Chapter 1

Questions to think about

1.

Chapter 2

Keywords

- 1. Double dimer model
- 2. Ising model
- 3. Hexahedron recurrence, octahedron recurrence
- 4. Pfaffian
- 5. Kasteleyn determinant
- 6. lattice permutations
- 7. phase transition

Chapter 3

Survey

3.1 T-systems, networks and dimers [DF14]

3.1.1 Excerpts

• In the case of type A, the T-system equation is also known as the octahedron recurrence, and appears to be central in a number of combinatorial objects, such as the lambda-determinant and the Alternating Sign Matrices [24][8], the puzzles for computing Littlewood-Richardson coefficients [20], generalizations of Coxeter-Conway frieze patterns [5][1][3], and the domino tilings of the Aztec diamond [12][25]

3.2 Q-systems as Cluster Algebras II: Cartan Matrix of Finite Type and the Polynomial Property [FK09]

A new interpretation for the T-system arose from realizing that the corresponding discrete evolution could be viewed as a particular mutation in a suitably defined cluster algebra

- 3.3 Arctic curves of the octahedron equation [DFSG14]
- 3.4 Double-dimers, the Ising model and the hexahedron recurrence [KP13]
- 3.5 Perfect Matchings and the Octahedron Recurrence [Spe04]
- 3.6 Uniformly positive correlations in the dimer model and phase transition in lattice permutations in \mathbb{Z}^d , d>2, via reflection positivity [Tag19]

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