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Unit IV (Application of PD)
=> Tangent Plane & Normal line:
    Equation of tangent plane at P(x0, y0, z0)
      to the surface f(x,y,z)=0 is
    cn-re) fre (re, 40, 20) + (4-40) fly (20, 40 20)
     + (z-20) fz (xo, yo, 20) = 0
    where fx (no, yo, zo) = of (no, yo, zo)
     fy (20, 40, 20) = of (110, 40, 20)
     8 fy (no, yo, 20) = 2f
8 z (no, yo, 20)
  Normal line
   7-70 = 4-40. = Z-20
+x(Mo, 50, 20) +x(Mo, 50, 70) +x(Mo, 50, 70)
   find the equation of tangent plane and mormal line to the surface myz = c at (1, 2,3)
   myz = 6 at (1,2,3)
  f(21, y, z) = xyz-6
   fx = 42 => fx (1,2,3) = 6
   fy = xz => fy(1,2,3) = 3
   fz = xy => fz (1,2,3) = 2
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3)
$$\pi^2 + y^2 + z^2 = 3$$
 at the point (1,1,1)
 $\pi + y + \chi = \chi = 0$

$$f(n) = 2n \Rightarrow f(1,1,1) = 2$$

$$f(z) = 2z \Rightarrow f(1,1,1) = 2$$

$$2n-2+2y-2+2z-2=0$$

 $2n+2y+2z-6=0$
 $n+y+z-3=0$

$$\frac{2^{-1}}{2^{-1}} = \frac{y-1}{2^{-1}} = \frac{z-1}{2^{-1}}$$

Clissifiate (

$$(x-1)(6) + (y-2)(3) + (z-3)(2) = 0$$

 $6x - 6 + 3y - 6 + 2z - 6 = 0$
=7 $6x + 3y + 2z - 18 = 0$

i

$$\frac{3(-1)}{f_{11}(1,2,3)} - \frac{9-2}{f_{21}(1,2,3)} - \frac{2-3}{f_{21}(1,2,3)}$$

$$= x-1 = y-2 = z-3$$
6 3 2

$$2x^2+y^2+z-9=0$$
 at $(1,2,4)$

$$f_{N} = 2\pi i + y + y + y = 3 \Rightarrow f(1,2,4) = 2 + 4 + y = 2$$

$$\frac{1}{4} = \frac{1}{12} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{1$$

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-1}{2}$$

4)
$$2n^2 + 4^2 + 2z = 3$$
 at $(2,1,-3)$

$$f(x,y,z) = 2x^2 + y^2 + 2z - 3$$

$$f(n) = 4\pi = f(2,1,-3) = 8$$

 $f(y) = 2y = f(2,1,-3) = 2$
 $f(z) = 2 = f(2,1,-3) = 2$

$$+(2) = 2 = +(2,1,-3) = 2$$

Normal line

$$n-2 = y-1 = z+3$$

$$5) \frac{n^2}{4} + y^2 + z^2 = 3 \quad a + (-2, 1, -3)$$

$$\frac{x+2}{-1} = \frac{y-1}{2} = \frac{z+3}{-2/3}$$

Local Extreme Values

(Maximum & minimum Values)

Norking Rule to determine extreme values

of a function f(n,y).

1. Solve $\partial t = 0$ & $\partial s = 0$ and $\partial s + \partial s$ the stationary point (α, b) 2. Obtain $r = \partial^2 f$, $s = \partial^2 f$, $s = \partial^2 f$ ∂n^2 , ∂n^2 3. i) If $n + -s^2 > 0$ & n < 0 (or t < 0) then

iii) If 91-52 LO then function has neither minimum non maximum value.

f(x, y) has maxlmum value at (a, b)

ii) It 91+-52>0 & x>0 (on +>0) then

+(my) has minimum value at (a,b)

which is f(a,b)

which is f (a,b).

- iv It nt -52 =0 then no conclusion can be made about the function and furthere investigation is required about the given function.
 - i) Discus maxima and minima of the punction n2+y2+on+12=0

f(x,y) = x2+y2+6x+12.

 $\frac{\partial f}{\partial n} = 2n + 6 = 0$

=> 2 n =-6

 $\frac{\partial f}{\partial y} = 2y = 0 \implies y = 0$

Stationary point (-3,0)

- 91 = 22+ = 2 sat (-3,0) => 9= 2
- S= 24 =0 Sat(-3,0) = 2
- $t = \frac{\partial^2 f}{\partial y^2} = Q$: t at (-3, 0) = 2

H+-52 = (2)(2) -0 = 4)0

91 at (-3,0) = 2>0

f(x,y) has min value

8 tmin (-3,0) = (-3)2+0+6(-3) +12

= 9-18+12=0

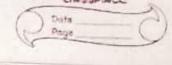
$$\begin{cases}
f(x,y) = 3n^2 - y^2 + x^3 \\
\frac{\partial f}{\partial n} = 6n + 3x^2
\end{cases}$$

$$\frac{\partial f}{\partial n} = 0 = 0$$
 $\frac{\partial n}{\partial n} + \frac{\partial n}{\partial n} = 0$
=) $\frac{\partial n}{\partial n} = 0$ $\frac{\partial n}{\partial n} = 0$ $\frac{\partial n}{\partial n} = 0$

$$n = \frac{\partial^2 f}{\partial n^2} = 6 + 6n$$

$$S = \delta^2 f = 6$$

$$t = \frac{\partial^2 f}{\partial y^2} = -2$$



$$9(4-52)$$
 = $-12 - 12(-2)$
= $-12 + 24$

$$n|_{(2,0)} = 6+6(-2)$$

= 6-12
=-6<0

3). Show that minimum value of
$$f(n)y) = xy + a^3$$
+ a^3 is $3a^2$

$$\frac{\partial f}{\partial n} = \frac{y - a^3}{n^2} = 0$$

$$\Rightarrow y = a^3$$

$$\frac{\partial f}{\partial y} = n - a^3 = 0$$

Colving (1) 2 (2)

$$(u,y)^{2} = (a,a)$$

$$8 = \frac{\partial^{2}f}{\partial n^{2}y}$$

$$5 = \frac{\partial^{2}f}{\partial n^{2}y}$$

$$1 = \frac{\partial^{2}f}{\partial n^{2}y}$$

$$2 =$$

$$\frac{\partial f}{\partial n} = 0 = 7n = 3n^2 - 3ay$$

$$\frac{\partial f}{\partial n} = 0 = 2y = a$$

$$\frac{\partial f}{\partial n} = -a^3$$

5)
$$f(n,y) = n^3 + 3\alpha y^2 - 3n^2 - 3y^2 + 4$$
.

$$\frac{\partial f}{\partial n} = 3n^2 + 3y^2 - 6n = 0$$

$$= 3\pi(x-2) + 3y^{2} = 0$$

$$y^{2} = -\pi(x-2)$$

$$xy - y = 0$$

 $y(x-0) = 0$
 $y = 0$, $x = 1$

$$y=0$$

$$\Rightarrow 0 = -n(n-2)$$

$$n = 0, 2$$

$$\begin{cases} (0,0), (2,0) \\ y^2 = -1(-1) = 1 \end{cases}$$

$$y^2 = -1(-1) = 1$$

$$y = \pm 1$$

$$\begin{cases} (1,1), (1,-1) \\ y = \pm 1 \end{cases}$$

$$\begin{cases} (1,1), (1,-1) \\ y = \pm 1 \end{cases}$$

$$\begin{cases} x = 2^{2}f = 6x - 6 \\ 3n^2 \end{cases}$$

$$\begin{cases} x = 2^{2}f = 6y \\ 3n y \end{cases}$$

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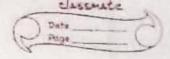
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$$\begin{cases} x = 2^{2}f = 6x$$

```
fman
P 1)
   -4
    0
P 3)
P 4)
      x3 + y3 - 63(x+y)+12xy
 6
       fman = 784
       tmin = -216.
```



Method of Lagrange Multipliers

Let f(x,y,z) be the function of three variable ie x,y,z and the variable be f(x,y,z) = 0connected by the relation.

Suppose we wish to find the values of x,y,z too which f(n)(y) is stationary i.e. man on min. For this purpose we evolte an auxillary eqn $f(n,y,z) + \lambda'$ $\phi(x,y,z) = 0$.

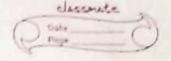
Differentiate wirt n, y, z > Of + 1 20 -0

$$\frac{\partial f}{\partial z} + \lambda \frac{\partial \phi}{\partial z} = 0$$

Q1 Find the minimum value of n2 + y2 subject to the condition ie an + by = c

$$A = -2x - 1$$

10. r. + y



Find the minimum value of nigz3 subject to condition 2n + y +3 = -a

P(x,y,2) + x \$ (x,y,2)=0

22 y 23+ 1 (2x+y+32-0)=0

w.r.t n wirt y

Z23 + A(A) .0 2 myz + 1 (2) =0 1 3-22 23

 $\lambda = -xyz^3$

w.7.t. 2

3 x4z2+ 1(3)=0

 $\lambda = -3 n^2 y^2$

=-22y-z.2

-> - xyz3 = - x2 z 3 = -x2yz2

=> n=y -(1) => z=n-6

from 0 x 2

an+y+32 - a

2n+x+3n=a

: y = 9 and z = a

$$\begin{array}{c} x^{2}y^{2} & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \\ & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \\ & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \\ & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} + \left(\frac{a}{6}\right)^{2} \\ & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} + \left(\frac{a}{6}\right)^{2} \\ & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \\ & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}\right)^{2} \\ & = \frac{1}{2} \left(\frac{a}{6}\right)^{2} \left(\frac{a}{6}$$

2(y-2) + 1(2y) = 0

2(Z-2) + A (2Z)=0

 $\lambda = \frac{2(y-2)}{2y}$

 $\lambda = 2(z-2)$

 $\lambda = \frac{y-2}{y}$

A = 2-2

 $\frac{1}{n} - 1 = \frac{2}{y} - 1 = \frac{2}{z} - 1$

 $\frac{1}{2} = \frac{2}{2} = \frac{2}{2}$

y = 2 2 Z = 2 x

n2+y2+ 22 = 36

 $9(2 + (9x)^2 + (2x)^2 = 36$

n2 + 4n2 + 4n2 = 36

9 n2 = 36

n2 = 4

N =12.

y = +4 , Z = +4.

: $B = \sqrt{(n-1)^2 + (y-2)^2 + (z-2)^2}$

: with n=& , y=4 , 2=4

D = \((2-1)^2 + (4-2)^2 + (4-2)^2

D = 3.

with
$$y = -2$$
, $y = -4$, $z = -4$

$$D = \sqrt{(-2-1)^2 + (-4-2)^2 + (-4-2)^3}$$

$$= 9$$

$$man = 9$$

$$min = 3$$