Programming Task: Interactive Quadric Surfaces in $\mathbb{R}^3$

Design an **interactive Python program** (and corresponding `.exe`) that visualizes and classifies all standard **quadric surfaces**. The user must be able to select the surface type, specify orientation, input parameters, and view the result in a fully interactive 3D window (mouse-controlled rotation and zoom).

General Requirements

- The program must run both as a Python script and as a Windows executable.
- Provide a clear, menu-driven or button-based interface.
- Validate all numeric inputs ($a, b, c > 0$).
- Use Matplotlib's 3D plotting environment.
- Include buttons for **Randomize**, **Plot**, and **Clear**.
- Display the equation, parameters, and qualitative description in an **Analysis Results** panel or console block.

Mode Selection

At startup, display:

```
Choose surface type:
[1] Ellipsoid
[2] Elliptic Cone
[3] Hyperboloid of One Sheet
[4] Hyperboloid of Two Sheets
[5] Elliptic Paraboloid
[6] Hyperbolic Paraboloid
[7] Cylinders (Elliptic / Hyperbolic / Parabolic)
```

Orientation Selection

For all surfaces *except the ellipsoid*, the user must specify the direction of the symmetry axis.

```
Choose orientation (axis of symmetry):
[1] Along z-axis
[2] Along y-axis
[3] Along x-axis
```

This step is skipped automatically for the ellipsoid, since its equation

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} + \frac{(z-l)^2}{c^2} = 1$$

is symmetric in all coordinate directions once a, b, c are chosen.

Input Parameters

Prompt the user for:

$$a > 0, \quad b > 0, \quad c > 0 \text{ (if applicable)}, \quad \text{center } (h, k, l).$$

Then request visible coordinate ranges:

$$x_{\min}, x_{\max}, \quad y_{\min}, y_{\max}, \quad z_{\min}, z_{\max}.$$

Default range: $[-10, 10]$ for all axes.

Standard Equations by Type and Orientation

1. Ellipsoid

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} + \frac{(z-l)^2}{c^2} = 1.$$

2. Elliptic Cone

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - \frac{(z-l)^2}{c^2} = 0.$$

Orientation variants:

$$\begin{aligned} \text{Along } z: \ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0, \\ \text{Along } y: \ \frac{x^2}{a^2} + \frac{z^2}{c^2} - \frac{y^2}{b^2} = 0, \\ \text{Along } x: \ \frac{y^2}{b^2} + \frac{z^2}{c^2} - \frac{x^2}{a^2} = 0. \end{aligned}$$

3. Hyperboloid of One Sheet

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} - \frac{(z-l)^2}{c^2} = 1.$$

Orientation variants:

$$\begin{aligned} \text{Along } z: \quad & \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \\ \text{Along } y: \quad & \frac{x^2}{a^2} + \frac{z^2}{c^2} - \frac{y^2}{b^2} = 1, \\ \text{Along } x: \quad & \frac{y^2}{b^2} + \frac{z^2}{c^2} - \frac{x^2}{a^2} = 1. \end{aligned}$$

4. Hyperboloid of Two Sheets

$$-\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} + \frac{(z-l)^2}{c^2} = 1.$$

Orientation variants:

$$\begin{aligned}
\text{Along } z: &\quad -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \\
\text{Along } y: &\quad -\frac{x^2}{a^2} - \frac{z^2}{c^2} + \frac{y^2}{b^2} = 1, \\
\text{Along } x: &\quad -\frac{y^2}{b^2} - \frac{z^2}{c^2} + \frac{x^2}{a^2} = 1.
\end{aligned}$$

5. Elliptic Paraboloid

$$\begin{aligned}
\text{Opens along } z: &\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = z, \\
\text{Opens along } y: &\ \frac{x^2}{a^2} + \frac{z^2}{c^2} = y, \\
\text{Opens along } x: &\ \frac{y^2}{b^2} + \frac{z^2}{c^2} = x.
\end{aligned}$$

6. Hyperbolic Paraboloid

$$\begin{aligned}
\text{Opens along } z: &\ \frac{y^2}{b^2} - \frac{x^2}{a^2} = z, \\
\text{Opens along } y: &\ \frac{z^2}{c^2} - \frac{x^2}{a^2} = y, \\
\text{Opens along } x: &\ \frac{z^2}{c^2} - \frac{y^2}{b^2} = x.
\end{aligned}$$

7. Cylinders

$$\begin{aligned}
\text{Elliptic:} &\quad \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \\
\text{Hyperbolic:} &\quad \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, \\
\text{Parabolic:} &\quad (y-k)^2 = 4p(x-h).
\end{aligned}$$

Output Information

- Selected surface type and (if applicable) orientation.
- Equation in canonical form (with substituted parameters).
- Intercepts with coordinate axes (if any).
- Automatically generated qualitative description (open/closed, single/two-sheeted, etc.).

Visualization Rules

- Use Matplotlib's 3D plotting environment:

```
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(X, Y, Z, rstride=1, cstride=1, cmap='viridis', alpha=0.7)
```

- Enable full interactive control: rotation and zoom with mouse.
- Plot coordinate axes and mark the surface center.
- **Traces:** A *trace* (or *cross-section*) is the curve obtained by intersecting the surface with a coordinate plane such as xy, xz, or yz. Each trace should be drawn in a **different color** (for example: red for the xy-plane, blue for xz, and green for yz) so that the student can see how the surface cuts through these planes.
- Optionally add a colorbar for height.

Randomizer, Range, and Clear Options

- **User-defined Ranges:** default $[-10, 10]$ for all axes.
- **Randomize:** generates random $a, b, c \in [1, 10]$.
- **Clear:** resets parameters and removes plots.
- **Auto-fit (optional):** adjusts axis limits automatically.

Example Output (Console Simulation)

```
--- Analysis Results ---
Mode: Elliptic Cylinder
Parameters: a=2, b=1
Range: x in [-5,5], y in [-5,5], z in [-5,5]

Equation: (x/2)^2 + (y/1)^2 = 1
Description: open, infinite along z-axis
```

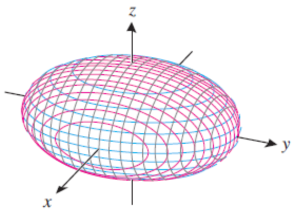
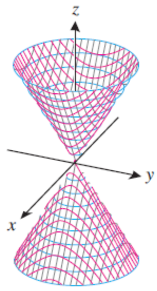
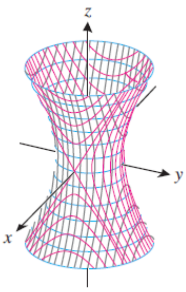
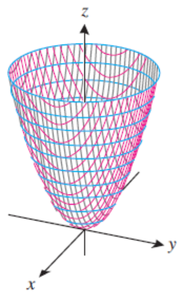
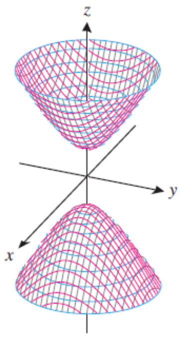
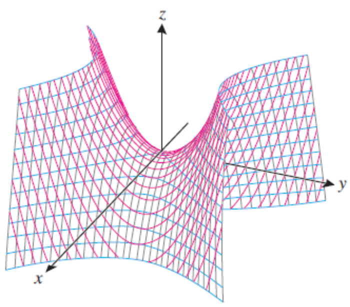
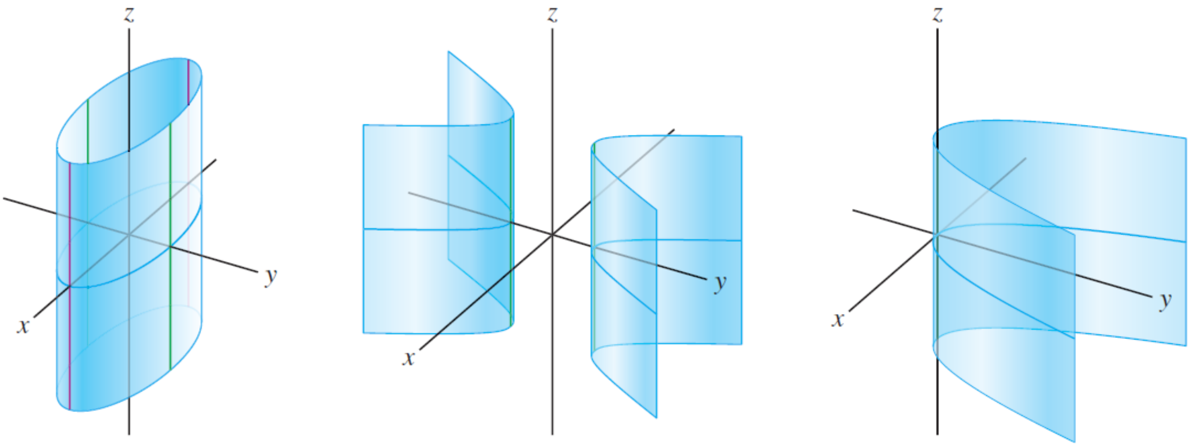
SURFACE	EQUATION	SURFACE	EQUATION
ELLIPSOID 	$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ The traces in the coordinate planes are ellipses, as are the traces in those planes that are parallel to the coordinate planes and intersect the surface in more than one point.	ELLIPTIC CONE 	$$z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$ The trace in the xy-plane is a point (the origin), and the traces in planes parallel to the xy-plane are ellipses. The traces in the yz- and xz-planes are pairs of lines intersecting at the origin. The traces in planes parallel to these are hyperbolas.
HYPERBOLOID OF ONE SHEET 	$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$ The trace in the xy-plane is an ellipse, as are the traces in planes parallel to the xy-plane. The traces in the yz-plane and xz-plane are hyperbolas, as are the traces in those planes that are parallel to these and do not pass through the x- or y-intercepts. At these intercepts the traces are pairs of intersecting lines.	ELLIPTIC PARABOLOID 	$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$ The trace in the xy-plane is a point (the origin), and the traces in planes parallel to and above the xy-plane are ellipses. The traces in the yz- and xz-planes are parabolas, as are the traces in planes parallel to these.
HYPERBOLOID OF TWO SHEETS 	$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ There is no trace in the xy-plane. In planes parallel to the xy-plane that intersect the surface in more than one point the traces are ellipses. In the yz- and xz-planes, the traces are hyperbolas, as are the traces in those planes that are parallel to these.	HYPERBOLIC PARABOLOID 	$$z = \frac{y^2}{b^2} - \frac{x^2}{a^2}$$ The trace in the xy-plane is a pair of lines intersecting at the origin. The traces in planes parallel to the xy-plane are hyperbolas. The hyperbolas above the xy-plane open in the y-direction, and those below in the x-direction. The traces in the yz- and xz-planes are parabolas, as are the traces in planes parallel to these.

Figure 1:



$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$
Elliptic cylinder

$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$
Hyperbolic cylinder

$$y = ax^2$$
Parabolic cylinder

Figure 2: