Page No.
Date : 24/7/23

Discrete Mathematics

(George Canton) 1897

- Any collection of elements is termed as set
- eg $A = \{1, 2, 3, 4\}$ = Set of first 4 natural numbers

$1 \in A$ (belongs to)　　　　$5 \notin A$ (does not belong to)

- N = set of Natural numbers $= \{1, 2, 3 \ldots\}$
- Z = Set of integers $= \{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$
- Q = set of rational number $= \left\{ \frac{a}{b} / a \in Z,\ b \in N \right\}$

- R = Set of real numbers

To represent a set:-

1) Roaster Form : eg $\{a, e, i, o, u\}$
2) Set Builder Form: eg $\{x \mid x$ is vowel in English alphabet$\}$

$A = \{x \mid x$ is integer & $-3 \leq x < 7\}$
$= \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

$B = \{x \mid x$ is prime and divisor of $60\}$
$= \{2, 3, 5\}$

$C = \{x \mid x$ is letter of word 'TRIGNOMETRY'$\}$
$\{T, R, I, G, N, O, M, E, Y\}$

$A = \{3, 6, 9, 12\}$
$\{x \mid x$ is multiple of 3 less than $15\}$

$B = \{1, 4, 9, 16, 25, \ldots, 100\}$
$\{x \mid x$ is square of natural no. less than $11\}$.

$C = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$
$\left\{ \frac{n}{n+1} \mid n \in N,\ n \leq 6 \right\}$

Types of Sets:

1) Null Set / Empty set: $(\emptyset)$; $\{\}$

Ex: $\{x \mid x \in \mathbb{N}, 5 < x < 6\}$

$\{x \mid x \in \mathbb{N}, 2x = 3\}$

2) Singleton set:

Ex: $\{x \mid x$ is an identity for addition$\}$

$\{0\}$

$\{x \mid x \in \mathbb{N}, 5 < x < 7\}$

$\{6\}$

3) Finite set:

Ex: $V = \{a, e, i, o, u\}$

4) Infinite set:

Ex: $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \dots\}$

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$

5) Subset of a set:

$A \subseteq B$

Eg: $A = \{1\ 2\ 3\}$

$B = \{1\ 2\ 3\ 4\ 5\ 6\}$

$A \subset B$ but $B \not\subset A$.

6) Equal sets:

$A = B$ OR $A \subset B$, $B \subset A$

- Subsets of $\{\}$ → only one subset $\{\}$
- Any non empty set has atleast 2 subsets.
  1) $\emptyset$ 2) set itself.
- Subsets of $\{x, y, z\}$

$P(A) \Rightarrow \{\ \emptyset\ \{x\}\ \{y\}\ \{z\}\ \{x, y\}\ \{x, z\}\ \{y, z\}\ \{x, y, z\}\ \}$

Total no. of subsets : 2^N

Power set : 2^N

Page No.
Date :

Power set :- The set of all subsets of given set A is the power set of A denoted by P(A)

1) Union of 2 sets A & B, denoted by $A \cup B$ defined as
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
if $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$A = \{1\ 2\ 3\}$ $B = \{1\ 2\ 3\ 4\ 6\ 8\ 11\ 12\}$
$A \cup B = \{1\ 2\ 3\ 4\ 6\ 8\ 11\ 12\}$

- $A \cup A = A$
- $A \cup B = B \cup A$
- $A \cup U = U$
- $A \cup \emptyset = A$
- $A \cup (B \cup C) = (A \cup B) \cup C$

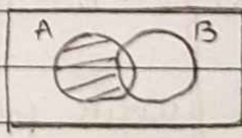
2) Intersection of 2 sets is denoted $A \cap B$. & defined as
$A \cap B = \{x \mid x \in A \ \& \ x \in B\}$
$x \in A \cap B \Rightarrow x \in A \ \& \ x \in B$

eg: $A = \{1\ 2\ 3\}$ $B = \{1\ 2\ 3\ 4\ 5\ 6\ 7\}$
$A \cap B = \{1\ 2\ 3\}$

- $A \cap A = A$
- $A \cap B = B \cap A$
- $A \cap U = A$
- $A \cap \emptyset = \emptyset$
- $A \cap (B \cap C) = (A \cap B) \cap C$

31/7/23

* Difference of 2 sets:
$A - B = \{x \mid x \in A \ \& \ x \notin B\}$



* Complement of set:
$A' = \{x \mid x \in U \ \& \ x \notin A\}$

* DeMorgan's Law:
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

Page No.
Date :

Q If $A=\{3,5,7,9,11\}$ $B=\{7,9,11,13\}$ $C=\{11,13,15\}$ $D=\{15,17\}$

Find ① $A\cap B$ ② $A\cap(B\cup D)$ ③ $(A\cap B)\cap(B\cup C)$ ④ $(A\cup D)\cap(B\cup C)$ ⑤ $A\cup B\cup D$ ⑥ $A\cap C\cap D$

1) $A\cap B=\{7,9,11\}$

2) $A\cap(B\cup D)=\{3,5,7,9,11\}\cap\{7,9,11,13,15,17\}$
$=\{7,9,11\}$

3) $(A\cap B)\cap(B\cup C)=\{7,9,11\}\cap\{7,9,11,13,15\}$
$=\{7,9,11\}$

4) $A\cup D=\{3,5,7,9,11,15,17\}$
$(A\cup D)\cap(B\cup C)=\{7,9,11,15\}$

5) $A\cup B\cup D=\{3,5,7,9,11,13,15,17\}$

6) $A\cap C\cap D=$ ~~{3,5,~~ $\{11\}\cap\{15,17\}$
$=\phi$

Q $A=\{3,6,9,12,15,18,21\}$ $B=\{4,8,12,16,20\}$
$C=\{2,4,6,8,10,12,14,16\}$ $D=\{5,10,15,20\}$

1) $A-B=\{3,6,9,15,18,21\}$

2) $A-C=\{3,9,15,18,21\}$

3) $A-D=\{3,6,9,12,18,21\}$

4) $B-D=\{4,8,12,16\}$

5) $D-B=\{5,10,15\}$

6) $C-D=\{2,4,6,8,12,14,16\}$

Q Write following as intervals:

1) $\{x \mid x\in R, -4<x\leq 6\}$ $(-4,6]$

2) $\{x \mid x\in R.\ 3\leq x\leq 4\}$ $[3,4]$

Q Write following as set builder form:

1) $(-3,0)$ $\{x \mid x\in R, -3<x<0\}$

2) $[6,12]$ $\{x \mid x\in R, 6\leq x\leq 12\}$

3) $[-23,5)$ $\{x \mid x\in R, -23\leq x<5\}$

Q Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1\ 2\ 3\ 4\}$ $B = \{2\ 4\ 6\ 8\}$ $C = \{3\ 4\ 5\ 6\}$

$A' = \{5\ 6\ 7\ 8\ 9\}$

$B' = \{1\ 3\ 5\ 7\ 9\}$

$A \cup C = \{1\ 2\ 3\ 4\ 5\ 6\}$

$(A \cup C)' = \{7\ 8\ 9\}$

$A \cup B = \{1\ 2\ 3\ 4\ 6\ 8\}$

$(A \cup B)' = \{5\ 7\ 9\}$

$(B - C) = \{2\ 8\}$

$(B - C)' = \{1\ 3\ 4\ 5\ 6\ 7\ 9\}$

Q Prove following results using Venn diagram

1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2) $(A \cup B)' = A' \cap B'$

- $(A')' = A$
- $\emptyset' = U$
- $U' = \emptyset$
- $A \cup A' = U$
- $A \cap A' = \emptyset$

1/8/23

Cartesian Product

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

$B \times A = \{(b, a) \mid b \in B, a \in A\}$

$A = \{a\ b\ c\}$ $B = \{1\ 2\ 3\}$

$A = \{$Guj, Mah, Kar$\}$ $B = \{01, 02, \ldots 09\}$

(Guj, 01}

$R \subset A \times B$

Relation is defined as subset of $A \times B$

It is denoted by symbol (R)

$R \subseteq A \times B$

eg: $a \leq b$

$l \perp m$

$A \subset B$

Types of Relation

1) Empty relation:

If $R = \emptyset$ then R is said to be empty relation

$A = \{1\ 2\}$ $B = \{3\ 4\}$ $R = A \times B$ $\{(a,b) \mid a > b\}$

$R = \emptyset$

2) Universal relation:

If $R = A \times B$

$A = \{1, 2\}$ $B = \{3, 4\}$

$R = \{(a, b) \mid a < b\}$

$R = \{(1,3)\ (1,4)\ (2,3)\ (2,4)\}$

3) Reflexive relation:

R is reflexive iff $(a,a) \in R\ \forall\ a \in A$

eg: '$\leq$' on $\mathbb{Z}$, '/' defined on $\mathbb{N}$

4) Symmetric relation:

if $(a,b) \in R \Rightarrow (b,a) \in R\ \forall\ a, b \in A$

eg: 1) "$\perp$" on set of lines

2) "//" " " " "

3) "$\cong$" defined on set of triangles

5) Transitive relation:

if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R\ \forall\ a, b, c \in A$

eg: "//" on set of lines

"$\leq$" on $\mathbb{Z}$

"$\cong$", '$\sim$' on triangles

6) Equivalence relation:

iff

1) R is reflexive, symmetric & transitive

eg: ~~'+' on set of $\mathbb{Z}$~~

~~1) relation "$\subset$ of subset of" defined on sets~~

'$\cong$' is congruent to set of triangles.

Page No.
Date :

1) $\triangle ABC \cong \triangle ABC$
2) $\triangle ABC \cong \triangle PQR \Rightarrow \triangle PQR \cong \triangle ABC$
3) $\triangle ABC \cong \triangle PQR$, $\triangle PQR \cong \triangle XYZ \Rightarrow \triangle ABC \cong \triangle XYZ$

Ex: $R = \{(a, b) / 2 \text{ divide } a-b\}$ R is defined $\mathbb{Z}$

1) 2 divide $a-a$ $\therefore$ R is reflexive
2) 2 divides $a-b$ $\therefore$ R is symmetric
3) 2 divides $a-b$ & 2 divides $b-c$ $\therefore$ R is transitive
$\Rightarrow$ 2 divides $a-c$

Ex: $R = \{(a,b) \mid a \leq b^2\}$ R is defined on $\mathbb{R}$

1) $a \not\leq a^2$ $\therefore$ R is not reflexive $\quad \frac{1}{2} \not\leq \frac{1}{4}$
2) $a \leq b^2$ so $b \leq a^2$ $\therefore$ R is not symmetric
3) $a \leq b^2$, $b \leq c^2 \Rightarrow a \leq c^2$ $\therefore$ R is not transitive
$(10, 4, 2)$
$10 \leq 16 \quad 4 \leq 4$
$10 \not\leq 4$

Ex: Set $A = \{x \in \mathbb{Z} ; 0 \leq x \leq 12\}$
R is relation defined on A
$R = \{(a,b) / |a-b| \text{ is a multiple of } 4\}$
$(0,4) (1,5) (2,6) (3,7) (4,8) (5,9) (6,10) (7,11) (8,12)$

1) $|a-a|$ is multiple of 4 $\therefore$ Reflexive
2) $|a-b|$ is multiple of 4 $\Rightarrow$ $\therefore$ Symmetric
$|b-a|$ is multiple of 4
3) $|a-b|$ is multiple of 4; $|b-c|$ is multiple of 4
$\Rightarrow$ $|a-c|$ is multiple of 4 $\therefore$ Transitive
$\therefore$ Equivalence relation

Equivalence Class

Partitioning of a ~~class~~ set

$A_1, A_2 \ldots A_n$ are said to be partition of set A

1) $A_i \neq \emptyset \quad \forall i = 1 \ldots n$
2) $A_1 \cup A_2 \ldots \cup A_n = A$
3) $A_i \cap A_j = \emptyset \quad i \neq j$

$R\{(a,b) / 2 \text{ divide } a-b ; a,b \in \mathbb{Z}\}$

$[0] = \{0, \pm 2, \pm 4, \pm 6 \ldots\} = [E]$

$[1] = \{\pm 1, \pm 3, \pm 5 \ldots\} = [O]$

2 equivalence classes

eg: $A = \{4, 5, 6, 7\}$

$R = \{(4\,4)\ (5\,5)\ (6\,6)\ (7\,7)\ (4\,6)\ (6,4)\}$

$[4] = \{4, 6\} \Rightarrow [6] = \{4, 6\}$

$[5] = \{5\} \Rightarrow [7] = \{7\}$

3 equivalence classes.

Equivalence Class

Consider an equivalence relation R on set A

The equivalence class of an element $a \in A$, is the set of all elements of A which are related to 'a'.

It is denoted by $[a]$

$R = \{(a,b) \mid$ either both a & b are even or both a & b are odd defined on $\mathbb{Z}\}$

Partial Order Relation (POSET) "$\le$"

1) Reflexive aRa
2) antisymmetric
aRb & $bRa \Rightarrow a=b$
3) Transitive
aRb & $bRc \Rightarrow aRc$

(1) eg: '$\le$' defined on $\mathbb{R}$ '$\geq$' defined on $\mathbb{R}$

A relation R defined on the set A is called a partial order relation if it satisfies the following properties.

$R_1 = \{(0\,2)\ (1\,2)\ (2\,0)\}$ ×
$R_2 = \{(0\,0)\ (0\,1)\ (0\,2)\ (1\,1)\ (1\,2)\}$ ✓

8/8/23

$A = \{1\ 2\ 3\ 4\}$
$R_1 = \{(1\,1)\ (1\,2)\ (1\,4)\ (2\,2)\ (2\,4)\ (3\,3)\ (4\,4)\} \rightarrow$ POSET

$R_2 = \{(1\,1)(1\,3)(2\,1)\ (2\,2)\ (3\,3)\ (3\,4)\ (4\,2)\ (4\,4)\}$ × P.O.R.
(A, R_2) is not POSET
R_2 is reflexive as $(1\,1)(2\,2)(3\,3)(4\,4) \in R_2$
R_2 is antisymmetric $(a,b) \in R_2$ but $(b,a) \notin R_2$
R_2 is not transitive as $(1\,3) \in R_2$ $(3\,4) \in R_2$ but $(1\,4) \notin R_2$

$R_3 = \{(1\,1)(1\,3)(1\,4)(2\,2)(3\,2)(3\,3)(3\,4)(4\,4)\}$
(A, R_3) Not POSET
$R_4 = \{(1\,1)\ (2\,2)\ (3\,3)\ (4\,4)\ (2\,3)\ (2\,4)\ (3\,2)\ (3\,4)\ (4\,3)\}$
(A, R_4) Not POSET

(2) eg: '/' defined on $\mathbb{N}$ $(\mathbb{N}, /)$ is a POSET
(3) eg: A = a non empty set
$P(A)$ = Power set of A
$(P(A), \subseteq)$ is a POSET

1) $A_1 \subseteq A_1\ \forall A_1 \in P(A)$
2) $A_1 \subseteq A_2$ & $A_2 \subseteq A_1 \Rightarrow A_1 = A_2$
3) $A_1 \subseteq A_2$ & $A_2 \subseteq A_3 \Rightarrow A_1 \subseteq A_3$
$\forall A_1\ A_2\ A_3 \in P(A)$

$R = \{(a,b) \mid a,b \in \mathbb{Z},\ a = b+1\} \longrightarrow$ ✗ POSET

$R = \{(a,b) \mid$ "a is ancestor of b"$\}$ defined on set of all people (provided each person is an ancestor of himself / herself) $\longrightarrow$ ✓ POSET.

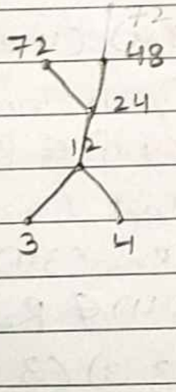
10/8/23

Hasse Digram

Hasse Diagram is a graphical representation of the relation of the elements of a partially ordered set, with an implied upward orientation

Draw a Hasse Diagram for

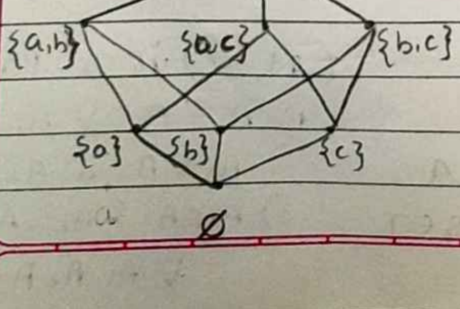
$(\{3, 4, 12, 24, 48, 72\}, /)$



$\{(3,3)\ (3,12)\ (3,24)\ (3,48)\ (3,72)$
$(4,4)\ (4,12)\ (4,24)\ (4,48)\ (4,72)$
$(12,24)\ (12,48)\ (12,72)\ (12,12)$
$(48,48)$
$(24,24)\ (24,48)\ (24,72)$
$(72,72)\}$

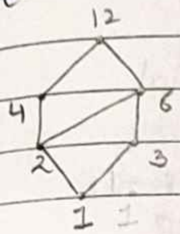
$A = (\{a,b,c\})$ $\quad (P(A), \subseteq)$

$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$



$(D_{12}, /)$ $\quad D_{12}$ = Divisors of 12

$\{1, 2, 3, 4, 6, 12\}$

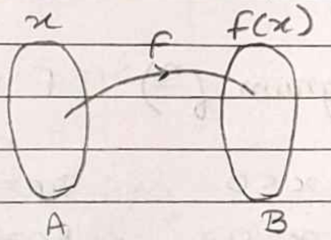


$\{(1\ 1)\ (1\ 2)\ (1\ 3)\ (1\ 4)\ (1\ 6)\ (1\ 12)$
$(2\ 2)\ (2\ 4)\ (2\ 12)\ (2\ 6)$
$(3\ 3)\ (3\ 6)\ (3\ 12)$
$(4\ 4)\ (4\ 12)$
$(6\ 6)\ (6\ 12)$
$(12\ 12)\}$

FUNCTIONS

A function is defined as a rule from a non-empty set A to non empty set B such that each element in A has unique image in B

$f(x)$ is said to be image of x under 'f'



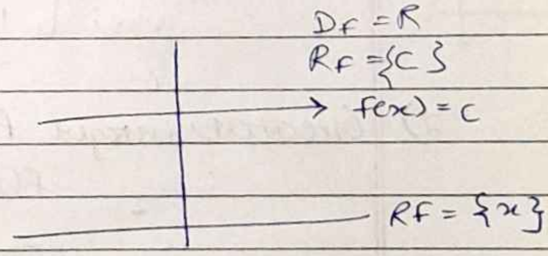
Types of function

1) Constant f^n :-

$f : R \to \mathbb{R}$

$f(x) = c \ (c \in \mathbb{R})$

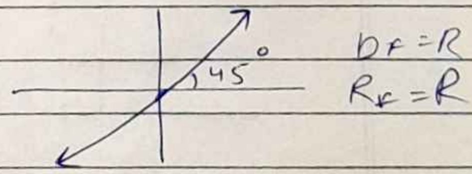
$D_f = \mathbb{R}$



2) Identity f^n :-

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = x$



3) Polynomial f^n :-

$f : R \to R$

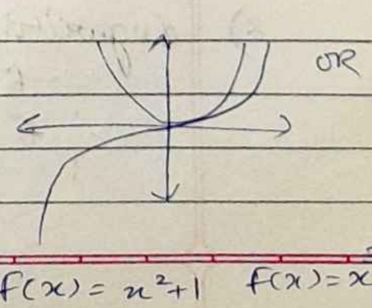
$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$

eg: $2x+3 = f(x)$

: $x^2+2x+5 = g(x)$

$D_f : R$

$R_f : R$



4) Rational f^n: $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \frac{p(x)}{q(x)} \; ; \; q(x) \neq 0$$

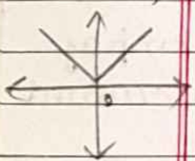
$p(x)$ and $q(x)$ are polynomial

eg: $f(x) = \dfrac{2x+3}{x-1}$

$D_f : \mathbb{R} - \{1\}$

$R_f : \mathbb{R}$

5) Modulus f^n: $f : \mathbb{R} \to \mathbb{R}$



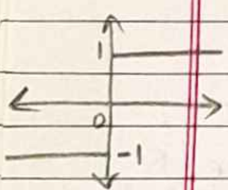
$f(x) = |x|$

$$f(x) = \begin{cases} x , & x > 0 \\ -x , & x < 0 \end{cases}$$

$D_f = \mathbb{R}$

$R_f = \mathbb{R}^+ \cup \{0\}$

6) ~~Integral part f^n~~ (Signum f^n) $f : \mathbb{R} \to \mathbb{R}$



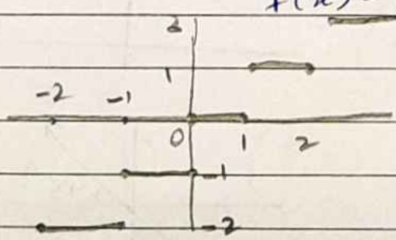
$$f(x) = \begin{cases} -1 \; ; & x < 0 \\ 0 \; ; & x = 0 \\ 1 \; ; & x > 0 \end{cases}$$

$D_f = \mathbb{R}$

$R_f = \{-1, 0, 1\}$

7) Greatest integer f^n: $f : \mathbb{R} \to \mathbb{R}$

$f(x) = [x]$



$[2.3] = 2$

$[5.4] = 5$

$[-2.3] = -3$

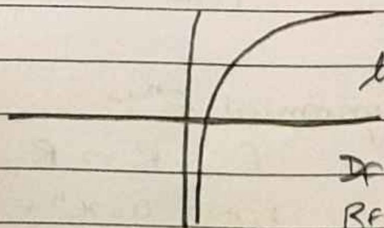
$D_f = \mathbb{R}$

$R_f = \mathbb{Z}$

8) Logarithmic f^n

$f : \mathbb{R}^+ \to \mathbb{R}$

$f(x) = \log(x)$

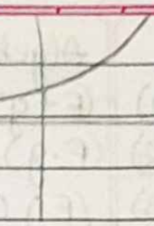


$D_f = \mathbb{R}^+$

$R_f = \mathbb{R}$

a) Exponential f^n : $f: \mathbb{R} \to \mathbb{R}$

$f(x) = e^x$

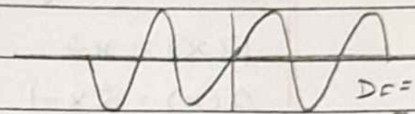


$D_f: \mathbb{R}$

$R_f: \mathbb{R}^+$

b) Trignometric f^n :

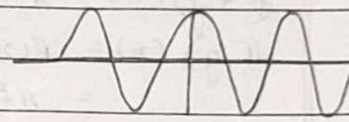
1) $f(x) = \sin x \quad f: \mathbb{R} \to \mathbb{R}$



$D_f = \mathbb{R}$

$R_f = [-1, 1]$

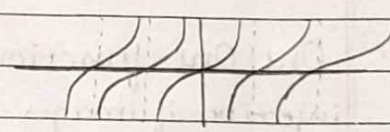
2) $f(x) = \cos x \quad f: \mathbb{R} \to \mathbb{R}$



$D_f = \mathbb{R}$

$R_f = [-1, 1]$

3) $f(x) = \tan x \quad f: \mathbb{R} \to \mathbb{R}$

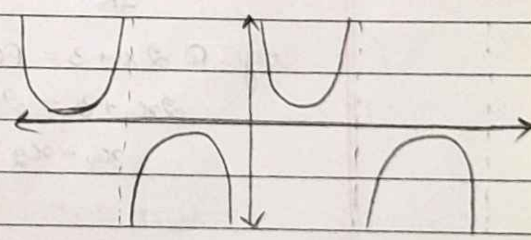


$D_f = \mathbb{R} - \left\{ (2n+1)\frac{\pi}{2} \right\}$

$n \in \mathbb{Z}$

$R_f = \mathbb{R}$

4) $f(x) = \operatorname{cosec} x \quad f: \mathbb{R} \to \mathbb{R}$



$D_f : \mathbb{R} - \{ n\pi / n \in \mathbb{Z} \}$

$R_f : (-\infty, -1) \cup (1, \infty)$

$: \mathbb{R} - [-1, 1]$

5) $f(x) = \sec x \quad f: \mathbb{R} \to \mathbb{R}$

$D_f = \mathbb{R} - \left\{ (2n+1)\frac{\pi}{2} ;\ n \in \mathbb{Z} \right\}$

$R_f = \mathbb{R} - [-1, 1]$

6) $f(x) = \cot x$

$D_f : \mathbb{R} - \{ n\pi,\ n \in \mathbb{Z} \}$

$R_f = \mathbb{R}$

Algebra of function

1) $(f+g)(x) = f(x) + g(x)$
2) $(f \cdot g)(x) = f(x) \cdot g(x)$
3) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$; $g(x) \neq 0$

$f(x) = x^3$
$g(x) = 2x - 1$
$(f+g)(x) = x^3 + 2x - 1$
$(f \cdot g)(x) = f(x) \cdot g(x)$
$= x^3(2x-1)$

One One functions: A f^n $f: x \to y$ is defined one-one f^n/ injective if images of distinct elements of X under f are distinct

i.e $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

i.e. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

OR

eg: $2x + 3 = f(x)$
$2x_1 + 3 = 2x_2 + 3$
$x_1 = x_2$ (One One)

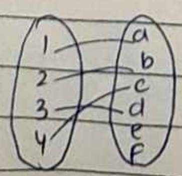
eg: $f(x) = x^2$
$f(1) = 1$; $f(-1) = 1$
~~$f(x_1) \neq f(x_2)$~~; (not one one)

(Surjective) Onto functions: A f^n $f: x \to y$ is said to be onto f^n if every element of Y is image of some element of X under f i.e. for each $y \in Y$ $\exists$ $x \in X$

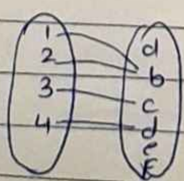
λ $f(x) = y$

OR

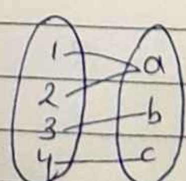
R_f = codomain.



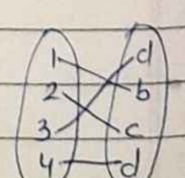
one one ✓
but onto ×
many one ×



one one ×
onto ×
many one ✓



many one ✓
onto ✓
one one ×



one one ✓
onto ✓
many one ×

Check for one-one or onto

Q $f: R \to R$ given by $f(x) = 2x$

$\to$ let $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 = 2x_2$

$x_1 = x_2$

f is one-one ✓

$f(x) = y = 2x$

$x = \cancel{f}\,\frac{y}{2}$

$y = \frac{x}{2} \in R$

onto ✓

Q $f: N \to N$ given by

$f(1) = f(2) = 1$ & $f(x) = x - 1$ for $x > 2$

$f(1) = f(2) = 1$

but $1 \neq 2$

$\therefore$ f is not one-one

$R_f : N$

$\therefore$ f is onto

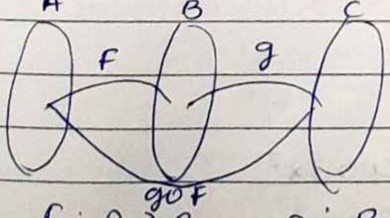
22/08/23 $f: R \to R$ $\quad R_f = R^+ > 1 \neq R$

$f(x) = 1 + x^2$ $\quad$ (not onto)

$f(1) = 2$

$f(-1) = 2$ (not one one)

Composite functions :



$f: A \to B$ $\quad g: B \to C$

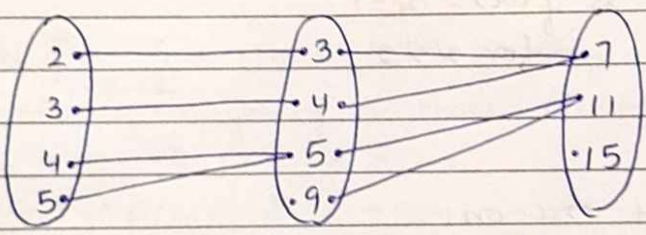
$gof(x): A \to C$ (f : codomain = g : domain)

$fog(x): C \to A$ (g : codomain = f : domain)

Composite of F^{ns}

Let : $F : A \to B$ & $g : B \to C$ be 2 functions then the composition of f & g is denoted by 'gof' & is defined as $gof(x) = g[F(x)]$ $\forall x \in A$

Q $F : \{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$
$g : \{3, 4, 5, 9\} \to \{7, 11, 15\}$
$f(2) = 3$; $f(3) = 4$; $f(4) = f(5) = 5$
$g(3) = g(4) = 7$; $g(5) = g(9) = 11$
Find $gof(x)$.



~~$gof : \{2, 3, 4, 5\} \to \{3, 4, 5\}$~~
~~gof~~

$gof : \{2, 3, 4, 5\} \to \{7, 11, 15\}$
$gof(2) = g[f(2)] = g(3) = 7$
$gof(3) = g(f(3)) = g(4) = 7$
$gof(4) = g(f(4)) = g(5) = 11$
$gof(5) = g(f(5)) = g(5) = 11$

Q Find fog and gof.
if : $F : R \to R$
& $g : R \to R$
$f(x) = \cos x$; $g(x) = 3x^2$
$f(g(x)) = f(3x^2) = \cancel{3\cos^2 x}$ $\cos 3x^2$
$g(f(x)) = g(\cos x) = \cancel{\cos 3x^2}$ $3\cos^2 x$
$fog \neq gof$

Page No.
Date :

Q Find fog and gof

1) $f(x) = |x|$ $g(x) = |5x-2|$

$fog = f(g(x)) = f(|5x-2|) = ||5x-2||$

$gof = g(f(x)) = g(|x|) = |5|x|-2|$

2) $f(x) = 8x^3$ $g(x) = x^{1/3}$

$f(g(x)) = f(x^{1/3}) = 8x^{3/3} = 8x$

$g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 2x$

Inverse function

A function $f: X \to Y$ is said to be invertible if $\exists$ a funⁿ $g: Y \to X$ such that $fog = gof = I$.

The function g is called inverse of f denoted by 'f^{-1}'

$\therefore$ f is invertible iff f is one-one & onto both

Q Let : $f: N \to Y$ $f(x) = 4x+3$

where $Y = \{x \in N / y = 4x+3$ for some $x \in N\}$

ST: 'f' is invertible and find its inverse

$f(x) = 4x+3$ $R_f : Y$ onto

Let $f(x_1) = f(x_2)$

$4x_1 + 3 = 4x_2 + 3$

$x_1 = x_2$ One-One

$f(x) = y = 4x+3$

$x = \frac{y-3}{4}$

$g(y) = \frac{y-3}{4}$ (inverse)

$g(x) = \frac{x-3}{4}$

$fog(x) = f(g(x))$

$= f\left(\frac{x-3}{4}\right)$

$= 4\left(\frac{x-3}{4}\right) + 3$

$= x$

$gof(x) = g(f(x))$

$= g(4x+3)$

$= \left(\frac{4x+3-3}{4}\right)$

$= x$

$gof = fog = x = I.$

Unit-2

Propositional Logic

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Proposition: Declarative sentence which can be either T/F not both.

1) Negation: If p is a proposition its negation is denoted by ~p

p	~p
T	F
F	T

2) Conjunction: for any 2 propositions p & q their conjuction is denoted by 'p∧q' which means 'p & q'

3) Disjunction: for any 2 propositions p & q, their disjunction is denoted by 'p∨q' which means p or q

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

p : Today is Thursday
q : It is raining today

4) Exclusive or :- for any 2 propositions p & q their exclusive or is denoted by 'p⊕q' which means either 'p' or 'q' but not both

p	q	p⊕q
T	T	F
T	F	T
F	T	T
F	F	F

5) Implication: For any 2 prepositions p & q, their implications is denoted by '$p \to q$' or '$p \Rightarrow q$' which means "if p the q" p is called hypothesis q is called conclusion or consequence

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

("p is sufficient for q")
("if p then q")
("q when p")
("p only if q")
("p, only if ~p")
("q follows from p")

6) Double Implication: for any 2 propositions, p & q the statement "p iff q" is called a biconditional statement denoted by "$p \Leftrightarrow q$". $\cong (p \to q) \wedge (q \to p)$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Nature of compound statement:-

1) Tautology: A compound statement is called tautology iff it is true for all possible values of its propositional variable.
2) Contradiction: A compound statement is called contradiction iff it is false for all possible values of its propositional variable. (fallacy)
3) Contigency: A compound statement is called contigency iff it is neither a tautology nor a contradiction.

Q Prepare a truth ~~trat~~ table

1) $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

2) $p \vee \sim q$

p	q	$\sim q$	$p \vee \sim p$
T	F	T	T
F	T	F	F
T	T	F	T
F	F	T	T

3) $(p \wedge q) \vee r$

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	T	T
F	T	T	F	T
T	F	T	F	T
F	F	T	F	T
F	T	F	F	F
T	F	F	F	F
F	F	F	F	F

4) $p \rightarrow (q \wedge r)$

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
F	T	T	T	T

F	F	T	F	T
F	T	F	F	T
T	F	F	F	F
F	F	F	F	T

5) $\sim p \wedge (\sim q \vee r)$

$\sim p$	$\sim q$	r	$\sim q \vee r$	$\sim p \wedge (\sim q \vee r)$
T	T	F	T	T
T	F	T	T	T
F	T	T	T	F
T	T	T	T	T
F	F	F	F	F
F	F	T	T	F
T	F	F	F	F
F	T	F	T	F

6) $(\sim p \wedge q) \Leftrightarrow \sim r$

$\sim p$	q	$\sim r$	$\sim p \wedge q$	$(\sim p \wedge q) \Leftrightarrow \sim r$
T	T	T	T	T
T	F	F	F	T
F	F	T	F	F
F	T	F	F	T
F	F	F	F	T
F	T	T	F	F
T	T	F	T	F
T	F	T	F	F

Page No.
Date :

7) $(p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T
T	F	F	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F
T	T	F	T	F	T
F	T	T	F	F	F
T	F	T	F	T	T

29/08/31 De Morgan's Law for propositional statement

1) $(A \cup B)' = A' \cap B'$
2) $(A \cap B)' = A' \cup B'$
3) $\sim(p \vee q) = \sim p \wedge \sim q$
4) $\sim(p \wedge q) = \sim q \vee \sim q$

Negation of a compound statement

Prove that $\sim(p \vee q) = \sim p \wedge \sim q$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Hence proved

Page No.
Date :

Q. p : The South-West monsoon is very good this year.
q : The rivers are rising.
Give verbal translations of :

1) $p \vee \sim q$
2) $\sim(p \vee \sim q) = \sim(\sim p) \wedge \sim(\sim q) = p \wedge q$

1) The South-West monsoon is very good this year but the rivers are not rising.
2) The south-west monsoon is very good this year and the rivers are rising.

Q. $p \Rightarrow (q \wedge r)$ Check for logically equivalent
$(p \Rightarrow q) \wedge (p \Rightarrow r)$

p	q	r	$q \wedge r$	$p \Rightarrow (q \wedge r)$	$(p \Rightarrow q)$	$(p \Rightarrow r)$	$(p \Rightarrow q) \wedge (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Same; logically equivalent.

$p \Rightarrow (q \wedge r) \cong (p \Rightarrow q) \wedge (p \Rightarrow r)$

Page No.
Date :

Q Check validity of statement: (all T)

1) $(p \wedge q) \Rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

<u>Valid</u>

<u>Tautology</u>

2) $(p \wedge q) \wedge \sim(p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

<u>Not Valid</u>

<u>Fallacy</u>

Unit-3

Recurrence Relation

* Sequence
  set of numbers following some rule
  $f : N \rightarrow R$

AP $\{1, 5, 9, 13, 17, \ldots\}$
GP $\{1, 2, 2^2, 2^3, \ldots\}$ 2^n - n^{th} term

* Series
  $\{1 + 5 + 9 + 13 + 17 + \ldots\}$

→ Recurrence Relation is only for sequence
For eg: $a_n = 2a_{n+1} + 3$
We can find n^{th} term using its previous term initial values is given.

Eg: $\{5, 8, 11, 14, 17, \ldots\}$
$a_n = a_{n+1} + 3$; $a_1 = 5$ $n \geq 2$

→ Order of Relation:-
order = highest - lowest subscript of n

- 1^{st} order
  $a_n = a_{n-1} + 3$
- 2^{nd} order
  $f_n = f_{n+1} + f_{n+2}$