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Particle Swarm Optimization applied to the design of water supply systems[☆]

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Abstract

In the past decade, evolutionary methods have been used by various researchers to tackle optimal design problems for water supply systems (WSS). Particle Swarm Optimization (PSO) is one of these evolutionary algorithms which, in spite of the fact that it has primarily been developed for the solution of optimization problems with continuous variables, has been successfully adapted in other contexts to problems with discrete variables.

In this work we have applied one of the variants of this algorithm to two case studies: the Hanoi water distribution network and the New York City water supply tunnel system. Both cases occur frequently in the related literature and provide two standard networks for benchmarking studies. This allows us to present a detailed comparison of our new results with those previously obtained by other authors.

Keywords: Water supply systems; Optimal design; NP-hard problem; Evolutionary method; Particle Swarm Optimization

1. Introduction

The high cost of water supply systems has motivated a great and intense effort to obtain more economically viable systems of this kind, which must still guarantee a satisfactory delivery of water in the amount, and of the quality, required by all the connected users.

The problem of design optimization for new water supply systems is defined by the best possible combination of reducing costs for its components, given design constraints, and such that all water demands are met even during the occurrence of particular system failures. In practice, this optimization can take numerous forms, depending on the various kinds of components which comprise a water supply system, the diverse criteria for correct functioning and the design constraints of such a network.

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Even though the design of new systems is important in itself, in practice, situations in which the interest focuses on the expansion, the renovation or even just the optimum operation of existing systems are much more frequent.

A general strategy to solve such optimization problems of water supply systems can be defined in terms of a balanced combination of: least cost for the layout and size using new components; the reuse or substitution of existing components; and a working system configuration which fulfills all the water demands and the design constraints, whilst guaranteeing, of course, a certain degree of reliability [1,2].

As a consequence, the objective function which will enter the optimization of water supply systems can take numerous forms depending on the intended design and type (system enlargement, rehabilitation, new design, operation). Obviously, there exists no unique set of such main factors, even for various approaches to one and the same specific problem. This implies that the most effective techniques of such optimization have to adapt themselves easily to whatever objective function is used.

Also, in comparison to other optimization problems, the constraints which have to be taken into account for design problems of water supply systems are of a distinct nature: the feasibility of the solutions can only be assessed after their full construction, and consequently, the constraints cannot explicitly be taken into account in the construction of the solution.

For these reasons, for the last decade, many researchers in the field have changed direction, leaving aside traditional optimization techniques based on linear and nonlinear programming and embarked on the implementation of Evolutionary Algorithms: Genetic Algorithms [3–6]; Ant Colony Optimization [7,8]; Simulated Annealing [9]; Shuffled Complex Evolution [10]; and Harmony Search [11], among others.

Among all the advantages which have spurred the popularity and usage of evolutionary algorithms in the optimized design of water supply systems, we emphasize the following:

(1) It is easier to deal with discrete problems, which permits, opposed to other optimization methods, the direct inclusion of commercial pipe diameters in the design.
(2) They only employ the information given by the objective function and thus avoid complications associated with the determination of derivatives and other auxiliary functions.
(3) They are global optimization procedures in the sense that they can adapt themselves easily to any given objective function.
(4) Due to the fact that they allow for a family of solutions, one can obtain various optimal solutions or solutions close to the optimal case.
(5) Within the process of optimal design, one can readily include an analysis for systems with different work loads.

As a result of the iterative nature of the generation of the solutions using the aforementioned algorithms, these can be intuitively interpreted as algorithms which continually search through the solution space. This process takes full advantage of all solutions found up to the moment and helps to guide the search. Evolutionary Algorithms and their search techniques are characterized by two fundamental ingredients [12]:

- Exploration, which is the capability of an algorithm to pursue a broad search within the solution space.
- Exploitation, which is the capability of an algorithm to search more specifically in a local subset of the solution space, close to where previously good solutions have been found.

One of the evolutionary algorithms which has shown its potential and good aspects for the solution of various optimization problems [13,24,23] is Particle Swarm Optimization (PSO). The PSO algorithm was developed by Kennedy and Eberhart [14] and inspired by the social behavior of a group of migrating birds trying to reach an unknown destination. This algorithm, with certain modifications, is used in the present work to find solutions for the optimal design of water supply systems.

In the following we will give a description of the optimal design problem for a water supply system, next an explanation of the algorithm, and finally the results of its application to two standard benchmark tests in the field, including a comparison with the results obtained by other authors.

2. Problem description and optimal design

The problem of economically and optimally designing a water supply system amounts to determining the values of all the variables involved in such a way that the investment and maintenance costs of the system are minimal; apart from other possible constraints which have to be taken into account [15].

The expenditures to be made for a water supply system are:

- Costs of the conduction elements
- Costs of the storage elements
- Costs of the pumping elements
- Costs of the elements for regulation, measurement and control
- Costs for energy and operation
- Costs for maintenance.

Furthermore, there are additional constraints which have to be considered during the design process:

- Geometrical constraints
- Constraints for maximal and minimal pressure in each node
- Constraints for the maximal and minimal fluid velocities in the pipeline system
- Compliance with the continuity equations
- Compliance with energy conservation
- Compliance with certain reliability criteria.

Apart from the basic variables of the problem, which are the diameters of the new pipes, one may require additional variables that depend on the design characteristics of the system: storage volumes, pump head, the type of renovation to be carried out for various parts of the network, *etc*. The estimation of individual costs will always depend on these variables. The correct approach to assess the costs for each element becomes important when defining the objective function, which has to be fully adapted to the problem under consideration: design, enlargement, renovation, operation design, *etc*. On the other hand, it is important that the objective function reflects with the utmost reliability the total cost of the system during its entire lifetime. Various authors have used, in their optimization, an objective function which only considers the costs of the pipelines [7,8] (new and/or additional, duplicated pipelines), and others have taken into account other different costs involved [5,16]. One very interesting approach to the objective function is presented by Martínez [17].

One should not lose out of sight the mathematical constraints when resolving optimization problems. In the case of evolutionary algorithms, which is the topic of this article, the constraints are included as penalty costs in the given objective function, such that the violation of one of the imposed constraints provokes an increase in the value of the objective function. This increase is sufficiently high to render the corresponding solutions unacceptable.

In this work, due to the nature of the two examples we have chosen, one uses an objective function which only takes into account the pipeline costs and penalty costs, if not complying with minimal values for the pressure at each node of the network. Nevertheless, a generalization to a broader class of objective functions is straightforward. The examples we use have traditionally been employed in the literature and provide a standardized and simplified environment to carry out a wide range of tests and analyses. Hence, in order to facilitate the comparison with results obtained by other authors, we employ the following objective function to estimate the costs:

$$F_{\text{obj}} = \sum_{i=1}^{\#\text{pipes}} c_i \, l_i + P, \tag{1}$$

where one sums over all individual pipes carrying index i. The costs per meter depending on the diameter of pipe i is given by c_i and its corresponding length by l_i. The penalty P only applies when the pressure in any node is less than a predetermined minimal value. For nodes with pressures larger than this minimal value, the associated individual penalties vanish, and one uses the usual Heaviside step function H in the explicit expression for P:

$$P = \sum_{i=1}^{\#\text{nodes}} H\left(p_{\min} - p_i\right) \cdot p \cdot \left(p_{\min} - p_i\right). \tag{2}$$

The factor p which multiplies with the pressure difference $\Delta p_i = p_{\min} - p_i$ represents a fixed value which becomes effective whenever the minimal pressure requirement is not met. Note that in this model the individual penalties grow linearly with Δp_i.

The variables in the problem are the diameters pertaining to the new pipes of the network or those of the additional, duplicated pipelines. One therefore deals with determining the values which minimize the total cost of the pipelines while complying with the minimal pressure requirements of the network.

Furthermore, this simple variant for the design of a water supply system forms an NP-complete problem; the solution space is so large that in practice the analysis of all possibilities is not feasible due to the huge amount of computational time required.

3. The PSO algorithm

Within the PSO algorithm, every solution is a bird of the flock and is referred to as a particle: in this framework the birds, besides having individual intelligence, also develop some social behavior and coordinate their movement towards a destination [18].

Initially, the process starts from a swarm of particles, in which each of them contains a solution to the hydraulic problem that is generated randomly, and then one searches the optimal solution by iteration. The i-th particle is associated with a position in an s-dimensional space, where s is the number of variables involved in the problem; the values of the s variables which determine the position of the particle represent a possible solution of the optimization problem. Each particle i is completely determined by three vectors: its current position X_i, its best position reached in previous cycles Y_i, and its velocity V_i:

$$\text{current position } X_i = (x_{i1}, x_{i2}, \ldots, x_{is}) \tag{3}$$

$$\text{best previous position } Y_i = (y_{i1}, y_{i2}, \ldots, y_{is}) \tag{4}$$

$$\text{flight velocity } V_i = (v_{i1}, v_{i2}, \ldots, v_{is})\,. \tag{5}$$

This algorithm simulates a flock of birds which communicate during flight. Each bird looks at a specific direction (its best ever attained position Y_i), and later, when they communicate among themselves, the bird which is in the best position is identified. With coordination, each bird moves also towards the best bird using a velocity which depends on its present velocity. Thus, each bird examines the search space from its current local position, and this process repeats until the bird possibly reaches the desired position. Note that this process involves as much individual intelligence as social interactivity; the birds learn through their own experience (local search) and the experience of their peers (global search).

In each cycle, one identifies the particle which has the best instantaneous solution to the problem; the position of this particle subsequently enters into the computation of the new position for each of the particles in the flock. This calculation is carried out according to

$$X_i' = X_i + V_i', \tag{6}$$

where the primes denote new values for the variables, and the new velocity is given by

$$V_i' = \omega V_i + c_1 \operatorname{rand}()\,(Y_i - X_i) + c_2 \operatorname{rand}()\,\left(Y^* - X_i\right). \tag{7}$$

Here, c_1 and c_2 are two positive constants which are called *learning factors or rates*; rand( ) represents a function which creates random numbers between 0 and 1 (two independent random numbers enter Eq. (7)); ω is a factor of inertia suggested by Shi and Eberhart [18] in order to control the impact which the histories of velocities has on the current velocity. The ω factor may vary from one cycle to the next. As it controls the balance between global and local search, it was suggested to have it decrease linearly with time, usually in a way to first emphasize global search and then, with each cycle of the iteration, to prioritize local search. Finally, Y^* is the best present solution of all Y_i.

The particles propagate through the solution space and are influenced by the best solution which was previously found individually, as well as the best particle of the entire swarm [19]. Thus, in Eq. (7), the second term represents the cognition or intrinsic knowledge of particle i, since it compares its current position X_i with its best previous position Y_i. The third term in this equation represents the social collaboration between the particles: it measures the difference between the current position X_i and the best solution of the entire system found up to the moment Y^*.

To control any change in the particle velocities, we introduce the respective upper and lower limits:

$$V_{\inf} \le V_i \le V_{\sup} \quad \forall i. \tag{8}$$

Once the current position is calculated, the particle directs itself towards a new position.

A schematic representation of the algorithm is shown below:

- Generate a family of N random solutions (particles).
- Search for the best particle.
- REPEAT the following block until the condition terminates:
  - ∘ Calculate the value of ω given by Eq. (11).
  - ∘ LOOP $i = 1, \ldots, N$:
    - ⋄ BEGIN
      Calculate the value of the objective function for particle i.
      IF particle i gives a better value for the objective function, let particle i be the best particle.
      Calculate the new velocity for particle i using Eq. (7).
      Calculate the new position for particle i using Eq. (6).
    - ⋄ END
- RETURN the solution of the best particle.

The termination condition for the algorithm is when either the maximum number of iterations is reached or the objective function assumes a preset value [20]. In this work, the algorithm searches for a solution until a certain number of iterations is reached without any further improvement of the best cost obtained.

The previously described algorithm can be considered as the standard PSO algorithm, which is applicable to continuous systems and cannot be used for discrete problems. Various approaches have been put forward to tackle discrete problems with PSO [21–23,20].

Essentially, this algorithm only takes integer parts of the flying velocity vector components into account; hence the new velocities V_i' are integer and consequently the new position vector components will also be integer (since the initial position vectors were generated with integer values).

According to this scheme, Eq. (7) turns out to be:

$$V_i' = \text{fix}\left(\omega V_i + c_1 \,\text{rand}()\,(Y_i - X_i) + c_2 \,\text{rand}()\left(Y^* - X_i\right)\right), \tag{9}$$

where fix() implies that we only take the integer part of the result. Furthermore, one has to take into account that the current velocity cannot pass the limits established by Eq. (8). In case $V_i' > V_{\text{sup}}$ we redefine $V_i' = V_{\text{sup}}$; and for $V_i' < V_{\text{inf}}$ we set $V_i' = V_{\text{inf}}$ for each iteration i.

The particle position is computed by Eq. (6), following exactly the standard procedure of PSO. If the new position $X_i' < 0$, we redefine $X_i' = 0$. If for the case of discrete problems it should be $X_i' > X_{\text{sup}}$, where X_{sup} is a predefined maximum position, then in full analogy to the velocity case we set $X_i' = X_{\text{sup}}$. All the remaining issues related to the standard PSO algorithm remain unchanged.

Note that for a hydraulic problem the most convenient selection of the pipe diameters poses additional constraints on the pressures which should exist in the individual nodes of the network. The PSO algorithm – like all evolutionary algorithms – cannot directly handle the constraints of the optimization problem. As described in the previous section, the standard technique to convert the constrained system into a free system uses an appropriate penalty function P in Eq. (1). To direct the search away from the excluded domains towards the allowed regions, the penalty function increases correspondingly to exclude any unwanted solutions.

4. Applications of the PSO algorithm

In the following, we present two case studies to illustrate our method. Some of the parameters we use for the PSO algorithm have been adapted from previous work by other authors [23,24,20] in different contexts. For applications of discrete PSO to the design of water supply systems, however, no references in the research literature seem to exist. Adopting the notation of the previous sections, we will employ the following parameters to carry out our computations:

$$c_1 = 3, \qquad c_2 = 2, \tag{10}$$

$$\omega = \frac{1}{2}\left[1 + \frac{1}{\ln k + 1}\right] \quad (k \text{ is the iteration number}), \tag{11}$$

$$V_{\text{sup}} = 50\% \text{ of the total variable range}, \tag{12}$$

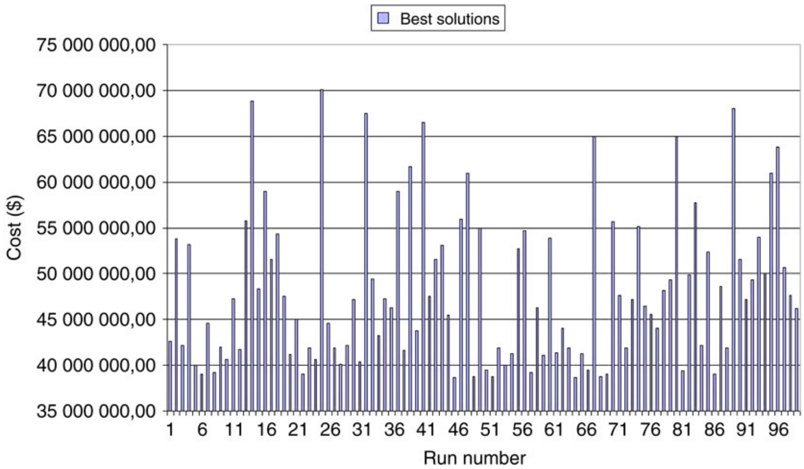

Fig. 1. Representation of the best solution for each of the 100 runs in the Hanoi water distribution problem.

Table 1
Results for the design of the Hanoi network by various researchers

Cost $\times 10^6$ \$	6093	6182	6195	6367	6133
Reference	Matías 2003 Ref. [5]	Wu et al. 2001 Ref. [4]	Savic and Walters 1995 Ref. [3]	Zecchin et al. 2005 Ref. [8]	The present work
Method	Genetic algorithms	Genetic algorithms	Genetic algorithms	$AS_{\text{i-best}}$ [a]	PSO

[a] $AS_{\text{i-best}}$ is and algorithm based on Ant Colony Optimization (ACO) but which adopts a different scheme for the evaluation of pheromones.

$$V_{\text{inf}} = -V_{\text{sup}}, \tag{13}$$

$$N = 100 \quad \text{(number of particles)}. \tag{14}$$

As the termination condition of the algorithm we used a limit of 800 iterations if there was no further improvement in the solution of least cost.

4.1. The Hanoi water distribution network

The Hanoi water distribution problem has been attacked many times before in the literature [3,9,5,25,8,4], and to gauge the effectiveness of our proposed algorithm, we will apply it to this same problem.

The network consists of one fixed head source, 34 pipes and 31 demand nodes subject to a load condition. Furthermore, the network has three grids and two ramified branches. One has to find the diameters for the 34 pipes such that the total cost of this network is minimal and the pressure at each node of consumption is at least 30 m.

For this problem 100 executions of the full algorithm were performed. The costs of the best solution for each run appear in Fig. 1. Furthermore, Table 1 displays the cost of our best solution obtained compared with the results from other authors.

4.2. The New York city water supply tunnel network

Similarly to the Hanoi water distribution problem, the New York water supply network problem has been studied extensively by various researchers [7,5,26,3]. It is a network which has one reservoir, 21 pipes and 19 demand nodes subject to a load condition. For further details of the net layout we refer to Ref. [26].

This network had problems in the fulfilling the minimal-pressure requirement in nodes 16, 17, 18, 19 and 20. With hindsight, to at least reach a minimal value for the pressure, one has to modify the network by duplicating some of the already existing tunnels. Thus, the problem which has to be solved consists in determining the tunnels which have to be duplicated and finding the diameter for each new tunnel.

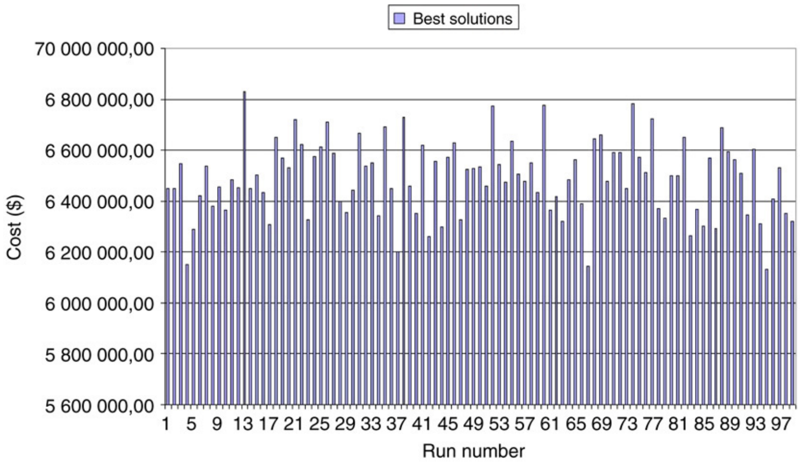

Fig. 2. Representation of the best solution for each of the 100 runs in the New York water supply problem.

Table 2
Results for the design of the New York network by various researchers

Cost × 10^6 $	38.64	38.8	38.64	40.42	38.64
Reference	Matías 2003 Ref. [5]	Dandy et al. 1996 Ref. [26]	Maier et al. 2003 Ref. [7]	Savic and Walters 1997 Ref. [3]	The present work
Method	Genetic algorithms	Genetic algorithms	ACO	Genetic algorithms	PSO

The algorithm was executed 100 times. Table 2 displays the total cost of the best solution we obtained in comparison with results from other authors. Fig. 2 details the costs of the best solution for each run.

Observe the reduced number of iterations which were necessary to obtain results with our algorithm applied to the New York network. The best solution found by this algorithm is obtained only after 24 iterations, and the costs given in Fig. 2 are all found with less than 40 iterations. Other authors mention for approximately the same numerical estimates a much higher number of required iterations: Maier et al. [7] used 13 928 iterations; Matías [5] found a solution in a run with 300 iterations.

5. Conclusions and outlook

The PSO algorithm has been subject to many different modifications in order to adapt to problems with discrete variables. The approach which was presented in this work has shown satisfactory performance when applied to water supply systems, especially for the optimal design of the networks provided by the Hanoi and New York City water distribution problems. In fact, the solutions we obtained with our method are among the best results compared to the published results of the same benchmark examples, but required a significantly lower number of iterations for its approximation.

Although the algorithm parameters have been used without any changes from previously published sources by other authors, and for different problems, all our case studies of the PSO method have given feasible solutions for the water supply problems under consideration. Presumably, a more elaborate analysis of the parameters for this case with some fine-tuning will deliver more improvements. Such an analysis of the PSO algorithm for the problem at hand would help to obtain better solutions in a more general framework and speed up the convergence of the procedure.

The application of PSO to problems related to the design of hydraulic networks seems to have a promising future, not only for water distribution as shown in this work, but also for waste water, according to previous findings by Montalvo et al. [27]. The PSO algorithm excels by its flexibility and adaptability in accommodating either discrete or continuous types of optimization variables. Its simplicity allows for a straightforward implementation, with relatively high execution speeds compared to other evolutionary algorithms, alongside a high convergence rate towards acceptable solutions. All these advantages motivate, without any doubt, the need for further research of the PSO algorithm within the field of hydraulic networks.

References

[1] I.C. Goulter, F. Bouchart, Quantitative approaches to reliability assessment in pipe networks, Journal of Transportation Engineering 112 (3) (1986) 287–301.

[2] I.C. Goulter, F. Bouchart, Reliability-constrained pipe network model, Journal of Hydraulic Engineering 116 (2) (1990) 211–229.

[3] D.A. Savic, G.A. Walters, Genetic operators and constraint handling for pipe network optimization, in: Evolutionary Computing, AISB Workshop, 1995, pp. 154–165.

[4] Z.Y. Wu, A.R. Simpson, Competent genetic-evolutionary optimization of water distribution systems, Journal of Computing in Civil Engineering 15 (2) (2001) 89–101.

[5] A.S. Matías, Diseño de redes de distribución de agua contemplando la fiabilidad, mediante algoritmos genéticos, Ph.D. Thesis, Universidad Politécnica de Valencia, Valencia, Spain. 2003.

[6] Z.Y. Wu, T. Walski, Self-adaptive penalty cost for optimal design of water distribution systems, Journal of Water Resources Planning and Management 131 (3) (2005) 181–192.

[7] H.R. Maier, A.R. Simpson, A.C. Zecchin, W.K. Foong, K.Y. Phang, H.Y. Seah, C.L. Tan, Ant colony optimization for design of water distribution systems, Journal of Water Resources Planning and Management 129 (3) (2003) 200–209.

[8] A.C. Zecchin, A.R. Simpson, H.R. Maier, J.B. Nixon, Parametric study for an ant algorithm applied to water distribution system optimization, IEEE Transactions on Evolutionary Computation 9 (2) (2005) 175–191.

[9] M.D.C. Cunha, J. Sousa, Water distribution network design optimization: Simulated annealing approach, Journal of Water Resources Planning and Management 125 (4) (1999) 214–221.

[10] S.-Y. Liong, M. Atiquzzaman, Optimal design of water distribution network using shuffled complex evolution, Journal of The Institution of Engineers, Singapore 44 (1) (2004) 93–107.

[11] Z.W. Geem, Optimal cost design of water distribution networks using harmony search, Engineering Optimization 38 (3) (2006) 259–280.

[12] A. Colorni, M. Dorigo, F. Maffioli, V. Maniezzo, G. Righini, M. Trubian, Heuristics from nature for hard combinatorial optimization problems, 1996.

[13] Y. Dong, B.X.J. Tang, D. Wang, An application of swarm optimization to nonlinear programming, Computers & Mathematics with Applications 49 (11–12) (2005) 1655–1668.

[14] J. Kennedy, R.C. Eberhart, Particle swarm optimization, in: IEEE International Conference on Neural Networks, Perth, Australia, IEEE Service Center, Piscataway, NJ, 1995.

[15] J. Izquierdo, R. Pérez, P.L. Iglesias, Mathematical models and methods in the water industry, Mathematical and Computer Modelling 39 (11–12) (2004) 1353–1374.

[16] G.C. Dandy, M.O. Engelhardt, Multi-objective trade-offs between cost and reliability in the replacement of water mains, Journal of Water Resources Planning and Management 132 (2) (2006) 79–88.

[17] J.B. Martínez, Quantifying the economy of water supply looped networks, Journal of Hydraulic Engineering 133 (1) (2007) 88–97.

[18] Y. Shi, R. Eberhart, A modified particle swarm optimizer, in: Evolutionary Computation Proceedings, IEEE World Congress on Computational Intelligence, 4–9 May 1998, 1998, pp. 69–73.

[19] M.S. Voss, Social programming using functional swarm optimization, in: Proceedings of the 2003 IEEE Swarm Intelligence Symposium (SIS03), Purdue University, Indianapolis, Indiana, 2003.

[20] X.H. Shi, Y.C. Liang, H.P. Lee, C. Lu, Q.X. Wang, Particle swarm optimization-based algorithms for tsp and generalized tsp, Information Processing Letters 103 (5) (2007) 169–176.

[21] B. Al-Kazemi, C.K. Mohan, Multi-phase discrete particle swarm optimization, in: JCIS, 2002, pp. 622–625.

[22] R. Rastegar, M.R. Meybodi, K. Badie, A new discrete binary particle swarm optimization based on learning automata, in: 2004 International Conference on Machine Learning and Applications, 2004.

[23] C.-J. Liao, C.-T. Tseng, P. Luarn, A discrete version of particle swarm optimization for flowshop scheduling problems, Computers and Operation Research 34 (10) (2007) 3099–3111.

[24] Y.-X. Jina, H.-Z. Chenga, J.-Y. Yanb, L. Zhang, New discrete method for particle swarm optimization and its application in transmission network expansion planning, Electric Power Systems Research 77 (3–4) (2007) 227–233.

[25] A. Zecchin, H. Maier, A. Simpson, A. Roberts, M. Berrisford, M. Leonard, Max-min ant system applied to water distribution system optimisation, in: Proceedings MODSIM 2003: International Congress on Modelling and Simulation, Townsville, Queensland, Australia, 2003.

[26] G.C. Dandy, A.R. Simpson, L.J. Murphy, An improved genetic algorithm for pipe network optimization, Water Resources Research 32 (2) (1996) 449–458.

[27] J. Izquierdo, I. Montalvo, R. Pérez, V.S. Fuertes, Design optimization of wastewater collection networks by PSO, Computers and Mathematics with Applications 56 (3) (2008) 777–784.