

SIS 4 IV

1. The CLT (Central limit theorem) shows that the distribution of sample means approaches a normal dist. as the sample size grows, regardless of the population dist.
2. The sample size n must be sufficiently large (usually $n \geq 30$) for the CLT to apply.
3. The expected value of \bar{X}_n is the population mean, so $E(\bar{X}_{100}) = 50$
4. The variance of \bar{X}_n is $\sigma^2/n = 10^2/100 = 1$
5. The variance of the sample mean decreases as n increases, making estimates more precise.
6. The distribution of the sample means will be approx. normal in shape, even though the original dist. is skewed.
7. According to the law of Large numbers, the proportion of heads should be close to 0.5
8. Since the expected value of a die roll is 3.5, with 1000 rolls the avg. will almost certainly fall between 3 and 4 due to law of large numbers

11. "Convergence in probability" means that as the sample size increases, the sample mean gets arbitrarily close to the true population mean with high probability.
12. LLN (Law of large numbers) guarantees that the sample mean will approach the true population mean as the sample size grows.
13. The sample mean \bar{X}_n will be normally distributed with mean 0 and variance $\frac{1}{n}$, i.e. $\bar{X}_n \sim N(0, \frac{1}{n})$
14. CLT doesn't require a normal population only i.i.d.
so its False
15. CLT makes the simplifying dist. of \bar{X} approx. $N(\mu, \frac{\sigma^2}{n})$, some can form confidence intervals and hypothesis tests even when the population shape is unknown.
16. by LLN, the avg of many die rolls will be close to the - expected value 3.5
17. LLN justifies generalization: the empirical (training) avg loss converges to the expected loss as sample size increases.

19. In quality control, X-bar charts use the (approx.) normal sample mean to set control limits (e.g., $\mu \pm \frac{3\sigma}{\sqrt{n}}$) and detect shifts in a manufacturing process.

20. For exp. lifetimes with the mean A , $\text{Var}(\bar{X}_n) = \frac{A^2}{n}$
Require $P(|\bar{X}_n - A| \leq 0.05A) \geq 0.90 \Rightarrow 0.05\sigma_p \geq$
 $\geq Z_{0.95} \approx 1.645$, so $\sigma_n \geq 32.9$ and $n \geq 108.3$