

$$\textcircled{1} C_n^0 \cdot C_m^k + C_n^1 \cdot C_m^{k-1} + \dots + C_n^k \cdot C_m^0 = C_{n+m}^k, \quad m, k \in \mathbb{N}$$

$\exists k=1$ , тогда  $C_m^k + C_n^1 \cdot C_m^{k-1} + \dots + C_n^k \cdot C_m^0 = C_{n+m}^k$   
 равносильно верно, н.з. все  $C_m^0 = 1 \Rightarrow C_n^k + C_m^k = C_{n+m}^k$   
 $+ C_m^k = C_{n+m}^k$

Для  $k$  - верно, тогда проверим для  $k+1$

$$C_m^{k+1} + C_n^1 \cdot C_m^{k+1-1} + \dots + C_n^{k+1} \cdot C_m^0 = C_{n+m}^{k+1}$$

$$C_m^{k+1} + C_n \cdot C_m^k + \dots + C_n^{k+1} = C_{n+m}^{k+1}$$

$$C_m^{k+1} + C_n^{k+1} = C_{n+m}^{k+1}$$

н.з.г

$$\textcircled{2} \sqrt{\sum_{k=1}^n (x_k - y_k)^2} \leq \sqrt{\sum_{k=1}^n x_k^2} + \sqrt{\sum_{k=1}^n y_k^2}$$

$$\sqrt{\sum_{k=1}^n (x_k^2 - 2x_k y_k + y_k^2)} \leq \sqrt{\sum_{k=1}^n x_k^2} + \sqrt{\sum_{k=1}^n y_k^2}$$

$$\sqrt{\sum_{k=1}^n (x_k^2)} - \sqrt{\sum_{k=1}^n (2x_k y_k)} + \sqrt{\sum_{k=1}^n (y_k^2)} \leq \sqrt{\sum_{k=1}^n x_k^2} + \sqrt{\sum_{k=1}^n y_k^2}$$

$$-\sqrt{\sum_{k=1}^n 2x_k y_k} \leq 0$$



$$\textcircled{3} \lim_{n \rightarrow \infty} \left(256 - \frac{5}{n^3}\right)^{-\frac{1}{4}} = \frac{1}{4}$$

По определению  $\forall \varepsilon > 0 \exists N > 0 \forall n: |n| > N \Rightarrow |f(n) - A| < \varepsilon$

$$\begin{aligned} & \left| \left(256 - \frac{5}{n^3}\right)^{-\frac{1}{4}} - \frac{1}{4} \right| < \varepsilon \\ & \left| \frac{1}{\sqrt[4]{256 - \frac{5}{n^3}}} - \frac{1}{\sqrt[4]{256}} \right| = \left| \frac{\sqrt[4]{256} - \sqrt[4]{256 - \frac{5}{n^3}}}{\sqrt[4]{256 - \frac{5}{n^3}} \cdot \sqrt[4]{256}} \right| = \left| \frac{4 - \sqrt[4]{256 - \frac{5}{n^3}}}{4 \cdot \sqrt[4]{256 - \frac{5}{n^3}}} \right| \\ & = \frac{1}{4} \left| \frac{4 - \sqrt[4]{256 - \frac{5}{n^3}}}{\sqrt[4]{256 - \frac{5}{n^3}}} \right| < \varepsilon \\ & \left| \frac{4}{\sqrt[4]{256 - \frac{5}{n^3}}} - 1 \right| < 4\varepsilon \end{aligned}$$

при  $n < -N$

$$-\frac{4}{\sqrt[4]{256 - \frac{5}{n^3}}} + 1 < 4\varepsilon$$

$$\frac{4}{\sqrt[4]{256 - \frac{5}{n^3}}} > -4\varepsilon + 1$$

$$\left(\frac{4}{\sqrt[4]{256 - \frac{5}{n^3}}}\right)^4 > (-4\varepsilon + 1)^4$$

$$256 - \frac{5}{n^3} > \left(\frac{4}{-4\varepsilon + 1}\right)^4$$

$$-\frac{5}{n^3} > \left(\frac{4}{-4\varepsilon + 1}\right)^4 - 256 \quad / \cdot (-1)$$

$$n < \sqrt[3]{\frac{5}{256 - \left(\frac{4}{-4\varepsilon + 1}\right)^4}} = -N(\varepsilon)$$

$$\Rightarrow N(\varepsilon) = \sqrt[3]{\frac{5}{256 - \left(\frac{4}{-4\varepsilon + 1}\right)^4}}$$

при  $n > N$

$$\frac{4}{\sqrt[4]{256 - \frac{5}{n^3}}} - 1 < 4\varepsilon$$

$$\frac{4}{\sqrt[4]{256 - \frac{5}{n^3}}} < 4\varepsilon + 1$$

$$\left(\frac{4}{\sqrt[4]{256 - \frac{5}{n^3}}}\right)^4 < (4\varepsilon + 1)^4$$

$$\frac{5}{n^3} < \left(\frac{4}{4\varepsilon + 1}\right)^4 - 256$$

$$n^3 < \frac{5}{\left(\frac{4}{4\varepsilon + 1}\right)^4 - 256}$$

$$n < \sqrt[3]{\frac{5}{\left(\frac{4}{4\varepsilon + 1}\right)^4 - 256}}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt[4]{256 - \frac{5}{n^3}}} = \frac{1}{4}$$

с.т.г.

$$\lim_{n \rightarrow \infty} \frac{1}{4 - \sqrt[4]{\frac{5}{n^3}}} = \frac{1}{4}$$

$$\frac{1}{4 - \sqrt[4]{\frac{5}{\infty}}} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4}$$



$$(4) \lim_{n \rightarrow \infty} a^{\frac{r}{n+p}} = t, a > 0, p > 0$$

$$a^{\frac{r}{\infty}} = t$$

$$a^0 = t$$

$$1 = t$$

Darüber:  $\lim_{n \rightarrow \infty} a^{\frac{r}{n+p}} = 1$

$$(5) \lim_{n \rightarrow \infty} \frac{\sqrt[n]{a^m} - 1}{\sqrt[n]{a^k} - 1}, a > 1, k, m \in \mathbb{N}$$

$$\lim_{n \rightarrow \infty} \frac{a^{\frac{m}{n}} - 1}{a^{\frac{k}{n}} - 1}$$

$$\frac{a^{\frac{m}{\infty}} - 1}{a^{\frac{k}{\infty}} - 1} = \frac{a^0 - 1}{a^0 - 1} = \frac{1 - 1}{1} = 0$$

Darüber:  $\lim_{n \rightarrow \infty} \frac{\sqrt[n]{a^m} - 1}{\sqrt[n]{a^k} - 1} = 0$

$$(6) \lim_{n \rightarrow \infty} \frac{n \cdot \sin n!}{n \sqrt{n^2 + 2n + 1}} = \left[ \frac{\infty}{\infty} \right]$$

$$\lim_{n \rightarrow \infty} \frac{\frac{n \cdot \sin n!}{n \sqrt{n}}}{\frac{n \sqrt{n}}{n \sqrt{n} + \sqrt{\frac{n}{(n \sqrt{n})^2 + 1}}}} = \lim_{n \rightarrow \infty} \frac{\frac{\sin n!}{\sqrt{n}}}{1 + \sqrt{\frac{1}{n^2} + 1}} = \frac{0}{1 + 1}$$

$$= \frac{0}{2} = 0$$

Darüber:  $\lim_{n \rightarrow \infty} \frac{n \cdot \sin n!}{n \sqrt{n^2 + 2n + 1}} = 0$



$$\textcircled{7} \{x_n\} = \sum_{k=1}^n \frac{\sin k\alpha}{k(k+1)}, \alpha \in \mathbb{R}$$

$$\{x_n\} = \frac{\sin \alpha}{2} + \frac{\sin 2\alpha}{6} + \dots + \frac{\sin n\alpha}{n(n+1)}, \alpha \in \mathbb{R}$$

$$\forall \varepsilon > 0 \exists N(\varepsilon): \forall n > N(\varepsilon), \forall p > 0: |x_{n+p} - x_n| < \varepsilon$$

$$\Rightarrow \left| \frac{\sin \alpha}{2} + \dots + \frac{\sin(n+p)\alpha}{(n+p)(n+p+1)} - \left( \frac{\sin \alpha}{2} + \dots + \frac{\sin n\alpha}{n(n+1)} \right) \right| < \varepsilon$$

$$\Rightarrow \left| \frac{\sin(n+1)\alpha}{(n+1)(n+2)} + \dots + \frac{\sin(n+p)\alpha}{(n+p)(n+p+1)} \right| < \varepsilon$$

$$\sin \alpha \leq 1$$

$$\left| \frac{\sin(n+1)\alpha}{(n+1)(n+2)} + \dots + \frac{\sin(n+p)\alpha}{(n+p)(n+p+1)} \right| \leq \left| \frac{1}{(n+1)(n+2)} + \dots + \frac{1}{(n+p)(n+p+1)} \right|$$

$$\left| \frac{1}{(n+p+1)(n+p+2)} \right| < \varepsilon$$

$$\left| \frac{1}{(n+1)(n+2)} + \dots + \frac{1}{(n+p)(n+p+1)} \right| < \varepsilon$$

модуль всегда больше 0

$$\frac{1}{(n+1)(n+2)} + \dots + \frac{1}{(n+p)(n+p+1)} < \varepsilon$$

применим все св-во:  $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$  в пер-во

$$\frac{1}{n+1} - \frac{1}{n+2} + \dots + \frac{1}{n+p} - \frac{1}{n+p+1} < \varepsilon$$

$$\frac{1}{n+1} - \frac{1}{n+p+1} < \varepsilon$$



Формула пер-во

$$\frac{\varepsilon}{n+1} - \frac{1}{n+p+1} < \frac{1}{n+1} < \varepsilon$$

$$\frac{1}{n+1} < \varepsilon$$

Формула пер-во до n

$$\frac{1}{n+1} < \frac{1}{n} < \varepsilon$$

$$\frac{1}{n} < \varepsilon$$

$$n > \frac{1}{\varepsilon}$$

по формуле  $N(\varepsilon) = \left[ \frac{1}{\varepsilon} \right]$  какое бы  $n > N$  не взяли,  
 $n \geq N(\varepsilon) + 1 > N(\varepsilon) = \left[ \frac{1}{\varepsilon} \right] \geq \frac{1}{\varepsilon}$

А тогда произвольное n

$$N > \frac{1}{\varepsilon}$$

то для произвольного p

$$|x_{n+p} - x_n| < \varepsilon$$

$\Rightarrow$  послед-ть  $x_n$  явл-ся фундаментальной, значит сс-дится.

$$\textcircled{8} \{x_n\} = \left\{ 1; \frac{1}{10}; \frac{2}{10}; \dots; \frac{9}{10}; \frac{1}{10^2}; \frac{2}{10^2}; \frac{99}{10^2}; \dots; \frac{1}{10^n}; \frac{2}{10^n}; \dots; \frac{10^n - 1}{10^n}; \dots \right\}$$

$$y_1 = \frac{1}{10^n}$$

$$y_2 = \frac{10^n - 1}{10^n}$$

$$\lim_{n \rightarrow \infty} y_1 = \lim_{n \rightarrow \infty} \frac{1}{10^n} = 0$$

$$\lim_{n \rightarrow \infty} y_2 = \lim_{n \rightarrow \infty} \frac{10^n - 1}{10^n} = \lim_{n \rightarrow \infty} \frac{10^n}{10^n} - \lim_{n \rightarrow \infty} \frac{1}{10^n} = 1 - \frac{1}{10^\infty} = 1$$

$$\lim_{n \rightarrow \infty} x_n = 0$$

$$\lim_{n \rightarrow \infty} x_n = 1$$



9)  $\lim_{x \rightarrow a} \frac{(x^n - a^n) - n \cdot a^{n-1} \cdot (x-a)}{(x-a)^2} = \left[ \frac{0}{0} \right]$ , применим Л'О-  
миталя.

$$\lim_{x \rightarrow a} \frac{(x^n - a^n - n \cdot a^{n-1} \cdot (x-a))'}{(x-a)^2}' = \lim_{x \rightarrow a} \frac{n x^{n-1} - 0 - n \cdot a^{n-1}}{2(x-a)} =$$

$$\lim_{x \rightarrow a} \frac{(n x^{n-1} - n \cdot a^{n-1})'}{(2x - 2a)'} = \lim_{x \rightarrow a} \frac{(n \cdot \frac{x^2}{x} - n \cdot a^{n-1})'}{(2x - 2a)'} =$$

$$\left( \frac{u}{v} \right)' = \frac{u'v - v'u}{v^2} \text{ в числителе}$$

$$= \lim_{x \rightarrow a} \frac{n^2 x^{n-1} \cdot x - n x^{n-1}}{x^2} \cdot \frac{1}{2} = \lim_{x \rightarrow a} \frac{n^2 x^n - n x^n}{x^2} = \frac{n^2 \cdot a^n - n \cdot a^n}{a^2}$$

Ответ:  $\lim_{x \rightarrow a} \frac{(x^n - a^n) - n \cdot a^{n-1} \cdot (x-a)}{(x-a)^2} = \frac{n^2 \cdot a^n - n \cdot a^n}{a^2}$



$$10) \lim_{x \rightarrow 0} \frac{\sqrt[5]{1+2x} - \sqrt[3]{1+5x}}{\sqrt[5]{1+5x} - \sqrt[5]{1+2x}} = \left[ \frac{0}{0} \right]$$

$$\lim_{x \rightarrow 0} \frac{\sqrt[5]{1+2x} - 1 + 1 - \sqrt[3]{1+5x}}{\sqrt[5]{1+5x} - \sqrt[5]{1+2x}} = \lim_{x \rightarrow 0} \frac{\sqrt[5]{1+2x} - 1}{\sqrt[5]{1+5x} - \sqrt[5]{1+2x}} + \lim_{x \rightarrow 0} \frac{1 - \sqrt[3]{1+5x}}{\sqrt[5]{1+5x} - \sqrt[5]{1+2x}}$$

Применяем формулы:  $a^2 - b^2 = (a-b)(a+b)$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt[5]{1+2x} - 1)(\sqrt[5]{1+2x} + 1)(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{(\sqrt[5]{1+x} - \sqrt[5]{1+2x})(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})(\sqrt[5]{1+2x} + 1)} +$$

$$+ \lim_{x \rightarrow 0} \frac{(1 - \sqrt[3]{1+5x})(1 + \sqrt[3]{1+5x} + \sqrt[3]{(1+5x)^2})(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{(\sqrt[5]{1+x} - \sqrt[5]{1+2x})(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})(1 + \sqrt[3]{1+5x} + \sqrt[3]{(1+5x)^2})}$$

$$= \lim_{x \rightarrow 0} \frac{(1+2x-1)(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{(1+x-1-2x)(\sqrt[5]{1+2x} + 1)} + \lim_{x \rightarrow 0} \frac{(1-1-5x)}{(1+x-1-2x)}$$

$$\frac{(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{(1 + \sqrt[3]{1+5x} + \sqrt[3]{(1+5x)^2})} = \lim_{x \rightarrow 0} \frac{2x(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{-x(\sqrt[5]{1+2x} + 1)} +$$

$$+ \lim_{x \rightarrow 0} \frac{-5x(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{-x(1 + \sqrt[3]{1+5x} + \sqrt[3]{(1+5x)^2})} = \lim_{x \rightarrow 0} \frac{2(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{-(\sqrt[5]{1+2x} + 1)}$$

$$\frac{(\sqrt[5]{1+2x})^4}{(\sqrt[5]{1+2x} + 1)} + \lim_{x \rightarrow 0} \frac{5(\sqrt[5]{(1+x)^4} + \dots + \sqrt[5]{(1+2x)^4})}{(1 + \sqrt[3]{1+5x} + \sqrt[3]{(1+5x)^2})} = -\frac{10}{2} + \frac{25}{3} = \frac{-30+50}{6} =$$

$$= \frac{20}{6} = \frac{10}{3}$$

Аналогично  $\lim_{x \rightarrow 0} \frac{\sqrt[5]{1+2x} - \sqrt[3]{1+5x}}{\sqrt[5]{1+5x} - \sqrt[5]{1+2x}} = \frac{10}{3}$

$$11) \lim_{x \rightarrow 0} \left( \frac{\sqrt[3]{1-3x} - \sqrt{1-2x}}{1 - \cos \pi x} \right) = \left[ \frac{0}{0} \right] \Rightarrow \text{используем мет. Лопиталя}$$

метод  $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$

$$\lim_{x \rightarrow 0} \frac{(\sqrt[3]{1-3x} - \sqrt{1-2x})'}{(1 - \cos \pi x)'} = \left[ (u - v)' = (u' - v') \right]$$

$$\sqrt[3]{1-3x}' - \sqrt{1-2x}' = [u'(x) = n \cdot u^{n-1}(x) \cdot u'(x)] \Rightarrow \frac{1}{3} \cdot \frac{1}{\sqrt[3]{(1-3x)^2}} \cdot$$

$$\cdot (-3x)' - \frac{1}{2} \cdot \frac{1}{\sqrt{1-2x}} \cdot (-2x)' \Rightarrow [(au - bu)' = au' - bu'] =$$

$$= \frac{1}{3\sqrt[3]{(1-3x)^2}} \cdot (-3(x)' + (1)') - \frac{1}{2\sqrt{1-2x}} \cdot (-2(x)' + (1)') = \frac{1}{\sqrt[3]{(1-3x)^2}} - \frac{1}{\sqrt{1-2x}}$$



$$-\frac{1}{\sqrt[3]{(1-3x)^2}}$$

$$2) (1 - \cos \pi x)' = [(u-v)]' = u' - v' = 0 + \sin \pi x \cdot (\pi x)' =$$

$$= \pi \sin \pi x$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt[3]{1-2x}} - \frac{1}{\sqrt[3]{1-3x}}}{\pi \sin \pi x} = \lim_{x \rightarrow 0} \frac{\left(\frac{1}{\sqrt[3]{1-2x}} - \frac{1}{\sqrt[3]{1-3x}}\right)'}{(\pi \sin \pi x)'} =$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{2} \frac{1}{(1-2x)^{5/3}} - \frac{1}{2} \frac{1}{(1-3x)^{5/3}}}{\pi^2 \cos(\pi x)} = \left[ \cos \pi x - 1 \right] = \lim_{x \rightarrow 0} \frac{(1-2x)^{5/3}}{\pi^2}$$

$$= -\frac{1}{\pi^2} \frac{5}{3} = -\frac{5}{3\pi^2}$$

$$\text{Ответ: } -\frac{5}{3\pi^2}$$

17)  $x_n$  - вып. ряд - мб

$$\text{Д-мб: } \lim_{n \rightarrow \infty} (n \cdot (x_{n+1} - x_n)) \neq +\infty$$

$$\bar{x}_n - \text{огранич.} \Rightarrow \forall n \in \mathbb{N} \exists M : |x_n| \leq M$$

$$\text{Д-б: } \text{Погреш. } \lim_{n \rightarrow \infty} (n(x_{n+1} - x_n)) = +\infty$$

$$\text{Погреш. } \forall M \exists N \in \mathbb{N} : n \geq N \Rightarrow (n \cdot (x_{n+1} - x_n)) > M$$

$$\forall n |x_{n+1} - x_n| = |x_n - x_{n+1}| \leq |x_n| + |x_{n+1}| \leq M + M \leq 2M$$

$$n \cdot |x_{n+1} - x_n| \leq n \cdot 2M \leq 2Mn$$

$\Rightarrow$  1)

$$12) \lim_{x \rightarrow \pi} \frac{e^{\sin x} - e^{\sin 2x}}{\sqrt[3]{\pi x^2} - \pi} = \frac{e^0 - e^{\sin 2\pi}}{\sqrt[3]{\pi \cdot \pi^2} - \pi} = 0$$

Ответ: 0

$$15) \lim_{x \rightarrow +\infty} \sqrt{x^4 + 2x^2 + 3x + 4} - \alpha x^2 - \beta x - \gamma = 0$$

$$\alpha = \frac{-\sqrt{x^4 + 2x^2 + 3x + 4} + \beta x + \gamma}{x^2}$$

$$\beta = \frac{-\sqrt{x^4 + 2x^2 + 3x + 4} + \alpha x^2 + \gamma}{x}$$

$$\gamma = \frac{-\sqrt{x^4 + 2x^2 + 3x + 4} + \alpha x^2 + \beta x}{1}$$



$$(13) \lim_{x \rightarrow 1} (4^x - \sqrt{x+8})^{\operatorname{tg}(\frac{\pi x}{2})} = \lim_{x \rightarrow 1} (4^x - \sqrt{x+8})^{\frac{\sin \pi x}{\cos \pi x}} = (4 - \sqrt{9})^{\frac{\sin \pi}{\cos \pi}} = (4 - 3) = 1$$

Далее:  $\lim_{x \rightarrow 1} (4^x - \sqrt{x+8})^{\operatorname{tg}(\frac{\pi x}{2})} = 1$

$$(14) \lim_{x \rightarrow 0} \operatorname{ctg}^2 x \cdot (e^{-x} + \sin x - 1) = [0 \cdot \infty]$$

$$\begin{aligned} \lim_{x \rightarrow 0} \operatorname{ctg}^2 x \cdot \frac{e^x \sin^x - e^x - 1}{e^x} &= \lim_{x \rightarrow 0} \frac{1}{e^x} \cdot \lim_{x \rightarrow 0} \operatorname{ctg}^2 x (e^x \sin^x - e^x + 1) \\ &= \lim_{x \rightarrow 0} \operatorname{ctg}^2 x = \frac{\cos^2 x}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{\cos^2 x (e^x \sin^x - e^x + 1)}{\sin^2 x} = \\ &= \lim_{x \rightarrow 0} \cos^2 x \cdot \lim_{x \rightarrow 0} \frac{e^x \sin^x - e^x + 1}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{e^x \sin^x - e^x + 1}{\sin^2 x} = \\ &= \lim_{x \rightarrow 0} \frac{e^x \cdot x \cdot \frac{\sin x}{x} - e^x + 1}{x^2 \left( \frac{\sin x}{x} \right)^2} = \lim_{x \rightarrow 0} \frac{e^x \cdot x - 1 + 1}{x^2} = \lim_{x \rightarrow 0} \frac{e^x (x-1) + 1}{x^2} \\ &\quad \left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \end{aligned}$$

Мемог Ломманд:

$$\lim_{x \rightarrow 0} \frac{(e^x (x-1) + 1)'}{(x^2)'} =$$

$$\left[ \begin{aligned} f'(x) &= (e^x (x-1) + 1)' = e^x (x-1) + e^x \\ g'(x) &= (x^2)' = 2x \end{aligned} \right]$$

$$= \lim_{x \rightarrow 0} \frac{e^x (x-1) + e^x}{2x} = \lim_{x \rightarrow 0} \frac{e^x \cdot x - 1 + 1}{2x} = \lim_{x \rightarrow 0} \frac{1}{2}$$

$$\frac{1}{2} \lim_{x \rightarrow 0} e^x = \frac{1}{2} \lim_{x \rightarrow 0} 1 = \frac{1}{2}$$

Далее:  $\frac{1}{2}$

(16) Выясните прав. часть  $C(1-x^2)$  при  $x \rightarrow 1$  у  $p$ -ли

$$f(x) = \frac{\operatorname{tg} \frac{\pi x}{2}}{\sqrt{1-x^2}}$$

Решение:



$$\text{no Th 1 } \lim_{x \rightarrow 1} \frac{f(x)}{C(1-x)^a} = 1 \Leftrightarrow \lim_{x \rightarrow 1} \frac{\log \frac{\pi x}{2}}{C(1-x)^a (\sqrt[3]{1-\sqrt[3]{5x}})} = 1$$

$$\text{no Th 2 } \left[ \log \frac{\pi x}{2} \sim \frac{\pi x}{2}, \sqrt[3]{1-\sqrt[3]{5x}} \sim 1 - \sqrt[3]{5x} \sim 7x, (1-x)^a \sim 7x \right]$$

$$\sim x] = \lim_{x \rightarrow 1} \frac{\frac{\pi}{2}}{C \cdot 7x} = [7x^2 \sim 17x] = \lim_{x \rightarrow 1} \frac{\frac{\pi}{2}}{C \cdot 7x^2}$$

$$[7x^2 \sim 7] = \lim_{x \rightarrow 2} \frac{\frac{\pi}{2}}{7 \cdot C} \Rightarrow C = \frac{7\pi}{2}$$

$$\text{Answer: } C = \frac{7\pi}{2}$$