Exercise 1

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Question 2

a) $p(\text{apple}) = 0.2 \cdot \frac{3}{3+4+3} + 0.6 \cdot \frac{3}{3+3+4} + 0.2 \cdot \frac{1}{1+1+0}$

$= 0.34$

b) $p(g \mid o) = \dfrac{p(o \mid g)\, p(g)}{P(o)} = \dfrac{\frac{3}{3+3+4} \cdot 0.6}{0.2 \cdot \frac{4}{10} + 0.6 \cdot \frac{3}{10} + 0.2 \cdot \frac{1}{2}} = 0.5$

Question 3

a) Minimize the expected Loss:

$$\mathbb{E}[L] = \sum_k \sum_j \int_{R_j} L_{kj}\, p(x, C_k)\, dx$$

$$= \sum_j \int_{R_j} \sum_k L_{kj}\, p(x, C_k)\, dx$$

$$= \sum_j \int_{R_j} \underbrace{\sum_k L_{kj}\, p(C_k \mid x)\, p(x)}_{\text{find the min}}\, dx$$

$\because p(x)$ is normalized factor

$$\Rightarrow \arg\min_j \left\{ \sum_k L_{kj}\, p(C_k \mid x)\, p(x) \right\} = \arg\min_j \left\{ \sum_k L_{kj}\, p(C_k \mid x) \right\}$$

According to " $p(C_j \mid x) \geq p(C_k \mid x) \Rightarrow j \neq k$

$$\Rightarrow \arg\min_j \left\{ l_s \sum_{k \neq j} p(C_k \mid x) \right\}$$

$\because l_s$ is constant $\Rightarrow \arg\min_j \left\{ \sum_{k \neq j} p(C_k \mid x) \right\}$

$$= \arg\min_j \left\{ 1 - p(C_j \mid x) \right\}$$

$$= \underset{j}{\operatorname{argmax}} \{ p(C_j | x) \}$$

"when the rejection risk is lower than the risk of assignment to each class $C_k \in C$, ..."

$$l_s \sum_{k \neq j} p(C_k | x) \leq l_r$$

$$\Rightarrow \quad l_s (1 - p(C_j | x)) \leq l_r$$

$$\Rightarrow \quad 1 - p(C_j | x) \leq l_r / l_s$$

$$\Rightarrow \quad p(C_j | x) \geq 1 - \frac{l_r}{l_s}$$

b) when $l_r = 0$ $\quad p(C_j | x) \geq 1$

unless $p(C_i | x) = 1$ alway reject

c) when $l_r > l_s$ $\quad p(C_j | x) \geq$ negative

never reject

Question 4

$$p(x \mid \theta) = \theta^2 x \exp(-\theta x) g(x) \quad \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

Assumption: $L(\theta) = p(X \mid \theta) = \prod_{n=1}^{N} p(x_n \mid \theta)$

Log-likelihood: $E(\theta) = \ln L(\theta) = \ln \left(\prod_{n=1}^{N} p(x_n \mid \theta) \right) = \sum_{n=1}^{N} \ln p(x_n \mid \theta)$

Maximize Log-likelihood:

$$\frac{\partial}{\partial \theta} E(\theta) = \frac{\partial}{\partial \theta} \sum_{n=1}^{N} \ln p(x_n \mid \theta) = \sum_{n=1}^{N} \frac{\frac{\partial}{\partial \theta} p(x_n \mid \theta)}{p(x_n \mid \theta)} \stackrel{!}{=} 0$$

$$\Rightarrow \frac{\partial}{\partial \theta} E(\theta) = \sum_{n=1}^{N} \frac{\frac{\partial}{\partial \theta} p(x_n \mid \theta)}{p(x_n \mid \theta)}$$

$$= \sum_{n=1}^{N} \frac{x_n g(x_n) \cdot [2\theta \exp(-\theta x_n) - \theta^2 x_n \exp(-\theta x_n)]}{\theta^2 x_n \exp(-\theta x_n) g(x_n)}$$

$$= \sum_{n=1}^{N} \frac{\exp(-\theta x_n)(2\theta - \theta^2 x_n)}{\theta^2 \exp(-\theta x_n)}$$

$$= \sum_{n=1}^{N} \left(\frac{2}{\theta} - x_n \right)$$

$$= \frac{2N}{\theta} - \sum_{n=1}^{N} x_n \stackrel{!}{=} 0$$

$$\Rightarrow \hat{\theta} = \frac{2N}{\sum_{n=1}^{N} x_n}$$