Tutorial : 1

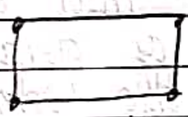
Ex : 1 Draw all simple graphs of one, two, three and four vertices.

One vertice

two vertice

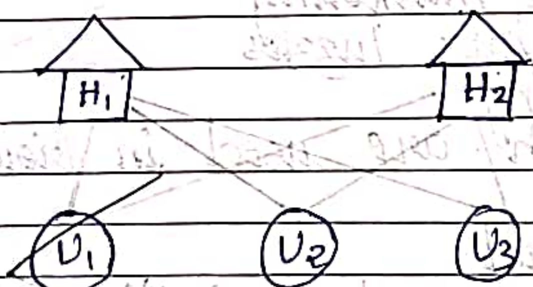
three vertices

four vertices

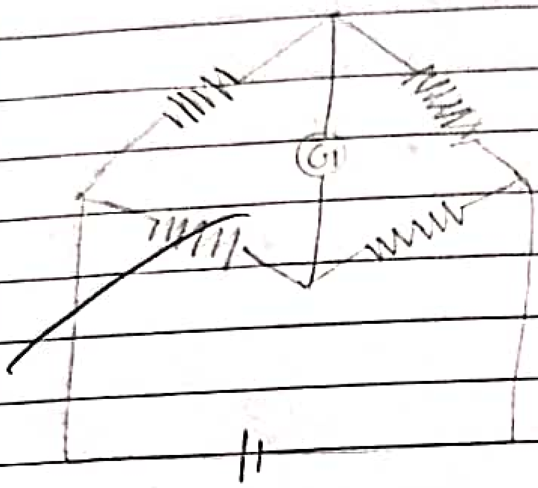


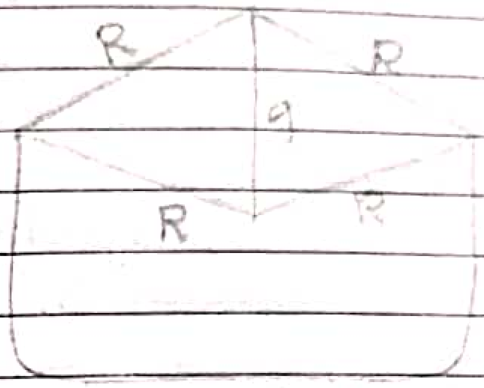
Ex 2 Draw graphs representing problems are

a) two houses & three utilities



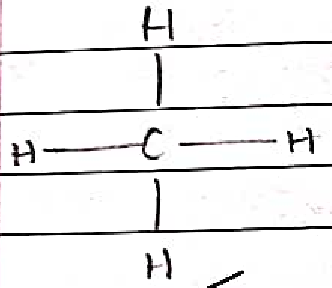
Ex 4 Draw the graph of wheatstone bridge circuit.

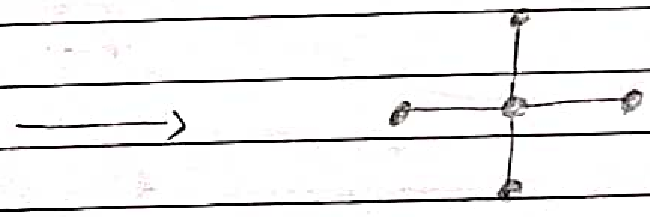




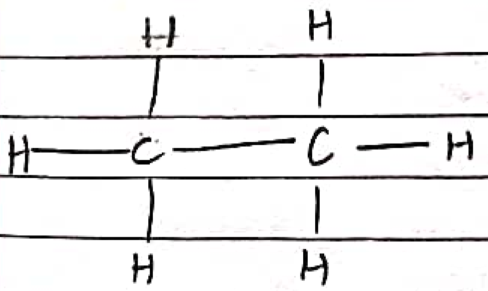
Ex 5 Draw graph of the following chemical compound

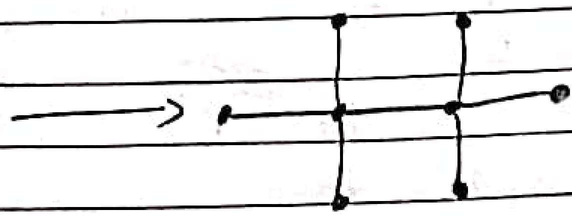
(i) CH_4





(ii) C_2H_6





case 3 :
Transitive

If 3 divides $(a-b)$ & 3 divides $(b-c)$
$\therefore (a,b) \in R \Rightarrow 3$ divides $a-b \Rightarrow a-b = 3d$

$(b,c) \in R \Rightarrow 3$ divides $b-c \Rightarrow b-c = 3H$

$(a-b)+(b-c) = 3(d+H)$

$a-c = 3(d+H)$, where $d+H \in Z$

3 divides $a-c$

Thus, $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

$\therefore R$ is transitive

3. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a,b) \mid b = a+1\}$ is reflexive, symmetric (or) transitive.

- $\{1, 2, 3, 4, 5, 6\}$

$R = \{(1,2), (2,3), (3,4), (4,5), (5,6), (6,7)\}$

2. Show that a relation R in given R = {(a,b) | divides a-b} is an equivalence Relation.

– Given :

R = {(a,b) : 3 divides a-b}

case I :

~~Reflexive~~
since if we consider (a,a)

$a - a = 0$

and

3 divides 0

$\therefore \frac{0}{3} = 3$

(a, 0) is reflexive

case 2 :

~~symmetric~~
If 3 divides a-b,
then 3 divides − (a−b)
(b−a)

Hence if (a,b) ∈ R, (b,a) ∈ R
∴ R is symmetric

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* Reflexive :

A relation is reflexive where if $a \in R$, the relation has (a, a)

But the above relation does not have $(1,1)(4,$

$\therefore$ Relative is not reflexive

* Symmetric :

A symmetric relation is where

$(a,b) \in R$
and $(b,a) \in R$

But here $(1,2) \in R$ but $(4,1) \notin R$

* Transitive :

A transitive relation is where

$(a,b) \in R$, $(b,c) \in R$
and $(a,c) \in R$

$\therefore$ The relation is not transitive.

4. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) \mid (a-b)$ is even$\}$ is an equivalance relation.

- $\{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3),$
$(2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (5,1), (5,2),$
$(5,3), (5,4), (5,5)\}$

outcomes which satisfy the relation:

$R = \{(1,1), (1,3), (1,5), (4,2), (2,4), (3,1), (3,3), (3,5)$
$(4,2), (4,4), (5,1), (5,3), (5,5)\}$

Reflexive:
$(1,1) \in R$
$(4,2) \in R$
∴ The relation is a reflexive relation.

Symmetric:
$(1,3) \in R$
& $(3,1) \in R$
∴ The relation is symmetric relation.

Transitive:
$(1,3) \in R$ also, $(1,5) \in R$ $(3,5) \in R$
∴ The relation is transitive relation

The above given relation is equivalence relation.

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$(T_1, T_2), (T_2, T_3) \in R$

then $(T_1, T_3) \in R$

$\therefore$ R is transitive

$\therefore$ The relation is reflexive, symmetric & transitive.

Hence, the given relation is an equivalence relation.

6. Check whether the relation $R = \{(x, y) \mid x \geq y\}$ defined on set of positive integers is a partial ordered relation (or) not.

- Antisymmetric:

xRy and yRx

$x = y$ $\quad x \geq y$ and $y \geq x$

Reflexive:

$x \geq y$

$\therefore$ It is reflexive

Transitive:

xRy $\;$ yRz is, xRz

$x \geq y$ $\;$ $y \geq x$

i.e. $x \geq y$

$\therefore$ xRz

∴ The relation is partial order relation.

7. Show that the relation 'divides' defined on N is a partial ordered relation.

- We have a divides a $a \in N$.

$\therefore (a,a) \in R$

Hence it is a reflexive relation.

- Let $a,b,c \in N$ such that a divides b.
It means b divides a if $a = b$

∴ It is anti symmetric relation.

Let $a,b,c \in N$ such that a divides b and b divides c.

If this happens, then a divides c.

$(a,b) \in R$
$(b,c) \in R$
$\therefore (a,c) \in R$

It is a transitive relation.

Hence, the relation is partial order relation.

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9. Draw Hasse diagram for following.

(i) $(D_{48}, |)$
D_{48} = divisors of 48
$|$ = relation divides

- $D_{48} = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12), (1,16),$
$(1,24), (1,48), (2,2), (2,4), (2,6), (2,8), (2,48),$
$(2,12), (2,16), (2,24), (3,3), (3,6), (3,48), (3,12),$
$(3,24), (4,4), (4,8), (4,12), (4,48), (4,16), (4,24),$
$(6,6), (6,12), (6,24), (6,48), (8,8), (8,16), (8,24), (12,12),$
$(12,48), (24,48), (12,24), (16,16), (24,24), (48,48)$

Reflexive:
$(a,a) \in R$

$\therefore (4,4) \in R$
Hence, relation is reflexive

Antisymmetric:

$(a,b) \in R$ then $(b,a) \notin R$
$(1,3) \in R$
but $(3,1) \notin R$
$\therefore$ Relation is antisymmetric.

Transitive :

If $(a,b) \in R$, $(b,c) \in R$

then $(a,c) \in R$

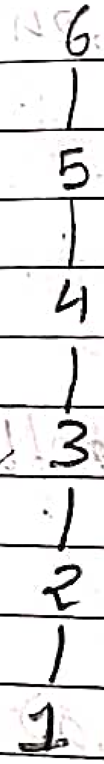
$(1,3) \in R$

$(3,6) \in R$

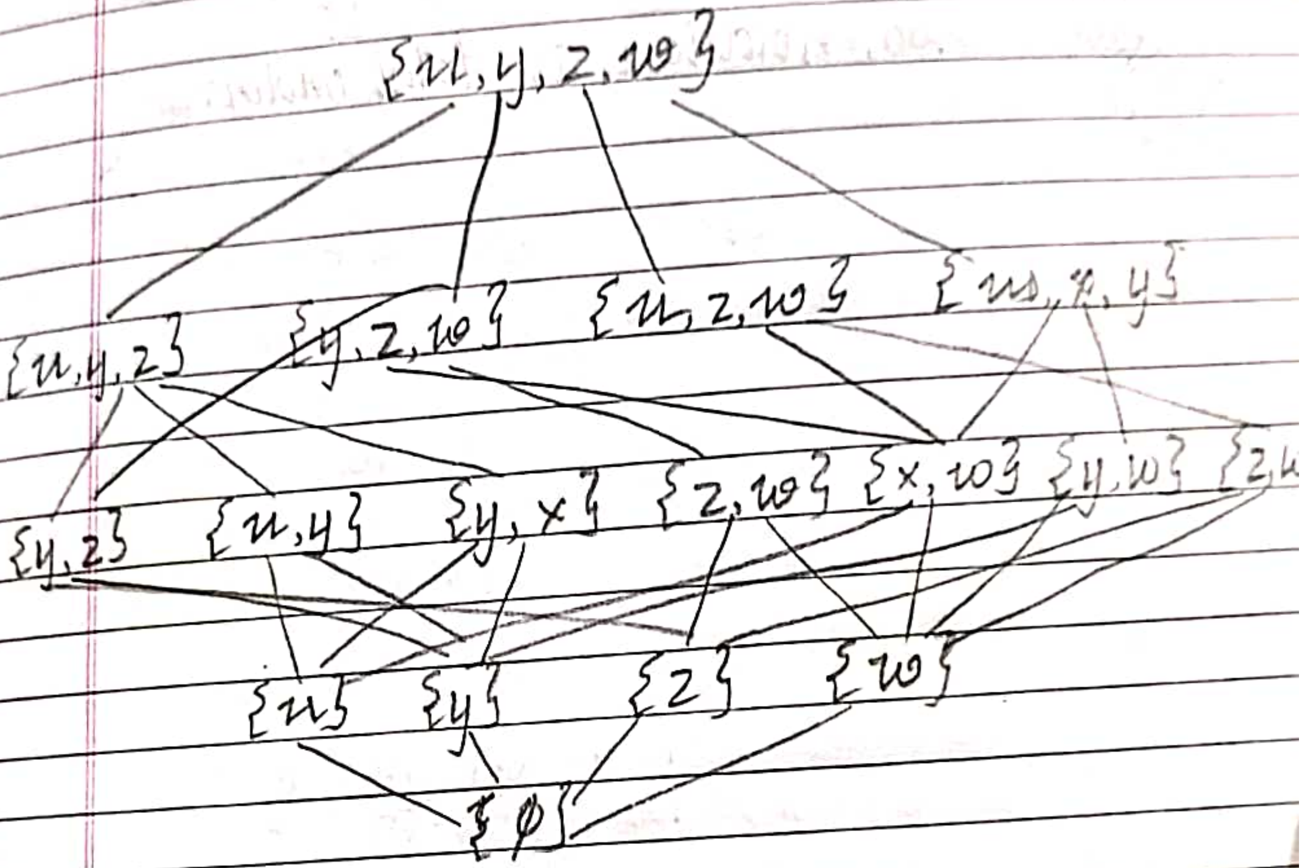
also, $(1,6) \in R$

$\therefore$ The relation is transitive.

(ii) $(\{1,2,3,4,5,6\}, \leq)$



(iii) $(P(A), \subseteq)$ where $A = \{x, y, z, w\}$



10. Give an example of a relation which is not an equivalence relation neither a partial order relation, but a total order relation.

Relation '<' defined on N

let $a \in N$, $a < a$ is not true

$\therefore$ '<' is not reflexive.

Tutorial: 5

1. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - 2a_{n-2}$ for $n \geq 2$. If $a_0 = 3$ & $a_1 = 5$. Find a_2, a_3, a_4, a_5, a_6

Here $a_0 = 3$, $a_1 = 5$

$a_n = a_{n-1} - 2a_{n-2}$ ——— (1)

put $n=2$ in (1)

$a_2 = a_1 - 2a_0$

$= 5 - 2(3)$

$\boxed{a_2 = -1}$

put $n=3$ in (1)

$a_3 = a_2 - 2a_1$

$= -1 - 2(5)$

$\boxed{a_3 = -11}$

put $n=4$ in (1)

$a_4 = a_3 - 2a_2$

$= -11 - 2(-1)$

$\boxed{a_4 = -9}$

put $n=5$ in (1)

$a_5 = a_4 - 2a_3$

$= -9 - 2(-11)$

$= 13$

put $n=6$ in (1)

$a_6 = a_5 - 2a_4$

$= 13 - 2(-9)$

$= 31$

2. Write recurrence relation for sequence
0, 2, 6, 12, 20, 30, 40

Given $a_0 = 0$ & $a_1 = 2$

$a_1 = a_0 + n(2)$
$= 0 + 1(2)$
$\boxed{a_1 = 2}$

$\therefore a_2 = a_1 + n(2)$
$= 2 + 2(2)$
$\boxed{a_2 = 6}$

$a_3 = a_2 + n(2)$
$= 6 + 3(2)$
$\boxed{= 12}$

$\therefore$ Recurrence Relation is
$a_n = a_{n-1} + 2n$

3. Find a recurrence relation & initial conditions for
1, 5, 17, 53, 161, 485, ...

Here $a_0 = 1$ & $a_1 = 5$

$a_1 = 3a_0 + 2$
$= 3(1) + 2$
$\boxed{a_1 = 5}$

$a_3 = 3a_2 + 2(3)$
$= 3(7) + 6$
$\boxed{a_3 = 27}$

$a_4 = 3a_3 + 2(4)$
$= 3(27) + 8$
$= 81 + 8$
$\boxed{a_4 = 89}$

$a_5 = 3a_4 + 2(5)$
$= 3(89) + 10$
$= 267 + 10$
$\boxed{a_5 = 277}$

$a_6 = 3a_5 + 2(6)$
$= 3(277) + 12$
$= 831 + 12$
$\boxed{a_6 = 843}$

$1, 7, 27, 89, 277, 843, \ldots$

5. Solve foll^n recurrence relation by iteration method.

(1) $T(P) = T(P-1) + 3, \; p \geq 1, \; T_0 = 2$

$\rightarrow T_p = T_{p-1} + 3$ ——— ①

$T_{p-1} = T_{p-2} + 3$ ——— put in ①

$\therefore T_p = (T_{p-2} + 3) + 3$
$T_p = T_{p-2} + 2(3)$ ——— ②

$T_{p-2} = T_{p-3} + 3$ ——— put in ②

$\therefore T_p = (T_{p-3} + 3) + 2(3)$
$T_p = T_{p-3} + 3(3)$ ——— ③

$\therefore$ We can conclude that
$T_p = T_{p-k} + k(3)$ ——— ④

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Now we have $T_0 = 2$

$\therefore$ let $p-k = 0$

$\therefore p = k$

put $k=p$ in (*)

$\therefore T_p = T_{p-p} + P(3)$

$T_p = T_0 + P(3)$

$T_p = 2 + 3P$

(2) $T_n = 3T_{n-1} + 1$, $T_1 = 1$

$T_n = 3T_{n-1} + 1$ ——— (1)

$T_{n-1} = 3T_{n-2} + 1$ ——— put in (1)

$\therefore T_n = 3[3T_{n-2} + 1] + 1$

$= 3^2 T_{n-2} + [3(1) + 1]$ ——— (2)

$T_{n-2} = 3T_{n-3} + 1$ ——— put in (2)

$\therefore T_n = 3^2[3T_{n-3} + 1] + [3(1) + 1]$

$T_n = 3^3 T_{n-3} + [3^2(1) + 3(1) + 1]$

$\therefore$ We can conclude

$T_n = 3^k T_{n-k} + [3^{k-1}(1) + 3^{k-2}(1) + \dots 1]$ ——— (A)

$$a_n = a \cdot 2^n + b \cdot 5^n$$

$a_0 = 2$

$\therefore \quad a_0 = a \cdot 2^0 + b \cdot 5^0$

$2 = a + b$ —— ①

$a_1 = 3$

$\therefore \quad a_1 = a \cdot 2^1 + b \cdot 5^1$

$2a + 5b = 3$ —— ②

Multiply eq. ① by ② & subtract from ②

$\cancel{2a} + 5b = 3$

$\cancel{2a} + 2b = 4$

$- \quad -$

$3b = -1$

$b = -\frac{1}{3}$ — put in ①

$a + \left(-\frac{1}{3}\right) = 2$

$a = 2 + \frac{1}{3}$

$a = \frac{7}{3}$

$\therefore$ $\boxed{\text{char. Eq}^n \quad a_n = \frac{7}{3}(2^n) - \frac{1}{3}(5^n)}$

(3) $a_n = 3a_{n-1} + 4a_{n-2}$ $a_0 = 2,\ a_1 = 3$

$\longrightarrow a_n - 3a_{n-1} - 4a_{n-2} = 0$

$\therefore r^2 - 3r - 4 = 0$

$r^2 - 4r + r - 4 = 0$

$r(r-4) + 1(r-4) = 0$

$\therefore r = -1, 4$

Roots are ~~equal~~ real & distinct

$\therefore$ characteristic eq^n

$a_n = a.r_1^n + b.r_2^n$

$a_n = a.(-1)^n + b.4^n$ ——— (*)

given $a_0 = 2$

$\therefore a_0 = a.(-1)^0 + b.4^0$

$2 = a + b$ ——— (1)

given $a_1 = 3$

$a_1 = a.(-1)^1 + b.4^1$

$3 = -a + 4b$ ——— (2)

Add (1) & (2)

$$\begin{array}{r} \cancel{a} + b = 2 \\ -\cancel{a} + 4b = 3 \\ \hline 5b = 5 \end{array}$$

$b = 1$ ——— put in (1)

$\therefore a = 1$

$\therefore$ from (*)

$\boxed{a_n = 1.(-1)^n + (1.(4)^n)}$

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$\therefore r = 2$

$$\begin{array}{c|cccc} 2 & 1 & 1 & -4 & -4 \\ & 0 & 2 & 6 & 4 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$\therefore r^2+3r+2 = 0$

$r^2+2r+r+2=0$

$r(r+2)+1(r+2)=0$

$r = -1, -2$

$\therefore$ characteristic eq^n

$a_n = a.r_1^n + b.r_2^n + c.r_3^n$

$a_n = a.2^n + b.(-1)^n + c(-2)^n$ —— (*)

given $a_0 = 8$

$\therefore a_0 = a.2^0 + b.(-1)^0 + c(-2)^0$

$8 = a+b+c$ —— (1)

$a_1 = 6 \therefore a_1 = a.(2)^1 + b.(-1)^1 + c(-2)^1$

$6 = 2a-b-2c$ —— (2)

$a_2 = 26$

$\therefore a_2 = a.(2)^2 + b(-1)^2 + c(-2)^2$

$26 = 4a+b+4c$ —— (3)

Add (1) & (2)

$$\begin{array}{r} a+\cancel{b}+c = 8 \\ 2a-\cancel{b}-2c = 6 \\ \hline 3a-c = 14 \end{array}$$ —— (4)

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$a_1 = (b_1 + (1) b_2) 3^1$ ——— (2)

$2 = b_1 + b_2$

$\therefore \boxed{b_2 = 1}$

from (*)

$a_n = (b_1 + n.b_2) 3^n$

$\boxed{a_n = (1 + n.(1)) 3^n}$

(9) $a_n - 5a_{n-1} + 6a_{n-2} = 1$ ——— (*)

→ gen^n sol^n $a_n = a_n^{(h)} + a_n^{(p)}$

for Homogeneous sol^n: $a_n^{(h)}$

characteristic eq^n $r^2 - 5r + 6 = 0$

$\therefore r = 2, 3$

Roots are real & distinct

$\therefore a_n = a.r_1^n + b.r_2^n$

$a_n = a.2^n + b.3^n$

for a_n^p, ——— (1)

$a_n - 5a_{n-1} + 6a_{n-2} = 1$

$f(n) = 1 = A$

$\left.\begin{array}{l} \text{let } a_n = A \\ a_{n-1} = A \\ a_{n-2} = A \end{array}\right\}$ put in (*)

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given $a_1 = -\frac{1}{2}$

$\therefore a_1 = a.3^1 + b.(-5)^1$

$3a - 5b = \frac{-1}{2}$ —— ②

Multiply ① × 5 & add to ②

$5a + \cancel{5b} = 5$

$3a - \cancel{5b} = \frac{-1}{2}$

$8a = \frac{9}{2}$

$a = \frac{9}{16}$ —— put in ①

$\frac{9}{16} + b = 1 \quad \therefore \quad b = 1 - \frac{9}{16}$

$= \frac{7}{16}$

$\therefore$ from * $a_n = \frac{9}{16} . (3)^n + \frac{7}{16} (-5)^n$

Now for $a_n^{(P)}$

$f(n) = 6n + 10$

$$\left.\begin{aligned} \text{put } a_n &= A_0 + A_1 n \\ a_{n+1} &= A_0 + A_1 (n+1) \\ a_{n+2} &= A_0 + A_1 (n+2) \end{aligned}\right\} \text{put in eq. an}$$

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Roots are real & distinct

$a_n = a.r_1^n + b.r_2^n$

$= a.(0)^n + b.2^n$

$a_n = b.2^n$

given $a_1 = 2$

$\therefore a_1 = b.2^1$

$a_1 = 2.b$

$1 = 2b$

$b = \frac{1}{2}$

$\therefore a_n = a(0)^n + \frac{1}{2}(2)^n$

$a_n^{(p)}, f(n) = 6n$

$\left.\begin{array}{l} \therefore a_n = A_0 + A_1 n \\ a_{n+1} = A_0 + A_1(n-1) \end{array}\right\}$ ~~put in a_n~~

$\therefore a_n^{(p)}, f(n) = 6n$

$\therefore a_n = A_0 + A_1 n$

$a_{(n-1)} = A_0 + A_1(n-1)$

$\therefore a_n = A_0 + A_1 n - 2[A_0 + A_1(n-1)]$

$= A_0 + A_1 n - 2A_0 - 2A_1 n + 2A_1$

$= (-A_0 + 2A_1) + (-A_1)n$

$= 0 + 6n$

$a_n - A_0 + A_1(n+2) + 2[A_0 + A_1(n+1)] - 15[A_0 + A_1 n]$

$= A_0 + A_1 n + 2A_1 + 2A_0 + 2A_1 n + 2A_1 - 15A_0 - 15A_1 n$

$= (-12A_0 + 4A_1) + (-12A_1)n$

$= A \quad = 10 + 6n$

Comparing coefficients

$-12A_1 = 6$

$\boxed{A_1 = \frac{-1}{2}}$

$-12A_0 + 4A_1 = 10$

$-6A_0 + 2A_1 = 5$

$-6A_0 + 2\left(-\frac{1}{2}\right) = 5$

$-6A_0 - 1 = 5$

$-6A_0 = 6$

$\boxed{A_0 = -1}$

$\therefore A_0 = -1 \quad A_1 = \frac{-1}{2}$

$a_n^{(p)} = -1 - \frac{1}{2}n$

Req. solⁿ $a_n = \frac{9}{16}(3)^n + \frac{7}{16}(-5)^n + \left(-1 - \frac{1}{2}n\right)$

(12) $a_n - 2a_{n-1} = 6n, \quad a_1 = 2$

General solⁿ: $a_n = a_n^{(h)} + a_n^{(p)}$

for $a_n^{(h)}$: $a_n - 2a_{n-1} = 0$

$r^2 - 2r = 0$

$r(r-2) = 0$

$$A - 5A + 6A = 1$$
$$2A = 1$$
$$A = 1/2$$

$\therefore$ from (*)

$$\boxed{a_n = a.2^n + b.3^n + \frac{1}{2}}$$

(10) $a_{n+2} + 2a_{n+1} - 15a_n = 6n + 10,\ a_0 = 1\ a_1 = \frac{-1}{2}$

$\rightarrow$ General sol^n: $a_n = a_n^{(h)} + a_n^{(p)}$

for $a_n^{(h)}$: $a_{n+2} + 2a_{n+1} - 15a_n$

$\therefore$ characteristic eq^n $r^2 + 2r - 15 = 0$

$$r^2 + 5r - 3r - 15 = 0$$
$$r(r+5) - 3(r+5) = 0$$
$$r = 3, -5$$

Roots are real & distinct

$\therefore a_n = a_n . r_1^n + b . r_2^n$

$\therefore a_n = a.3^n + b.(-5)^n$ —— *

Given $a_0 = 1$

$\therefore a_0 = a.3^0 + b.(-5)^0$

$1 = a + b$ —— (1)

Add (2) & (3)

$$2a - \cancel{b} - 2c = 6$$
$$4a + \cancel{b} + 4c = 26$$
$$6a + 2c = 32$$
$$3a + c = 16 \quad \text{——— (5)}$$

Add (4) & (5)

$$3a - \cancel{c} = 14$$
$$3a + \cancel{c} = 16$$
$$6a = 30$$
$$a = 5 \quad \text{——— put in (4)}$$
$$\cancel{1}5 - c = \cancel{1}4$$
$$c = 1 \quad \text{——— put int (1)}$$

$$0 + 5 + 1 = \cancel{8}$$
$$b = 2$$

from (4)

$$\boxed{a_n = 5 \cdot 2^n + 2 \cdot (-1)^n + 1 \cdot (-2)^n}$$

(8) $a_n = 6a_{n-1} - 9a_{n-2}$ $a_0 = 1$, $a_1 = 6$

→ $a_n - 6a_{n-1} + 9a_{n-2}$

$$r^2 - 6r + 9 = 0$$
$$\therefore r = 3, 3$$

Roots are real & equal

$$\therefore a_n = (b_1 + n \cdot b_2) 3^n \quad \text{——— (0)}$$

given $a_0 = 1$

$$\therefore a_0 = (b_1 + (0) b_2) 3^0$$
$$1 = b_1 \quad \text{——— (1)}$$

from ①

$$\boxed{a_n = \frac{19}{7}(-2)^n + \frac{9}{7}(5)^n}$$

(5) $a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$

$\longrightarrow r^3 - 8r^2 + 21r - 18 = 0$

for $r = 2$ it satisfies solⁿ

$\therefore a = 2$

2	1	-8	21	-18
	0	2	-12	18
	1	-6	9	0

$\therefore r^2 - 6r + 9 = 0$

$\therefore r = 3, 3$

$\therefore r = 2, 3, 3$

$\therefore$ characteristic eqⁿ

$a_n = a \cdot r_1^n + (b_1 + nb_2)3^n$

$$\boxed{a_n = a \cdot 2^n + (b_1 + n \cdot b_2)3^n}$$

(6) $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$, $a_0 = 8$, $a_1 = 6$, $a_2 = 26$

$a_n + a_{n-1} - 4a_{n-2} - 4a_{n-3} = 0$

$r^3 + r^2 - 4r - 4 = 0$

2 satisfies give eqⁿ

(4) $a_n - 3a_{n-1} + 10a_{n-2}$. $a_0 = 4$, $a_1 = 1$

$\rightarrow a_n - 3a_{n-1} \cancel{+} - 10a_{n-2} = 0$

$\therefore r^2 - 3r \cancel{+} - 10 = 0$

$r^2 - 5r + 2r - 10 = 0$

$r(r-5) + 2(r-5) = 0$

$r = -2, 5$

Roots are real & distinct $\therefore$ characteristic eq^n

$a_n = a \cdot r_1^n + b \cdot r_2^n$

$a_n = a \cdot (-2)^n + b \cdot (5)^n$ —— (*)

given $a_0 = 4$

$\therefore a_0 = a \cdot (-2)^0 + b \cdot (5)^0$

$4^0 = a + b$ —— (1)

given $a_1 = 1$

$\therefore a_1 = a \cdot (-2)^1 + b \cdot (5)^1$

$1' = -2a + 5b$ —— (2)

Multiply (1) × 2 & add to (2)

$$\begin{aligned} \cancel{2a} + 2b &= 8 \\ -\cancel{2a} + 5b &= 1 \\ \hline 7b &= 9 \end{aligned}$$

$b = \frac{9}{7}$

$\therefore$ from (1) $4 = a + \frac{9}{7}$

$a = 4 - \frac{9}{7}$

$a = \frac{19}{7}$

(2) $a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1$; $a_1 = 4$

$\rightarrow a_n - 6a_{n-1} + 9a_{n-2} = 0$

$r^2 - 6r + 9 = 0$

$(r-3)(r-3) = 0$

$\therefore r = 3, 3$

Roots are equal & real

$\therefore$ characteristic equation

$a_n = (b_1 + nb_2)3^n$ ——— (A)

$a_0 = 1$

$a_0 = (b_1 + (0)b_2)3^0$

$1 = b_1$

$a_1 = 4$

$\therefore a_1 = (b_1 + b_2)3^1$

$\frac{4}{3} = b_1 + b_2$

put $b_1 = 1$

$\therefore \frac{4}{3} = 1 + b_2$

$b_2 = \frac{4}{3} - 1$

$b_2 = \frac{1}{3}$

put in *

$$\boxed{a_n = \left(1 + \frac{4}{3} \cdot n\right) 3^n}$$

$a_{n-2} = 3a_{n-3} + 2$ —— put in (2)

$a_n = 3^2[3a_{n-3} + 2] + [3(2) + 2]$

$a_n = 3^3 a_{n-3} + 3^2(2) + 3(2) + 2$

$\therefore$ We can conclude

$a_n = 3^k a_{n-k} + [3^{k-1} + 3^{k-2} + \ldots\ldots + 1]2$ —— (A)

We have $a_0 = 1$

$\therefore$ let, $n-k = 0$

$\therefore n = k$ —— put in (A)

$a_n = 3^n a_0 + [3^{n-1} + 3^{n-2} + \ldots\ldots + 1]2$

$a_n = 3^n + [3^{n-1} + 3^{n-2} + \ldots\ldots + 1]2$

$= 3^n + \left(\frac{3^n - 1}{\cancel{3-1}}\right)\cancel{2}$

$= 3^n + 3^n - 1$

$= 2(3^n) - 1$

6. Solve by characteristic Root Method.

(1) $a_n = 7a_{n-1} - 10a_{n-2}$ with $a_0 = 2$, $a_1 = 3$

$\rightarrow a_n - 7a_{n-1} + 10a_{n-2}$

$r^2 - 7r + 10 = 0$

$r^2 - 5r - 2r + 10 = 0$

$(r-5)(r-2) = 0$

$\therefore r = 2, 5$

Roots are real & distinct

$\therefore$ characteristic equation -

$a_n = ar_1^n + br_2^n$

Here we have $T_1 = 1$

$\therefore n-k=1$

$k = n-1$ —— put in (A)

$T_n = 3^{n-1} T_{n-(n-1)} + [3^{n-2} + 3^{n-3} + \ldots\ldots 1]$

$= 3^{n-1} T_1 + [3^{n-2} + 3^{n-3} + \ldots\ldots 1]$

$T_n = 3^{n-1} + 3^{n-2} + 3^{n-3} + \ldots\ldots + 1$

Here $r = \frac{3^{n-2}}{3^{n-1}} = 3^{n-2-n+1} = 3^{-1}$

(3) $a_n = a_{n-1} + n \quad , \quad a_0 = 4$

$a_n = a_{n-1} + n$ —— (1)

$a_{n-1} = a_{n-2} + (n-1)$ —— put in (1)

$a_n = [a_{n-2} + (n-1)] + n$

$a_n = a_{n-2} - 1 + 2(n)$ —— (2)

$a_{n-2} = a_{n-3} + (n-2)$ —— put in (2)

$\therefore a_n = [a_{n-3} + (n-2)] - 1 + 2(n)$

$a_n = a_{n-3} + 3(n) - (2+1)$ —— (3)

$a_{n-3} = a_{n-4} + (n-3)$ —— put in (3)

$a_n = [a_{n-4} + (n-3)] + 3(n) - (2+1)$

$= a_{n-4} + 4(n) - (3+2+1)$

(4) $a_n = 3a_{n-1} + 2 \quad , \quad a_0 = 1$

$a_n = 3a_{n-1} + 2$ —— (1)

put, $n = n-1$

$\therefore a_{n-1} = 3a_{n-2} + 2$ —— put in (1)

$\therefore a_n = 3[3a_{n-2} + 2] + 2$

$a_n = 3^2 a_{n-2} + 3(2) + 2$ —— (2)

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$a_2 = 3a_1 + 2$
$= 3(5) + 2$
$\boxed{a_2 = 17}$

$\therefore$ We can conclude that
$a_n = 3a_{n-1} + 2$

4. Find first five terms of each of the following recurrence relation.

(1) $a_n = n(a_{n-1})^2, \quad a_0 = 1, \quad n \geq 1$

$\rightarrow$ put $n = 1$
$a_n = n(a_{n-1})^2$
$a_1 = 1(a_0)^2$
$\boxed{a_1 = 1}$

$a_2 = 2(a_1)^2$
$= 2(1)^2$
$\boxed{a_2 = 2}$

$a_3 = 3(a_2)^2$
$= 3(2)^2$
$\boxed{a_3 = 12}$

$a_4 = 4(a_3)^2$
$= 4(12)^2$
$= 4(144)$
$\boxed{a_4 = 576}$

$a_5 = 5(a_4)^2$
$= 5(576)^2$
$\boxed{a_5 = 1,658,880}$

(2) $a_k = 3a_{k-1} + 2k, \quad k \geq 2, \quad a_1 = 1$

put $k = 2$
$\therefore a_2 = 3a_1 + 2(2)$
$= 3(1) + 4$
$\boxed{a_2 = 7}$

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As '$<$' is not reflexive; '$<$' is neither equivalence nor a partial order relation.

As $\forall$ $a, b \in N$, we have either
$a < b$ (or) $b < a$

So, relation is total order.

Vs Vidoshw.
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8. The 'subset' relation defined on a power set of a non empty set A is a partial order relation. Show that

- Reflexive:

$A \subseteq A$

$\therefore (A, A) \in R$

Hence, the relation is reflexive.

Antisymmetric:

If $B \subseteq C$ & $C \subseteq B$
then $B = C$
But $B \neq C$

Transitive:

If $B \subseteq C$, $C \subseteq D$

then $B \subseteq B$

$(B, C) \in R$
$(C, D) \in R$

$\therefore (B, D) \in R$

The relation is transitive
Hence, the given relation is partially ordered relation.

5. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) \mid T_1$ is similar to $T_2\}$ is an equivalence relation.

- $R = \{(T_1, T_2) : T_1$ is similar to $T_2\}$

Reflexive:

Every triangle is similar to itself

$\therefore (T, T) \in R$

$\therefore$ Reflexive

Symmetric:

If we consider two $\triangle$'s T_1 & T_2

$T_1 \sim T_2$

$\therefore T_2 \sim T_1$

So, $(T_1, T_2) \in R$, then $(T_2, T_1) \in R$

$\therefore$ Relation is symmetric.

Transitive:

If T_1 is similar to T_2 and T_2 is similar to T_3

$\therefore T_1 \sim T_3$

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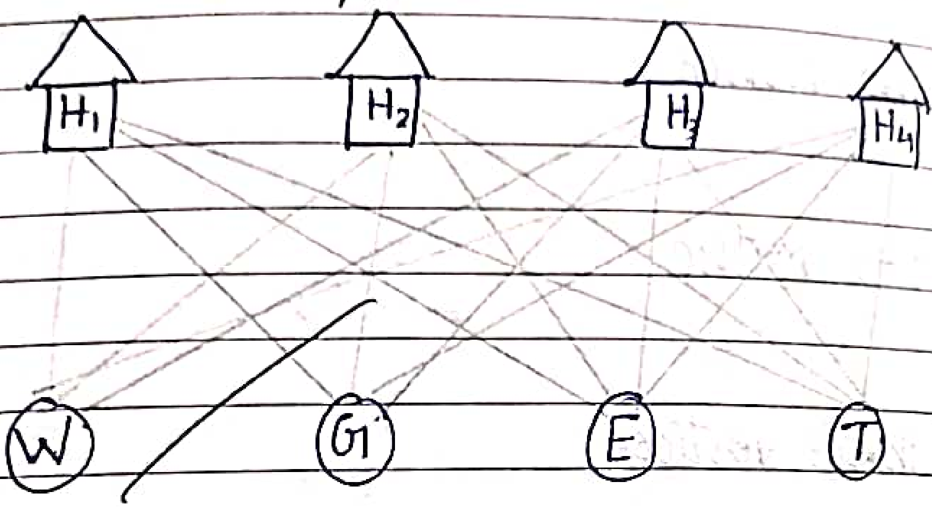
1. Define:
(i) An equivalence relation
(ii) A Partial ordered relation
(iii) A total ordered relation

(i) An Equivalence relation:
A relation which is both reflexive and symmetric as well as being transitive is known as equivalence relation.
Eg - "Equal to" relation.

(ii) Partial ordered relation:
A Partial ordered relation is a homogeneous relation that is Transitive and anti-symmetric. A relation R on set A is called a partial order relation if it satisfies the following three properties:
1) Reflexive
2) Antisymmetric
3) Transitive

(iii) A Total ordered relation:
A Total ordered relation is a partial order in which every element of the set is comparable with every other element of the set. All total orders are partial orders but not all partial orders are total order.

b) Four house & Four utilities say water, gas, electricity & telephone



Ex 3 Name 10 situations (game, activities life problems etc) that can be represented by means of graphs explain. What the vertices and the edges.

1) Airline scheduling
2) Direction in a map
3) Social media marketing
4) Solving Sudoku's Puzzles
5) City Planning
6) linear graph are used in biology and chemistry.
7) Traffic Control
8) Transportation and navigation
9) mobile phone network
10) webpage searchings