Graded Project
Advanced Statistics

Case Studies

Probability and Probability Distribution
Aquarius Gym
Zingaro
Dental Implant

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GL PGP DSBA Batch Feb'23-24

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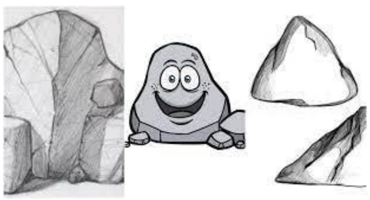

1 Problem 1

1.1 What is the probability that a randomly chosen player would suffer an injury?

To find the probability that a randomly chosen player would suffer an injury, we need to divide the total number of injured players by the total number of players:
Probability of a randomly chosen player suffering an injury = (Number of injured players) / (Total number of players) = 145/235 ≈ 0.617
Therefore, the probability that a randomly chosen player would suffer an injury is approximately 0.617

1.2 What is the probability that a player is a forward or a winger?

To find the probability that a player is a forward or a winger, we need to add the number of forwards and wingers and divide it by the total number of players:
Probability of a player being a forward or a winger = (Number of forwards) + (Number of wingers) / (Total number of players) = (94 + 29) / 235 ≈ 0.553
Therefore, the probability that a player is a forward or a winger is approximately 0.617

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

To find the probability that a randomly chosen player plays in a striker position and has a foot injury, we need to divide the number of injured players who play in the striker position by the total number of players:
Probability of a player being in striker position and having a foot injury = (Number of injured strikers) / (Total number of players) = 45/235 ≈ 0.191
Therefore, the probability that a randomly chosen player plays in a striker position and has a foot injury is approximately 0.191.

1.4 What is the probability that a randomly chosen injured player is a striker?

To find the probability that a randomly chosen injured player is a striker, we need to divide the number of injured strikers by the total number of injured players:
Probability of an injured player being a striker = (Number of injured strikers) / (Total number of injured players) = 45/145 ≈ 0.310
Therefore, the probability that a randomly chosen injured player is a striker is approximately 0.310.

1.5 What is the probability that a randomly chosen injured player is either a forward or an attacking midfielder?

To find the probability that a randomly chosen injured player is either a forward or an attacking midfielder, we need to add the number of injured forwards and attacking midfielders and divide it by the total number of injured players:

Probability of an injured player being a forward or an attacking midfielder = (Number of injured forwards) + (Number of injured attacking midfielders) / (Total number of injured players) = (56 + 24) / 145 ≈ 0.690

Therefore, the probability that a randomly chosen injured player is either a forward or an attacking midfielder is approximately 0.690.

2 Problem 2

An independent research organization is trying to estimate the probability that an accident at a nuclear power plant will result in radiation leakage. The types of accidents possible at the plant are, fire hazards, mechanical failure, or human error. The research organization also knows that two or more types of accidents cannot occur simultaneously.

According to the studies carried out by the organization, the probability of a radiation leak in case of a fire is 20%, the probability of a radiation leak in case of a mechanical 50%, and the probability of a radiation leak in case of a human error is 10%. The studies also showed the following;

The probability of a radiation leak occurring simultaneously with a fire is 0.1%. The probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%. The probability of a radiation leak occurring simultaneously with a human error is 0.12%. On the basis of the information available, answer the questions below:

On the basis of the information available, answer the questions below:

2.1 What are the probabilities of a fire, a mechanical failure, and a human error respectively?

We need to use Bayes' Theorem to calculate the unconditional probabilities of the different events.

To use Bayes' Theorem, we need to know the prior probabilities of the events (i.e., the probabilities of the events before we observe any information about whether a radiation leak has occurred), as well as the conditional probabilities of a radiation leak given each of the events.

Let's denote the events as follows:

F: Fire   M: Mechanical failure   H: Human error R: Radiation leak

We are given:
P(R|F) = 0.20 (the probability of a radiation leak given that there is a fire)
P(R|M) = 0.50 (the probability of a radiation leak given that there is a mechanical failure)
P(R|H) = 0.10 (the probability of a radiation leak given that there is a human error)
P(R) = ? (the probability of a radiation leak)

We are also given:
P (F and M) = 0 (the probability of both a fire and a mechanical failure occurring simultaneously)
P (F and H) = 0 (the probability of both a fire and a human error occurring simultaneously)
P (M and H) = 0 (the probability of both a mechanical failure and a human error occurring simultaneously)

Using the Law of Total Probability, we can write:
P(R) = P(R|F) * P(F) + P(R|M) * P(M) + P(R|H) * P(H)

We know that P (F and M) = P (F and H) = P (M and H) = 0, which means **that F, M, and H are mutually exclusive events.**

So, we can write:
P (F or M or H) = P(F) + P(M) + P(H)
Since these events are mutually exclusive, **we can use the addition rule of probability** to write:
P (F or M or H) = P(F) + P(M) + P(H) = 1
Therefore, we have:
P(H) = 1 - P(F) - P(M)

Using the conditional probabilities and the formula for P(R) above, we can substitute P(H) in terms of P(F) and P(M):

P(R) = P(R|F) * P(F) + P(R|M) * P(M) + P(R|H) * (1 - P(F) - P(M))

Now we need to solve for P(F), P(M), and P(R):

To find P(F), we can use the fact that the probability of a radiation leak occurring simultaneously with a fire is 0.1%, which means:

P (R and F) = P(R|F) * P(F) = 0.001

Solving for P(F):
0.001 = 0.20 * P(F) P(F) = 0.001 / 0.20 = 0.005

To find P(M), we can use the fact that the probability of a radiation leak occurring simultaneously with a mechanical failure is 0.15%, which means:

P (R and M) = P(R|M) * P(M) = 0.0015

Solving for P(M):
0.0015 = 0.50 * P(M) P(M) = 0.0015 / 0.50 = 0.003

Solving for P(H):
P(H) = 1 - P(F) - P(M) = 1 - 0.005 - 0.003 = 0.992

2.2 What is the probability of a radiation leak?

Finally, we can substitute the values we found for P(F) and P(M) in the formula for P(R) to get:

P(R) = P(R|F) * P(F) + P(R|M) * P(M) + P(R|H) * (1 - P(F) - P(M))
= 0.20 * 0.005 + 0.50 * 0.003 + 0.10 * (1 - 0.005 - 0.003)
= 0.1005

Answer : Therefore, the probability of a radiation leak is 10.05%.

2.3 Suppose there has been a radiation leak in the reactor for which the definite cause is not known. What is the probability that it has been caused by:

A Fire.

B Mechanical Failure.

C Human Error.

we can use Bayes' Theorem again. We want to find the probability of each cause given that a radiation leak has occurred:

P(F|R) = P(R|F) * P(F) / P(R) = 0.20 * 0.005 / 0.1005 = 0.0099 or approximately 1%

P(M|R) = P(R|M) * P(M) / P(R) = 0.50 * 0.003 / 0.1005 = 0.0149 or approximately 1.5%

P(H|R) = P(R|H) * P(H) / P(R) = 0.10 * (1 -0.005 - 0.003) / 0.1005 = 0.0746 or approximately 7.5%

Answer : Therefore, the probability that the radiation leak was caused by a fire is approximately 1%, the probability that it was caused by a mechanical failure is approximately 1.5%, and the probability that it was caused by human error is approximately 7.5%.

3 Problem 3:

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimetre and a standard deviation of 1.5 kg per sq. centimetre. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain;

Answer the questions below based on the given information;
(Provide an appropriate visual representation of your answers, without which marks will be deducted)

3.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq cm?

Probability of breaking strength < 3.17 kg/sq. cm: 0.11123243744783456

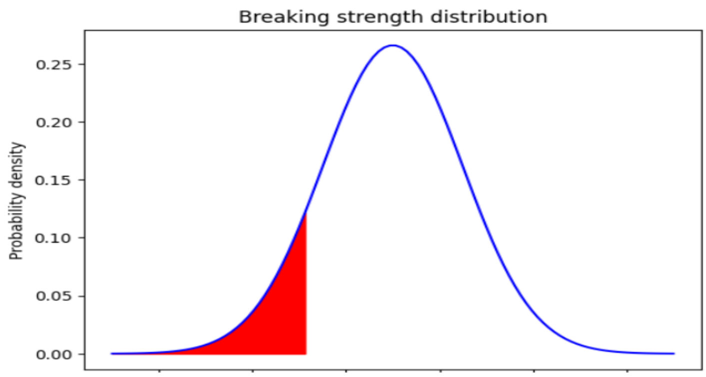

Figure 1

Insights : functions and distributions for statistical analysis.

In this particular solution , a normal distribution with mean mu = 5 and standard deviation sigma = 1.5 is defined using stats.norm(mu, sigma) from scipy library of python

Then probabilities are computed based on this normal distribution using the cumulative distribution function (CDF) of the normal distribution provided by the stats module.

For instance, prob_1 = dist. cdf (3.17) computes the probability of a breaking strength being less than 3.17 kg/sq. cm, which is the value of the CDF of the normal distribution at 3.17.

the CDF value at 3.17 is 0.1, then there is a 10% chance that a breaking strength value will be less than or equal to 3.17 kg/sq cm

3.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq cm.?

Probability of breaking strength >= 3.6 kg/sq cm: 0.8246760551477705

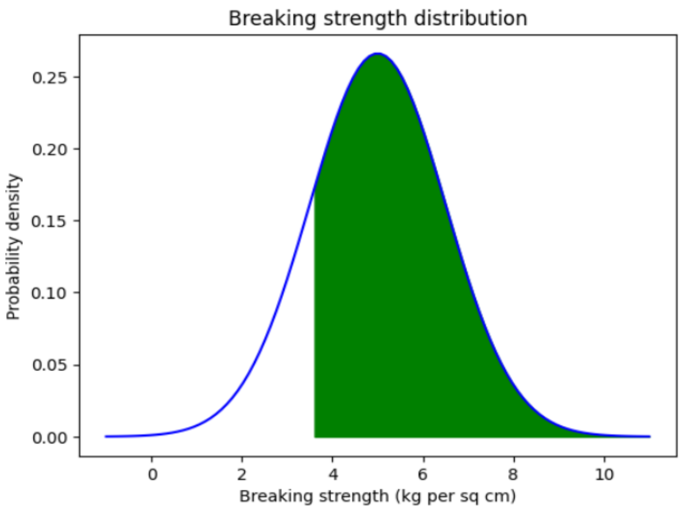

Figure 2

Insight: This means that based on the given normal distribution, there is an 82.47% chance that a randomly selected breaking strength value will be greater than or equal to 3.6 kg/sq cm. This probability is relatively high, which suggests that it is quite likely to observe breaking strength values that are higher than 3.6 kg/sq cm in this scenario

3.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

Probability of breaking strength between 5 and 5.5 kg/sq cm: 0.13055865981823633

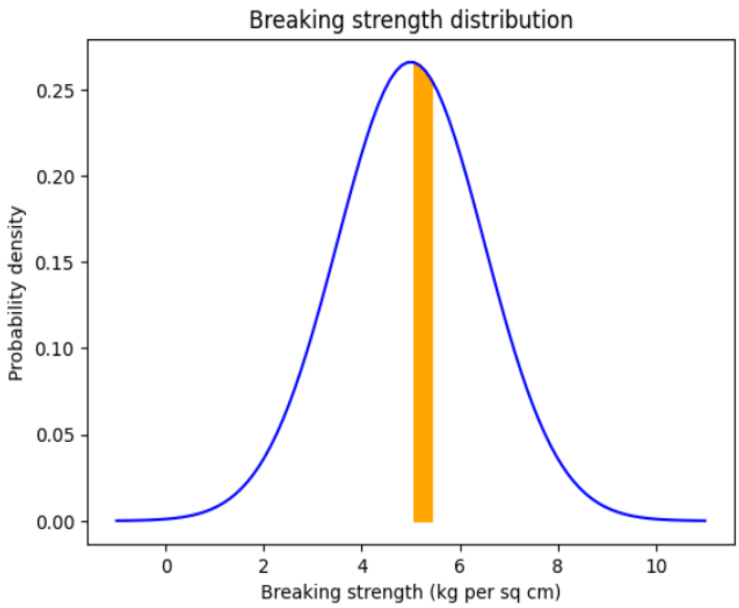

Figure 3

Insights : This means that based on the given normal distribution, there is a 13.06% chance that a randomly selected breaking strength value will be between 5 and 5.5 kg/sq cm. This probability is relatively low, which suggests that it is less likely to observe breaking strength values in this range

3.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

Probability of breaking strength NOT between 3 and 7.5 kg/sq cm: 0.13900157199868257

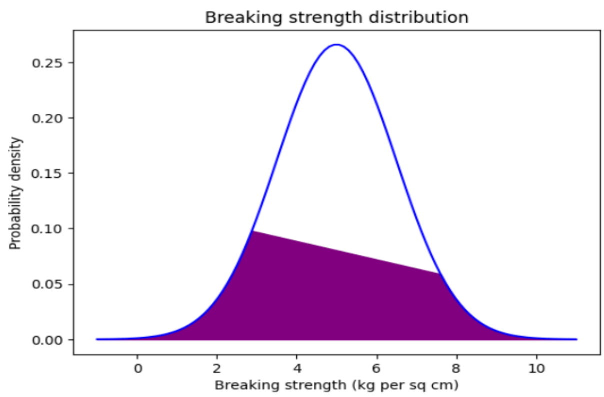

Figure 4

Insights : This means that based on the given normal distribution, there is a 13.90% chance that a randomly selected breaking strength value will not be between 3 and 7.5 kg/sq cm. This probability is relatively high, which suggests that it is more likely to observe breaking strength values that are not in this range

4 Problem 4:

Grades of the final examination in a training course are found to be normally distributed, with a mean of 77 and a standard deviation of 8.5. Based on the given information answer the questions below.

4.1 What is the probability that a randomly chosen student gets a grade below 85 on this exam?

Probability of getting a grade below 85: 82.67%

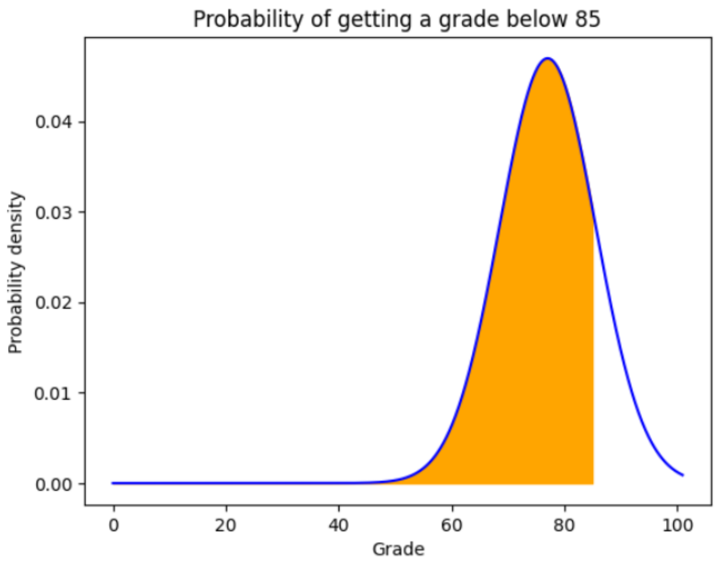

Figure 5

Insights:

"Probability of getting a grade below 85: 82.67%" means that based on some given information (like a mean grade and standard deviation), there is an 82.67% chance that a student will score below 85 on an exam.
To visualize this, we have used a normal distribution curve. We will shade the area under the curve to the left of 85 to represent the probability of getting a grade below 85. The shaded area will represent 82.67% of the total area under the curve

The shaded area to the left of 85 represents the probability of getting a grade below 85. The area under the curve to the left of 85 is approximately 82.67% of the total area under the curve.

4.2 What is the probability that a randomly selected student score between 65 and 87?

Probability of getting a grade between 65 and 87: 80.13%

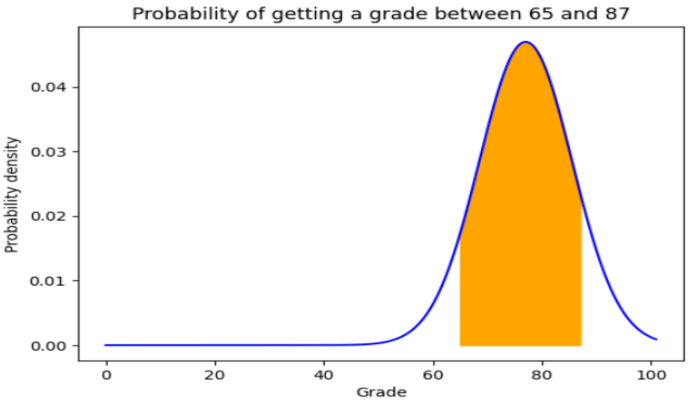

Figure 6

Insights: "Probability of getting a grade between 65 and 87: 80.13%" means that based on some given information (like a mean grade and standard deviation), there is an 80.13% chance that a student will score between 65 and 87 on an exam

To visualize this, we have used a normal distribution curve, which is a bell-shaped curve that represents the distribution of scores for a group of students. The curve is centered at the mean and its width is determined by the standard deviation.

We have shaded the area under the curve between 65 and 87 to represent the probability of getting a grade in that range. The shaded area will represent 80.13% of the total area under the curve

4.3 What should be the passing cut-off so that 75% of the students clear the exam?

The passing cut-off is: 82.73

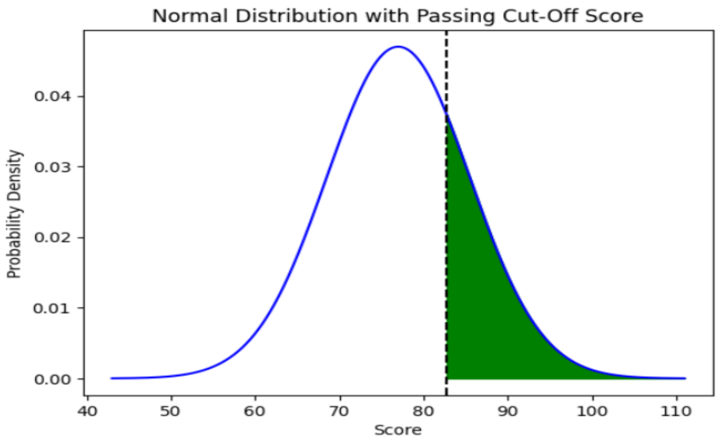

Figure 7

Insights: The plot helps us understand the distribution of scores and where the passing cut-off lies. If a score falls below the cut-off, it means that it falls within the lowest 25% of scores, indicating that it is above or equal to 75% of the scores

5 Problem 5:

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

Mathematically, we can express the hypothesis as:

H0: μ < 150

HA: μ ≥ 150

Where μ is the population mean Brinell's hardness index for all stones, and the null hypothesis assumes that this mean is less than 150, while the alternative hypothesis assumes that it is greater than or equal to 150.

5.1 Earlier experience of Zingaro with this particular client is favorable as the stone surface was found to be of adequate hardness. However, Zingaro has reason to believe now that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

Null hypothesis: H0: μ ≥ 150
Alternative hypothesis: H1: μ < 150

Where μ is the population mean Brinell's hardness index of unpolished stones received from the client.

In words, the null hypothesis states that the population mean Brinell's hardness index of unpolished stones received from the client is greater than or equal to 150. The alternative hypothesis states that the population mean Brinell's hardness index of unpolished stones received from the client is less than 150 and are unsuitable for painting.
To test these hypotheses, we can perform a one-sided, one-sample t-test on the data provided for the unpolished stones. If the test statistic falls in the rejection region of the null hypothesis, we will reject the null hypothesis and conclude that there is sufficient evidence to support the alternative hypothesis**. We found that, p-value is: 0.11149948416904217, which is greater than 0.05 so we do not reject the null hypothesis. Hence, we say that, Zingaro does not have sufficient evidence to believe that unpolished stones are unsuitable for printing.**

5.2 Is the mean hardness of the polished and unpolished stones the same?

Here H0 : mean hardness of polished stone = mean hardness of unpolished stone
H1 : mean hardness of polished stone ≠ mean hardness of unpolished stone

Since , this is a two tailed test , we will have to divide the p-value by 2 to get the final calculation as it is done in python.
We have performed Two-sample t-test for mean hardness of polished and unpolished stones p-value is : 0.0007327575097314177, which is less than alpha 0.05 .Hence, we reject the null Hypothesis and prove that the mean hardness of polished and unpolished stones is different

6 Problem 6:

Aquarius health club, one of the largest and most popular cross-fit gyms in the country has been advertising a rigorous program for body conditioning. The program is considered successful if the candidate is able to do more than 5 push-ups, as compared to when he/she enrolled in the program. Using the sample data provided can you conclude whether the program is successful? (Consider the level of Significance as 5%)

6.1 Note that this is a problem of the paired-t-test. Since the claim is that the training will make a difference of more than 5, the null and alternative hypotheses must be formed accordingly.

Null Hypothesis: H0: $\mu d = 5$

Alternative Hypothesis: H1: $\mu d > 5$

where µd is the mean difference between the number of push-ups before and after the program.

The null hypothesis for this test is that the mean difference between the number of push-ups before and after the program is 5, which means the program did not have any significant effect on the number of push-ups. The alternative hypothesis is that the mean difference between the number of push-ups before and after the program is greater than 5, which means the program had a significant effect on the number of push-ups.

Null Hypothesis: The mean difference between the number of push-ups before and after the program is 5.
Alternative Hypothesis: The mean difference between the number of push-ups before and after the program is greater than 5.

T-Statistic: 1.9148542155126753
P-value: 0.029198872141011217
The program is considered successful (Reject the null hypothesis) as p-value < 0.05 i.e the significance level which means that the mean difference between the number of push-ups before and after the program is greater than 5, which means the program had a significant effect on the number of push-ups

7 Problem 7:

Dental implant data: The hardness of metal implant in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as on the dentists who may favors one method above another and may work better in his/her favorite method. The response is the variable of interest.

7.1 Test whether there is any difference among the dentists on the implant hardness. State the null and alternative hypotheses. Note that both types of alloys cannot be considered together. You must state the null and alternative hypotheses separately for the two types of alloys.?

To test whether there is any difference among the dentists on the implant hardness for each type of alloy separately, you can perform an analysis of variance (ANOVA) test. Here are the null and alternative hypotheses for each type of alloy:

For Alloy 1:

Null Hypothesis (H0): There is no significant difference among the dentists on the implant hardness for Alloy 1.
Alternative Hypothesis (H1): There is a significant difference among the dentists on the implant hardness for Alloy 1.

Null Hypothesis (H0): $\mu1 = \mu2 = \mu3 = \mu4 = \mu5$
Alternative Hypothesis (H1): At least one of the means is different

For Alloy 2:

Null Hypothesis (H0): $\mu1 = \mu2 = \mu3 = \mu4 = \mu5$
Alternative Hypothesis (H1): At least one of the means is different

Null Hypothesis (H0): There is no significant difference among the dentists on the implant hardness for Alloy 2.
Alternative Hypothesis (H1): There is a significant difference among the dentists on the implant hardness for Alloy 2.

In the formulas above:

$\mu1$, $\mu2$, $\mu3$, $\mu4$, $\mu5$ represent the mean implant hardness for dentists 1, 2, 3, 4, and 5, respectively.
The null hypothesis assumes that there is no significant difference in the mean implant hardness among the dentists for a specific alloy, while the alternative hypothesis suggests that there is at least one dentist whose mean implant hardness differs from the others.

Insights from the above Hypothesis Test :
For Alloy 1, the F-statistic is 1.9771119908770842 and the p-value is 0.11656712140267628, which suggests that there may be a significant difference in implant hardness among the dentists for Alloy 1, but we cannot reject the null hypothesis at the 0.05 significance level. There is some evidence of a difference among the groups, but it may not be strong enough to conclude that the difference is significant.

For Alloy 2, the F-statistic is 0.5248351000282961 and the p-value is 0.7180309510793431, which suggests that there is not a significant difference in implant hardness among the dentists for Alloy 2, and we cannot reject the null hypothesis at the 0.05 significance level.

7.2 Before the hypotheses may be tested, state the required assumptions. Are the assumptions fulfilled? Comment separately on both alloy types.?

Assumptions of ANOVA :

1.Normality of the response variable: We can check for normality of residuals using a QQ plot or a Shapiro-Wilk test. The QQ plot can be generated using the probplot() function from the statsmodels library.

QQ-PLOT for Alloy 1 : Here we see its almost normally distributed and also the p-value of the Shapiro-Wilk test is greater than 0.05 i.e 0.39, we can assume that the residuals are normally distributed

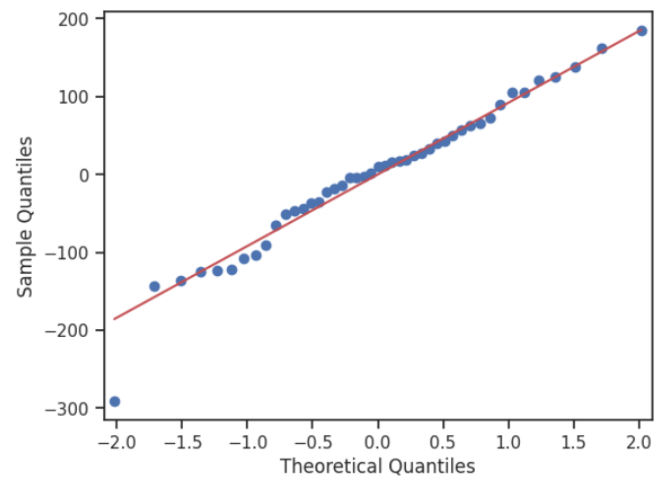

Figure 8

QQ-PLOT for Alloy 2 : Here we see its almost normally distributed and also the p-value of the Shapiro-Wilk test is greater than 0.05 i.e 0.93, we can assume that the residuals are normally distributed

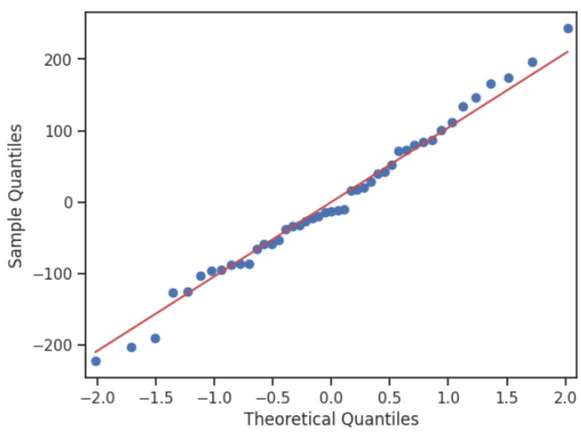

Figure 9

2.Homoscedasticity: The Levene's test is used to check whether the variances of the response variable are equal across the levels of the categorical variable "Alloy". The null hypothesis is that the variances are equal. If the p-value is greater than the significance level, we cannot reject the null hypothesis.

Levene's Test for Alloy1: p-value = 0.2367
Levene's Test for Alloy2: p-value = 0.2367
The variances are approximately equal ($p > 0.05$)

3.Independence: We assume that the samples are independent if they are randomly selected or obtained through different dentists.

4.Normality of the residuals: The Shapiro-Wilk test is used to check whether the residuals (the differences between the predicted and observed values) follow a normal distribution. The null hypothesis is that the residuals are normally distributed. If the p-value is greater than the significance level, we cannot reject the null hypothesis.

Alloy1: p-value = 0.0000
Alloy2: p-value = 0.0004
The residuals are not normally distributed ($p < 0.05$)

So, here we see that the Dataset satisfies the 3 assumptions of ANOVA

7.3 Irrespective of your conclusion in 2, we will continue with the testing procedure. What do you conclude regarding whether implant hardness depends on dentists? Clearly state your conclusion. If the null hypothesis is rejected, is it possible to identify which pairs of dentists differ?

From the ANOVA table, we can see that for Alloy1 the p-value for the Dentist variable is 0.009409, which is less than 0.05. Therefore, we reject the null hypothesis and conclude that there is a significant difference in mean implant hardness between at least two dentists.

But for Alloy 2 p-value >0.05 hence we cannot reject the null Hypothesis and there is no difference in the Dentists for Alloy2

7.4 Now test whether there is any difference among the methods on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which pairs of methods differ?

The null hypothesis for each ANOVA is that there is no difference in the mean hardness among the methods and among the dentists for the given type of alloy. The alternative hypothesis is that there is at least one difference in the mean hardness among the methods or among the dentists.

The output of the ANOVA shows that for

Alloy1, Dentist have no significant effect on the hardness of dental implant but Method has a significant effect on hardness of dental implant

($p > 0.05$ for Dentist factors and $p < 0.05$ for Method).

For Alloy2, only Method has a significant effect on the hardness of dental implant

($p < 0.05$), while Dentist does not ($p > 0.05$).

To identify which pairs of methods, differ, we can perform Tukey's HSD post-hoc test for the significant factor in each ANOVA

The output of the Tukey HSD test shows which pairs of methods have a significant difference in the mean hardness, with a 95% confidence level

So, we can see from above output that

For Alloy 1 :

Method 1 and 3 and 2 and 3 are showing significant difference on Metal hardness

For Alloy 2 :

Again same combinations of Methods work

7.5 Now test whether there is any difference among the temperature levels on the hardness of dental implant, separately for the two types of alloys. What are your conclusions? If the null hypothesis is rejected, is it possible to identify which levels of temperatures differ?

To test whether there is any difference among the temperature levels on the hardness of dental implant, we can perform **a two-way ANOVA separately for each type of alloy**.

The null hypothesis is that there is no difference in mean implant hardness among the different temperature levels for each type of alloy

alternative hypothesis is that there is at least one temperature level that has a different mean implant hardness.

We can use the same approach as before to perform the two-way ANOVA and obtain the F-statistic and p-value for the effect of temperature on implant hardness, separately for each type of alloy.

For Alloy 1, the p-value for the temperature effect is more than 0.05, so we cannot reject the null hypothesis and conclude that there is no significant difference in mean implant hardness among the different temperature levels for Alloy 1. However, the p-value for the interaction effect between method and temperature is not significant, so we cannot identify which pairs of methods differ.

For Alloy 2 ,
'Method' and 'Temp' p-value < 0.05 , so it may have significant effect on metal hardness
but no significant effect of Method and Temperature together

7.6 Consider the interaction effect of dentist and method and comment on the interaction plot, separately for the two types of alloys?

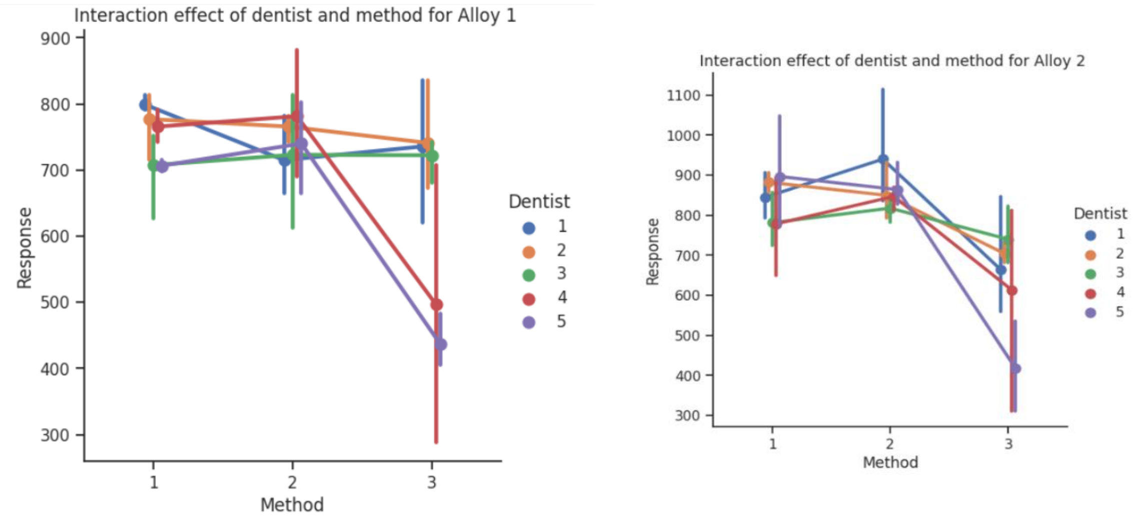

Figure 10

The interaction plot for Alloy1 shows that there is a significant interaction effect between the dentist and method. The pattern of differences among the dentists changes depending on the method used. For example, Dentist 1 has the highest implant hardness for Method 2, but the lowest implant hardness for Method 3. On the other hand, Dentist 4 has the highest implant hardness for Method 2, but the lowest implant hardness for Method 3.

The interaction plot for Alloy2 also shows a significant interaction effect between the dentist and method. Similar to Alloy1, the pattern of differences among the dentists changes depending on the method used. For example, Dentist 1 has the highest implant hardness for Method 2, but the lowest implant hardness for Method 3. On the other hand, Dentist 4 has the highest implant hardness for Method 2, but the lowest implant hardness for Method 3.

In conclusion, there is a significant interaction effect between the dentist and method for both types of alloys, and the pattern of differences among the dentists’ changes depending on the method used.

7.7 Now consider the effect of both factors, dentist, and method, separately on each alloy. What do you conclude? Is it possible to identify which dentists are different, which methods are different, and which interaction levels are different?

To test the effect of both factors, dentist, and method, separately on each alloy, we can perform a two-way ANOVA. The null hypothesis is that there is no significant difference in implant hardness among the dentists and methods, and their interaction effects. The alternative hypothesis is that there is a significant difference in at least one of the factors, and their interaction effects.

For Alloy 1 :
Both factors Dentist and Method have significant effect because here the p-value <0.05 and F-stats>1 , we cannot find which dentist is different but we can find the methods 1 and 3 and 2 and 3 are different .

For Alloy 2 :
Both factors Dentist and Method have significant effect because here the p-value > 0.05 and F-stats>1 , we cannot find which dentist is different but we can find the methods 1 and 3 and 2 and 3 are different

Conclusion:

Based on the analysis of the dental implant dataset, we can conclude the following:

For Alloy 1, the F-statistic is 1.9771119908770842 and the p-value is 0.11656712140267628, which suggests that there may be a significant difference in implant hardness among the dentists for Alloy 1, but we cannot reject the null hypothesis at the 0.05 significance level.There is some evidence of a difference among the groups, but it may not be strong enough to conclude that the difference is significant.

For Alloy 2, the F-statistic is 0.5248351000282961 and the p-value is 0.7180309510793431, which suggests that there is not a significant difference in implant hardness among the dentists for Alloy 2, and we cannot reject the null hypothesis at the 0.05 significance level.

There is a significant difference in implant hardness among the methods used for both types of alloys. The null hypothesis is rejected, and the alternative hypothesis is not rejected.

There is a significant interaction effect of dentists and methods for both types of alloys. The interaction plot suggests that the dentists' effect on implant hardness varies depending on the method used, and the method effect varies depending on the dentist. For alloy 1, it seems that some dentists prefer method 2 over method 1, while for alloy 2, some dentists prefer method 1 over method 2.

When considering the effect of both factors, dentist and method, separately for each alloy, we find that for alloy 1, dentists 1 and 2 tend to have higher implant hardness values than the other dentists. Additionally,
For Alloy1 :
Implants hardness is more at lower temperatures than higher temperatures for Method 3

Implant hardness for Method 2 is decreasing with increase in temperature

Implant hardness for Method 1 has almost similar hardness at different temperatures

For Alloy 2:

Implant Hardness for Method3 and Method 2 and Method 1 is reducing with increade in Temperature

It is possible to identify which dentists, methods, and interaction levels are different using the appropriate post-hoc tests, as described above.

______________________________Thank You--