}C_3} = \frac{3}{14}$$

$^{6}C_2$ $^{10}C_2$

2. B = 2 girls & 1 boy

$$P(B) = \frac{^{6}C_2 \times {}^{10}C_1}{^{16}C_3} = \frac{15}{56}.$$

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- Thm 4 (Addition Theorem)

$P(A \text{ or } B)$
$= P(A) + P(B) - P(A \cap B)$
$P(A \cup B)$

- For mutually exclusive events $P(A \cap B)$ is not possible.

- Statement: For any 2 events A & B the probability of occurence of ~~that~~ atleast 1 of the event i.e. A & B is denoted by $P(A \cup B)$ is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- For mutually exclusive events

$$P(A \cup B) = P(A) + P(B) \quad (\because P(A \cap B) = 0)$$

1. A card is drawn at random from a pack of 52 cards. What is prob. that it is either a spade or an ace?

→ Data or given: A = getting a spade card

$P(A) = \frac{13}{52} = \frac{1}{4}$

B = getting an ace card

$P(B) = \frac{4}{52}$

To find: $P(A \text{ or } B) = ?$

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According to addition thm

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{\cancel{16}\ 8}{\cancel{52}\ 26} = \frac{4}{13}$

2. The managing committee of a welfare association form a subcommittee of 5 persons whose profiles are
1) male age 40
2) female age 38
3) male age 65
4) male age 43
5) female age 27

What is the prob. that the chair person selected would be either a female or over 30 ?

→ A = female candidate is selected

$P(A) = \frac{2}{5}$

B = candidate has age over 30.

$P(B) = \frac{4}{5}$

$P(A \cap B) = \frac{1}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{5} + \frac{2}{5} - \frac{1}{5}$

$= 5/5 = \boxed{1}$

5 10 15 20 25 30 , 7,14,21,28 , 3,6,9,12,15,18,21,24,27

3. A bag contains 30 balls numbered from 1-30
1 ball is drawn at random find the prob. the
no. on ball is :
1) a multiple 5 or 7
2) " ~~8~~ " 3 or 7.

→ 1) $A$ = multiple of 5

$P(A) = \frac{6}{30}$

$B$ = multiple of 7

$P(B) = \frac{4}{30}$

$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B) = \frac{6}{30} + \frac{4}{30} = \frac{10}{30} = \frac{1}{3} = P(C)$

2) $\cancel{A}C$ = multiple of ~~8~~ 3

$P(\cancel{A}C) = \frac{9}{30}$

$B$ = multiple of 7 $\quad P(\cancel{A}B \cap C) = \frac{1}{30}$

$P(B) = \frac{4}{30}$

$P(\cancel{A}B \cup C) = \frac{10}{30} + \frac{4-1}{30} = \frac{13}{30}$ 30.

4. 2 cards are drawn from a pack of 52 cards. Find prob that

i) they will be both red or both pitchers

A= Both are red

$P(A) = \frac{^{26}C_2}{^{52}C_2} = \frac{325}{1326}$

B= Both are pitchers

$P(B) = \frac{^{12}C_2}{^{52}C_2} = \frac{66}{1326}$

$P(A \cap B) = \frac{^{6}C_2}{^{52}C_2} = \frac{15}{1326}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{325}{1326} + \frac{66}{1326} - \frac{15}{1326}$

$= \frac{376}{1326}$

$= \frac{188}{663}$

$= \cancel{94}$

$\frac{B}{4}$ - Multiplication Thm.

- For any 2 independent events A & B prob. of occurence of A & B is given by.

$$P(A \,\&\, B) = P(A) \times P(B)$$
$$P(A \cap B)$$

1. A man want's to marry a girl having qualities white complextion
Probability of getting such a girl is 1 in 20.
handsome dowry
" prob. is getting is " 1 in 50.
westernized manners & ettiqutes
prob is 1 in 100
Find prob of his getting married to a girl having all 3 qualities.

→ A = girl has white complextion

$P(A) = \frac{1}{20}$

B = girl bring's handsome dowry

$P(B) = \frac{1}{50}$

C = girl has westernized manners & ettiqutes

$P(C) = \frac{1}{100}$

$P(A \cap B) = P(A) \times P(B) \times P(C)$

$= \frac{1}{20} \times \frac{1}{50} \times \frac{1}{100} = \frac{1}{100000} = 0.00001$

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2. The prob. that student A solves a problem is 2/5 & that of student B is 2/3. What is prob. that
1) problem is solved
2) problem is not solved

A = student A solves prob.
$P(A) = 2/5$
B = student B solves it
$P(B) = 2/3$

A & B are independent events

1) A = problem is solved

$\cancel{P(A\cap B) = \frac{2}{5} \times \frac{2}{3}}$

$$P(A \cap B) = \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$$

$$P(A \cup B) = \frac{2}{5} + \frac{2}{3} - \frac{4}{15}$$

$$= \frac{6+10-4}{15}$$

$$= \frac{12}{15} = \boxed{\frac{4}{5}}$$

2) $P(\text{prob not solved}) = 1 - \frac{4}{5} = \boxed{\frac{1}{5}}$

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- $P(E/F) = \frac{P(E\cap F)}{P(F)}$

- $P(E/F) = \frac{P(E\cap F)}{P(F)}, P(F) \neq 0$

- $\therefore P(E\cap F) = P(F) \times P\left(\frac{E}{F}\right)$

OR

$= P(F) \times P\left(\frac{F}{E}\right)$

- Conditional Probability

For the two events E & F, F has already occured. Then prob. of E when F has already occured is known as conditional prob. denoted by $P(E/F)$ & defined by

$$P(E/F) = \frac{P(E\cap F)}{P(F)}, P(F) \neq 0.$$

Similarly, $P(F/E) = \frac{P(F\cap E)}{P(E)}, P(E) \neq 0$

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1. If P(A) is 7/13, P(B) = 9/13, $P(A \cap B) = 4/13$. Find P(A/B)

→ $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{4}{13}}{\frac{9}{13}}$

$= \boxed{\frac{4}{9}}$

children

2. A family has 2 children what is prob. that both are boys given that atleast 1 of them is boy.

→ S = {(b,b), (b,g), (g,b), (g,g)}

A has already occured

A = atleast one boy = {(b,b), (b,g), g(b)}

$P(A) = \frac{3}{4}$

$P(A \cap B) = 1/4$

B = both are boys = {(b,b)}

$P(B) = \frac{1}{4}$

$P(B/A) = \frac{1/4}{3/4} = \boxed{\frac{1}{3}}$

$\boxed{P(B/A) = \frac{1}{3}}$

3. A bag contains 10 black & 5 white balls. 2 balls are drawn at random, one after other without replacement. What is prob. that both balls drawn are black?

→ ~~Let P(BB)~~ B = taking a black ball

$P(B) = \frac{10}{15} = \frac{2}{3}$

$P(B/B) = \frac{9}{14}$

$P(B \cap B) = P(B) \times P(B/B)$

$P(BB) = \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{14}_{7}}$

$= \frac{3}{7}$

4. 3 cards are drawn successively without replacement from a pack of 52 cards, what is prob. that 1st 2 cards are king & 3rd card drawn is an ace.

→ $P(K) = \frac{4}{52}$ = prob. of drawing a king card in 1st draw

$P(K/K) = \frac{3}{51}$ = prob. of drawing king card in 2nd draw when 1st card drawn is king

$\boxed{P(A/KK) = \frac{4}{50}}$

$P(KKA) = P(K) \times P(K/K) \times P(A/KK)$

$= \frac{\cancel{4}^{1}}{\cancel{52}_{13}} \times \frac{\cancel{3}^{1}}{\cancel{51}_{17}} \times \frac{\cancel{4}^{2}}{\cancel{50}_{25}} = \boxed{\frac{2}{5525}}$

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5. A dice is ~~thra~~ thrown twice & sum of 2 nos. is observed to be 6. What is prob. that no. 4 has appeared atleast once?

→ $S = \{(1,1) \ldots\ldots\ldots (6,6)\}$

$E$ = sum of two nos. is '6'

$= \{(1,5), \underline{(2,4)}, (3,3), \underline{(4,2)}, (5,1)\}$ $P(E) = \frac{5}{36}$

$F = \{(2,4), (4,2)\}$

$P(F/E) = \frac{2}{5}$

or

$E \cap F = \{(2,4), (4,2)\}$ $P(E \cap F) = 2/36$

$P(F/E) = \frac{P(E \cap F)}{P(E)}$

$= \frac{\frac{2}{36}}{\frac{5}{36}}$

$= \boxed{\frac{2}{5}}$

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$\forall$ for all, $\exists$ there exist, $\ni$ such that

- Partition of a Sample Space

$E_1, E_2 \ldots\ldots\ldots E_n$ $\quad S$

1. $E_i \cap E_j = \phi \quad i \neq j$
2. $E_1 \cup E_2 \ldots\ldots \cup E_n = S$
3. $P(E_i) > 0 \quad \forall i = 1 \ldots\ldots n$ Priory probabilities

$P(A/E_i)$ = Posteriory probabilities.

$$P(E_i/A) = \frac{P(E_i).P(A/E_i)}{\sum_{i=1}^{n} P(E_i).P(A/E_i)}$$

~~Bayes Theorem.~~

- If a set of events $E_1, E_2 \ldots\ldots E_n$ is said to be partition of sample space S if & only if

- $E_1, E_2 \ldots\ldots E_n$ are called hypotheses.
- $P(E_1), P(E_2) \ldots\ldots P(E_n)$ are known as priory probabilities of hypotheses $E_1, E_2 \ldots\ldots E_n$.
- The conditional probability ~~$P(E_1/A), P(E_2/A) \ldots\ldots P(E_n/A)$ is~~ $P(E_i/A)$ is called posteriory probability.

- Total Theorem of Probability

Let A be any event associated with S then

$$P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + \ldots\ldots + P(E_n)(A/E_n)$$

which is known as "theorem of total probability."

- Baye's Thm

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + \ldots P(E_n)P(A/E_n)}$$

1. Bag 1 contain 3 red & 4 black balls while another bag 2 contains 5 red & 6 black balls. 1 ball is drawn at random from 1 one the bag & it is found to be red. Find prob. that it was drawn from bag 2.

→ Bag I: R – 3, B – 4

Bag II: R – 5, B – 6

$E_1$ = bag 1 is selected

$E_2$ = " 2 "

A = a ball drawn is of red colour

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

$P(A/E_1) = \frac{3}{7}$, $P(A/E_2) = \frac{5}{11}$

According to Baye's thm

$$P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{5}{11}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{5}{11}}$$

$$= \frac{\frac{5}{22}}{\frac{3}{14} + \frac{5}{22}} = \frac{\frac{5}{22}}{\frac{66+70}{308}} = \frac{5}{22} \times \frac{308}{136} = \boxed{\frac{35}{68}}$$

Conditional prob. = reverse prob.

2. In a factory which manufacture's bolts, machines A, B, C 25%, 35%, 40% of bolts resp. of their outputs 5, 4 & 2% are defective bolts resp. A bolt is drawn at random & is found to be defective, what is prob. that it's manufactured by machine B.

→

| | A | B | C |
|---|---|---|---|
| | 25% | 35% | 40% |
| Defective | 5% | 4% | 2% |

Let $E_1$ = Bolt manufactured by A

$E_2$ = " " B

$E_3$ = " " C

$P(E_1) = \frac{25}{100} = \frac{1}{4}$, $P(E_2) = \frac{35}{100} = \frac{7}{20}$, $P(E_3) = \frac{40}{100} = \frac{2}{5}$

A = bolt is defective.

$P(A/E_1) = \frac{5}{100} = \frac{1}{20}$, $P(A/E_2) = \frac{4}{100} = \frac{2}{50} = \frac{1}{25}$, $P(A/E_3) = \frac{2}{100} = \frac{1}{50}$

According to Baye's thm.

$$P(E_2/A) = \frac{P(E_2).P(A/E_2)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + P(E_3).P(A/E_3)}$$

$$= \frac{\frac{7}{20} \times \frac{1}{25}}{\frac{1}{4} \times \frac{1}{20} + \frac{7}{20} \times \frac{1}{25} + \frac{2}{5} \times \frac{1}{50}}$$

$$= \frac{\frac{7}{500}}{\frac{1}{80} + \frac{7}{500} + \frac{2}{250}} = \frac{\frac{7}{500}}{\frac{25+28+16}{2000}} = \frac{7}{500} \times \frac{2000}{69} = \frac{28}{69}$$

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## Summary

Total thm

$P(A) = P(E_1).P(A/E_1) + P(E_2)P(A/E_2) + \ldots\ldots + P(E_n)P(A/E_n)$

Bayes thm

$$P(E_i/A) = \frac{P(E_i) \times P(A/E_i)}{P(E_1).P(A/E_1) + \ldots\ldots P(E_n)P(A/E_n)}$$

1. Given 3 identical boxes I, II, III, each containing 2 coins. In box I both coins are gold coins, in box II both are silver coins & in box III there is 1 gold & 1 silver coin. A person chooses a box at random & takes a coin, if coin is of gold what is prob. that the other coin in box is also of gold?

→

| I | II | III |
|---|---|---|
| O → G | O → S | O → S |
| O → G | O → S | O → G |

$E_1$ = selecting box 1

$E_2$ = " " box 2

$E_3$ = " " box 3

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

A = selecting a gold coin

$A/E_1$ = selecting gold coin from box 1

$P(A/E_1) = \frac{2}{2} = 1$

$A/E_2$ = selecting gold coin from box 2

$P(A/E_2) = 0$

$A/E_3$ = selecting gold coin from box 3

$P(A/E_3) = \frac{1}{2}$

$$P(E_1/A) = \frac{P(E_1) \times P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3} \times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times 0 + \frac{1}{3}\times\frac{1}{2}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}}$$

$$= \frac{\frac{1}{3}}{\frac{3}{6}}$$

$$= \frac{1}{\cancel{3}} \times \frac{\cancel{6}^{2}}{3} = \boxed{\frac{2}{3}}$$

2. A doctor is to visit a patient. From the past experience it is known that the probabilites that he will come by train, bus, scooter or by other means of transport are ~~resp.~~ $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ & $\frac{2}{5}$ resp. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ & $\frac{1}{12}$ if he comes by train, bus & scooter. But if he comes by other means of transport then he will not be late.

When he arrives he is late. What is the prob. that he comes by train?

doctor
→ $E_1$ = comes by train
$P(E_1) = \frac{3}{10}$

$E_2$ = doctor comes by train
$P(E_2) = \frac{1}{5}$

$E_3$ = doctor comes by scooter
$P(E_3) = \frac{1}{10}$

$E_4$ = doctor comes by other means of transport
$P(E_4) = \frac{2}{5}$

A = doctor is late when
$P(A/E_1)$ = getting late by coming by train
$= \frac{1}{4}$

$P(A/E_2) = \frac{1}{3}$

$P(A/E_3) = \frac{1}{12}$

$P(A/E_4) = 0$

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + P(E_4)P(A/E_4)}$$

$$= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{5} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0}$$

$$= \frac{\frac{3}{40}}{\frac{3}{40} + \frac{1}{15} + \frac{1}{120}}$$

$$= \frac{\frac{3}{40}}{\frac{9+8+1}{120}}$$

$$= \frac{\cancel{3}}{\cancel{40}} \times \frac{\overset{3}{\cancel{120}}}{186} = \boxed{\frac{3}{62}}$$

3. Of three persons the chances that a politician, a businessman or an academician would be appointed as VC of university are 0.5, 0.3 & 0.2 resp. Probabilities that research is promoted by these persons if they are appointed as VC are 0.3, 0.7 & 0.8 resp. 1) Determine the prob. that research is promoted 2) If research is promoted then what is prob. that VC is academician.

→

$E_1$ = Politician is appointed as VC

$P(E_1) = 0.5$

$E_2$ = Businessman is " "

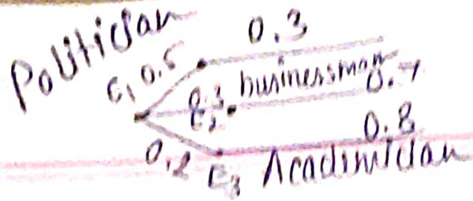
$P(E_2) = 0.3$

$E_3$ = Academician is " "

$P(E_3) = 0.2$

A = Research is promoted

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.7$, $P(A/E_3) = 0.8$




1) Total prob.

$= P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + P(E_3).P(A/E_3)$

$= 0.5 \times 0.3 + 0.3 \times 0.7 + 0.2 \times 0.8$

$= 0.15 + 0.21 + 0.16$

$P(A) = 0.52$

2) $P(E_3/A) = \dfrac{P(E_3).P(A/E_3)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + P(E_3)P(A/E_3)}$

$= \dfrac{0.2 \times 0.8}{0.5 \times 0.3 + 0.3 \times 0.7 + 0.2 \times 0.8}$

$= \dfrac{0.16}{0.15 + 0.21 + 0.16}$

$= \dfrac{0.16}{0.52}$

$\boxed{\dfrac{4}{13}} = 0.3.$

4. The probabilities of X, Y, Z becomes manager are $\frac{4}{9}, \frac{2}{9}$ & $\frac{1}{3}$ resp. The probabilities that the bonus scheme is introduced if X, Y, Z become manager are $\frac{3}{10}, \frac{1}{2}, \frac{4}{5}$ resp. What is prob. that

1) bonus scheme will be introduced by anyone person on becoming manager

2) bonus scheme has been introduced that manager appointed was X.

$E_1$ = prob. of X become's manager

$P(E_1) = \frac{4}{9}$

$E_2$ = Y become's manager

$P(E_2) = \frac{2}{9}$

$E_3$ = Z become's manager

$P(E_3) = \frac{1}{3}$

A = bonus scheme is introduced

$P(A/E_1) = \frac{3}{10}$

$P(A/E_2) = \frac{1}{2}$

$P(A/E_3) = \frac{4}{5}$

1) Bonus scheme is introduced by any person who become's manager

$P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + P(E_3).P(A/E_3)$

$= \frac{\cancel{4}2}{3\cancel{9}} \times \frac{\cancel{3}}{\cancel{10}5} + \frac{2}{9} \times \frac{1}{\cancel{2}} + \frac{1}{3} \times \frac{4}{5}$

$= \frac{2}{15} + \frac{1}{9} + \frac{4}{15}$

$= \frac{6+5+12}{45}$

$= \frac{23}{45}$

2) $P(E_1/A) = \dfrac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$

$$= \frac{\frac{4}{39} \times \frac{3}{105}}{\frac{24}{9} \times \frac{3}{105} + \frac{2}{9} \times \frac{1}{2} + \frac{1}{3} \times \frac{4}{5}}$$

$$= \frac{\frac{2}{15}}{\frac{2}{15} + \frac{1}{9} + \frac{4}{15}}$$

$$= \frac{\frac{2}{15}}{\frac{23}{45}}$$

$$= \frac{2}{\cancel{15}} \times \frac{\cancel{45}^{3}}{23} = \boxed{\frac{6}{23}}$$

22/2/23 # Random Variable & Probability Distribution

2 types of random variables

1. Discrete variable (finite)

e.g. children per family.
no. of flowers on plant
no. of car accident on road

2. Continuous variable (uncountably infinite value)

any real value.

e.g. height of student
weight of student.
temp. of room.

| $X$ | $x_1$ | $x_2$ | .... | $x_n$ |
|---|---|---|---|---|
| $P(X)$ | $P_1$ | $P_2$ | ..... | $P_n$ |

Table is known as Probability distribution

e.g.

- Tossing 3 coins (Discrete prob. distribution)

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$X$ = no. of heads

$X(HHH) = 3$

$X(HHT) = X(HTH) = X(THH) = 2$

$X(HTT) = X(THT) = X(TTH) = 1$

$X(TTT) = 0$

Random variable

| $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X=x)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

$\left.\begin{matrix} P(x_i) \geq 0 \\ \sum P(x_i) = 1 \end{matrix}\right\}$ condition

Imp. Q. Verify the following probability distribution.

1.

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | 0.4 | 0.3 | 0.2 | 0.3 |

→ Not a prob. distribution

2.

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{6}$ | $\frac{1}{4}$ |

→ $\Sigma P(x) = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{4} = \frac{12+8+4+6}{24} = \frac{30}{24} = \frac{15}{12} \neq 1$

Not a prob. distribution.

3.

| X | 0 | 1 | -2 | -3 |
|---|---|---|---|---|
| P(X) | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ |

→ X can have any +ve or -ve value but prob. should always be greater than 0 but here 1st is $-\frac{1}{2}$ so, not a prob. distribution

Q. A random variable has following distribution

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X=x)$ | a | 4a | 3a | 7a | 8a | 10a | 6a | 9a |

1. Find a
2. $P(X<3)$
3. $P(1 \leq x \leq 4)$
4. $P(X \geq 5)$

→ 1. $\sum P(X=x) = 1$

$\therefore a+4a+3a+7a+8a+10a+6a+9a = 1$

$\therefore 48a = 1$

$\therefore a = \boxed{\frac{1}{48}}$

2. $P(X<3)$

$= P(X=0)+P(X=1)+P(X=2)$

$= a+4a+3a$

$= 8a$

$= 8 \times \frac{1}{48} = \boxed{\frac{1}{6}}$

3. $P(1 \leq x \leq 4)$

$= P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 4a+3a+7a+8a$

$= 22a$

$= \frac{22}{48} = \boxed{\frac{11}{24}}$

4. $P(x \geq 5)$

$= P(x=5)+P(x=6)+P(x=7)$

$= 10a+6a+9a$

$= 25a$

$= \boxed{\frac{25}{48}}$

2. A random variable x has following probability distribution

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $P(X=x)$ | $k$ | $2k$ | $3k$ | $k^2$ | $k^2+k$ | $2k^2$ | $4k^2$ |

1. find k
2. $P(x<5)$
3. $P(0 \leq x \leq 5)$
4. $P(x>5)$

→ 1. $\sum P(x=x)=1$

$k+2k+3k+k^2+k^2+k+2k^2+4k^2=1$

$8k^2+7k=1$

$8k^2+7k-1=0$

$8k^2+8k-k-1=0$

$8k(k+1)-1(k+1)=0$

$(8k-1)(k+1)=0$

$8k=1$ or $k=-1$

$\boxed{k=\frac{1}{8}}$

2. $P(x<5)$

$= P(x=1)+P(x=2)+P(x=3)+P(x=4)$

$= k+2k+3k+k^2$

$= k^2 + 6k$

$= \frac{1}{64} + \frac{6}{8} = \frac{1+48}{64} = \boxed{\frac{49}{64}}$

3. $P(0 \le X \le 5)$

$= P(X=1) + P(X=2) + P(X=3) + P(X=3) + P(X=5)$

$= k + 2k + 3k + k^2 + k^2 + k$

$= 2k^2 + 7k$

$= \frac{2}{64} + \frac{7}{8} = \frac{1}{32} + \frac{7}{8} = \frac{28+1}{32} = \boxed{\frac{29}{32}}$

4. $P(X > 5)$

$= P(X=6) + P(X=7)$

$= 2k^2 + 4k^2$

$= 6k^2$

$= 6 \times \frac{1}{64} = \boxed{\frac{3}{32}}$

28/2/23

- Measures of central tendency
  - Mean
  - Median
  - Mode

- mean = $E(X) = \sum x p(x)$ - expected value of $x$.

Formulas
$$Variance(x) = E(x^2) - [E(x)]^2$$
$$E(x^2) = \sum x^2 p(x)$$
$$Std.\ deviation = \sqrt{Varience}$$

## Mathematical expectation

$$E(X) = x_1 p(x_1) + x_2 p(x_2) + \dots\dots + x_3 p(x_3)$$

1. A petrol pump propwriter sells ₹80000 worth of petrol on rainy days & an avg of ₹85,000 clear days. Statistics show that the prob is 0.76 for clear weather & 0.24 for rainy weather. Find the expected value for petrol sale.

| X | 80000 | 85000 |
|---|---|---|
| P(X) | 0.24 | 0.76 |

$$E(X) = x_1 p(x_1) + x_2 p(x_2)$$
$$= 80000 \times 0.24 + 85000 \times 0.76$$
$$= 19200 + 64600$$
$$\boxed{₹83800}$$

2. The probability that a man fishing at a particular place will catch 1, 2, 3, 4 fish are 0.4, 0.3, 0.2, & 0.1 resp. What is expected value of fish catch?

| $X$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X)$ | 0.4 | 0.3 | 0.2 | 0.1 |

$$E(X) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$$
$$= 1 \times 0.4 + 2 \times 0.3 + 0.2 \times 3 + 4 \times 0.1$$
$$= 0.4 + 0.6 + 0.6 + 0.4$$
$$= \boxed{2}$$

3. ~~A firm plans to bid ₹300 per kg~~ There are 3 alternative proposal before a businessman to start a new project. Proposal A: Profit of ₹5 lakh with prob. 0.6 or Loss of ₹ 80000 with prob 0.4

Proposal B: Profit of ₹10 lakh with prob. 0.4 or Loss of ₹ 2 lakh with prob. 0.6

Proposal C: Profit of ₹ 4.5 lakh with prob. 0.8 or Loss of ₹ 50000 with prob 0.2

If he wants to maximize the profit & minimize the loss which proposal should be preferred?

| ~~X~~ | ~~A~~ | ~~B~~ | ~~C~~ |
|---|---|---|---|
| ~~P(X)~~ | | | |

Expected value for A $= 500000 \times 0.6 - 80000 \times 0.4$
$$= 300000 - 32000$$
$$= 268000$$

Expected value for B = 1000000 × 0.4 − 200000 × 0.6
= 400000 − 120000
= 280000

Expected value for C = 4500000 × 0.8 − 50000 × 0.2
= 3600000 − 10000
= 3500000

4. A dealer of refrigerator estimates the probabilities of his selling refrigerators in a day are as follows:

| No. of ref. sold in a day | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| probability | 0.03 | 0.20 | 0.23 | 0.25 | 0.12 | 0.10 | 0.07 |

Find mean no. of ref. sold in a day.

→ Expected values

$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4) + x_5 p(x_5) + x_6 p(x_6)$

$= 0 \times 0.03 + 1 \times 0.20 + 2 \times 0.23 + 3 \times 0.25 + 4 \times 0.12 + 5 \times 0.10 + 6 \times 0.07$

$= 0.20 + 0.46 + 0.75 + 0.48 + 0.50 + 0.42$

$= 2.81 \simeq 3.$

5. A random variable x has following prob. distribution.

| x | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $p(x=x)$ | 1/36 | 3/36 | 5/36 | 7/36 | 9/36 | 11/36 |

Find
1. mean
2. variance
3. probabilities of $P(1<x<6)$

→ 1. Mean

$= x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4) + x_5 p(x_5) + x_6 p(x_6)$

$= 1 \times \frac{1}{36} + 2 \times \frac{3}{36} + 3 \times \frac{5}{36} + 4 \times \frac{7}{36} + 5 \times \frac{9}{36} + 6 \times \frac{11}{36}$

$= \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36}$

$= \boxed{\frac{161}{36}}$

2. Variance

$E(x^2) - [E(x)]^2$

$E(x^2) = x_1^2 p_1 + x_2^2 p_2 + x_3^2 p_3 + x_4^2 p_4 + x_5^2 p_5 + x_6^2 p_6$

$= 1 \times \frac{1}{36} + 4 \times \frac{3}{36} + 9 \times \frac{5}{36} + 16 \times \frac{7}{36} + 25 \times \frac{9}{36} + 36 \times \frac{11}{36}$

$= \frac{1+12+45+112+225+396}{36}$

$= \frac{791}{36}$

$\frac{791}{36} - \left(\frac{161}{36}\right)^2 = \frac{28476 - 25921}{1296} = \frac{2555}{1296} = \boxed{1.97}$

1.98

$\frac{24}{36}$

3. $P(1<x<6)$

$= \frac{3}{36} + \frac{5}{36} + \frac{7}{36} + \frac{9}{36} + \cancel{\frac{11}{36}}$

$= \frac{24}{36} = \boxed{\frac{2}{3}}$.