

Excercise

Q-1 What is the Time Complexity of following functions?

a.

```
void fun(int n, int arr[])
{
    int i = 0, j = 0;
    for (; i < n; ++i)
        while (j < n && arr[i] < arr[j])
            j++;
}
```

b.

```
void fun(int n, int arr[])
{
    int i = 0, j = 0;
    for (; i < n; ++i) {
        j = 0;
        while (j < n && arr[i] < arr[j])
            j++;
    }
}
```

Q-2 In a competition, four different functions are observed. All the functions use a single for loop and within the for loop, same set of statements are executed. Consider the following for loops:

A) **for**(i = 0; i < n; i++)

B) **for**(i = 0; i < n; i += 2)

C) **for**(i = 1; i < n; i *= 2)

D) **for**(i = n; i <= n; i /= 2)

Q-3 Consider the following functions:

$$f(n) = 2^n$$

$$g(n) = n!$$

$$h(n) = n^{\log(n)}$$

Which of the following statements about the asymptotic behavior of $f(n)$, $g(n)$, and $h(n)$ is true?

- (A) $f(n) = O(g(n))$; $g(n) = O(h(n))$
- (B) $f(n) = \Omega(g(n))$; $g(n) = O(h(n))$
- (C) $g(n) = O(f(n))$; $h(n) = O(f(n))$
- (D) $h(n) = O(f(n))$; $g(n) = \Omega(f(n))$

Q-4 In the following C function, let $n \geq m$.

```
int gcd(n, m)
{
    if (n % m == 0)
        return m;
    n = n % m;
    return gcd(m, n);
}
```

How many recursive calls are made by this function?

- (A) $\theta(\log(n))$
- (B) $\Omega(n)$
- (C) $\theta(\log(\log(n)))$
- (D) $\theta(\sqrt{n})$

Q-5 Consider the following functions

- $f(n) = 3n^{\sqrt{n}}$
- $g(n) = 2^{\sqrt{n} \log_2 n}$
- $h(n) = n!$

Which of the following is true? (GATE)

- (A) $h(n)$ is $O(f(n))$
- (B) $h(n)$ is $O(g(n))$
- (C) $g(n)$ is not $O(f(n))$
- (D) $f(n)$ is $O(g(n))$

Q-6 Let s be a sorted array of n integers. Let $t(n)$ denote the time taken for the most efficient algorithm to determine if there are two elements with sum less than 1000 in s . which of the following statements is true? (GATE)

- a) $t(n)$ is $O(1)$
- b) $n < t(n) < n \log_2 n$
- c) $n \log_2 n < t(n) < n^2$
- d) $t(n) = n^2$

Q-7 The recurrence equation

$$T(1) = 1$$

$$T(n) = 2T(n-1) + n, n \geq 2$$

evaluates to

- a. $2^{n+1} - n - 2$
- b. $2^n - n$
- c. $2^{n+1} - 2n - 2$
- d. $2^n + n$

Q-8 Consider the following three claims

1. $(n+k)^m = \Theta(n^m)$, where k and m are constants
2. $2^{n+1} = O(2^n)$

3. $2^{2n+1} = O(2^n)$

Q-9

Let $T(n)$ be a function defined by the recurrence $T(n) = 2T(n/2) + \sqrt{n}$ for $n \geq 2$ and $T(1) = 1$

Which of the following statements is TRUE?

$T(n) = \theta(\log n)$

$T(n) = \theta(\sqrt{n})$

$T(n) = \theta(n)$

$T(n) = \theta(n \log n)$

Q-10 Arrange the following functions in increasing asymptotic order:

A. $n^{1/3}$

B. e^n

C. $n^{7/4}$

D. $n \log^9 n$

E. 1.0000001^n

Q-11 If $T_1 = O(1)$, give the correct matching for the following pairs:

(M) $T_n = T_{n-1} + n$ (U) $T_n = O(n)$

(N) $T_n = T_{n/2} + n$ (V) $T_n = O(n \log n)$

(O) $T_n = T_{n-1} + \log n$ (W) $T = O(n^2)$

Q-12 Give the correct matching for the following pairs:

A. $O(\log n)$ 1. Selection sort

B. $O(n)$ 2. Insertion sort

C. $O(n \log n)$ 3. Binary search

D. $O(n^2)$ 4. Merge sort

Q-13 Which of the following is False?

- a. $100n \log n = O\left(\frac{n \log n}{100}\right)$
- b. $\sqrt{\log n} = O(\log \log n)$
- c. If $0 < x < y$ then $n^x = O(n^y)$
- d. $2^n \neq O(nk)$

Q-14 Given A, an array of size n, comprised of an increasing sequence of numbers followed immediately by a decreasing one. What is worst case time complexity of optimal algorithm to determine if a given number x is in the array?

- ☐ $\Theta(\log n)$
- ☐ $\Theta(n)$
- ☐ $\Theta(n^2)$
- ☒ $\Theta(\log n)^2$