

UFIFT-QG-24-09

Resumming Fermion Loops for Inflationary Gravity

A. J. Foraci* and R. P. Woodard†

*Department of Physics, University of Florida,
Gainesville, FL 32611, UNITED STATES*

ABSTRACT

We compute the 1-loop contribution to the graviton self-energy from a loop of massless fermions on a general cosmological background. The result is used to quantum-correct the linearized Einstein equation on de Sitter background and work out 1-loop corrections to gravitational radiation and to the response to a point mass. The renormalization group is employed to sum these to all orders for as long as the de Sitter phase persists.

PACS numbers: 04.50.Kd, 95.35.+d, 98.62.-g

* e-mail: aforaci@ufl.edu

† e-mail: woodard@phys.ufl.edu

1 Introduction

The inflationary production of long wavelength gravitons [1] induces large logarithmic corrections to the kinematics and long range forces carried by other fields [2–5]. Even if the particles associated with these other particles are not produced during inflation, the redshift of their vacuum energies can still cause secular changes in gravitational radiation and in the force of gravity [6–8]. This paper is devoted to working out the changes induced by a loop of massless fermions.

Section 2 computes the 1-loop contribution to the graviton self-energy from a loop of massless fermions on any cosmological background, slightly extending an old calculation on flat space background [9]. In section 3 we use this result to quantum-correct the linearized Einstein equation on de Sitter background. Solving this equation gives us 1-loop corrections to gravitational radiation and to the response to a static point mass. We also show how a variant of the renormalization group can be used to resum these results. Our conclusions comprise section 4.

2 The Graviton Self-Energy from Fermions

The 1PI (one-particle-irreducible) 2-point function for the graviton is known as the graviton self-energy $-i[\mu^\nu \Sigma^{\rho\sigma}](x; x')$. The diagrams representing the 1-loop fermion correction to it are shown in Figure 1.

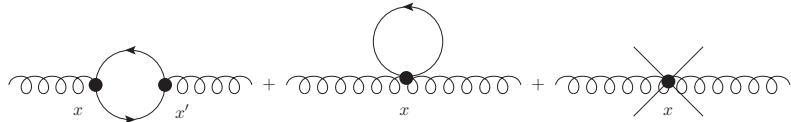


Figure 1: Fermionic contributions to the 1-loop graviton self-energy. Solid lines stand for fermions and curly lines for gravitons.

The background geometry of (D -dimensional) cosmology is,

$$ds^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x} = a^2[-d\eta^2 + d\vec{x} \cdot d\vec{x}] . \quad (1)$$

Here t is the co-moving time and η is the conformal time. We define the graviton field $h_{\mu\nu}(x)$ by conformally transforming the full metric,

$$g_{\mu\nu}(x) \equiv a^2 \tilde{g}_{\mu\nu} \equiv a^2 [\eta_{\mu\nu} + \kappa h_{\mu\nu}(x)] , \quad \kappa^2 \equiv 16\pi G . \quad (2)$$

Its indices are raised and lowered using the Minkowski metric $h^\mu_\nu \equiv \eta^{\mu\rho} h_{\nu\rho}$.

The massless Dirac Lagrangian for a general metric is,

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi} e_a^\mu \gamma^a \left(i\partial_\mu - \frac{1}{2} A_{\mu bc} J^{bc} \right) \psi \sqrt{-g} . \quad (3)$$

Here γ^a are the gamma matrices, $e_a^\mu(x)$ is the vierbein field, $A_{\mu bc}(x)$ is the spin connection and the J^{bc} are spin generators,

$$g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab} , \quad A_{\mu bc} \equiv e_b^\nu (e_{\nu c, \mu} - \Gamma_{\mu\nu}^\rho e_{\rho c}) , \quad J^{bc} \equiv \frac{i}{4} [\gamma^b, \gamma^c] . \quad (4)$$

If the local Lorentz gauge is fixed by requiring the vierbein to be symmetric $e_{\mu a} = e_{a\mu}$ there are no local Lorentz ghosts and one can regard the vierbein as a function of the graviton field [10],

$$e_{\mu b} = a \left[\sqrt{\tilde{g}\eta^{-1}} \right]_\mu^c \times \eta_{cb} = a \left[\eta_{\mu b} + \frac{\kappa}{2} h_{\mu b} - \frac{\kappa^2}{8} h_\mu^c h_{cb} + \dots \right] . \quad (5)$$

Because $\mathcal{L}_{\text{Dirac}}$ is conformally invariant for any dimension D , it has no dependence on the scale factor when both the fermion and the metric are conformally rescaled,

$$\psi \equiv a^{\frac{D-1}{2}} \tilde{\psi} , \quad \mathcal{L}_{\text{Dirac}} = \bar{\tilde{\psi}} \tilde{e}_a^\mu \gamma^a \left(i\partial_\mu - \frac{1}{2} \tilde{A}_{\mu bc} J^{bc} \right) \tilde{\psi} \sqrt{-\tilde{g}} . \quad (6)$$

This means that dependence on the scale factor can only enter the graviton self-energy through counterterms. Neither Einstein-Maxwell [11, 12] nor Einstein-Dirac [13] is perturbatively renormalizable, even at 1-loop order, but the divergences of any theory can be subtracted using BPHZ counterterms (Bogoliubov, Parasiuk [14], Hepp [15] and Zimmermann [16, 17]). The ones needed to renormalize any matter loop contribution to the graviton self-energy take the form [18],

$$\Delta\mathcal{L}_{\text{Einstein}} = c_1 R^2 \sqrt{-g} + c_2 C^{\alpha\beta\gamma\delta} C_{\alpha\beta\gamma\delta} \sqrt{-g} . \quad (7)$$

The fermion propagator from (6) can be expressed in terms of the massless scalar propagator on flat space $i\Delta(x; x')$,

$$\left\langle \Omega_0 \left| T[\tilde{\psi}_i(x) \bar{\tilde{\psi}}_j(x')] \right| \Omega_0 \right\rangle = i \gamma_{ij}^\mu \partial_\mu i\Delta(x; x') = -\frac{\Gamma(\frac{D}{2})}{2\pi^{\frac{D}{2}}} \frac{\gamma_{ij}^\mu \Delta x_\mu}{\Delta x^D} . \quad (8)$$

Here $\Delta x^2 \equiv \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\epsilon)^2$ is the conformal coordinate interval. The 3-point vertex is,

$$\frac{i\kappa}{2} h_{\mu\nu} \left[\eta^{\mu\nu} \bar{\tilde{\psi}} i\gamma^\alpha \partial_\alpha \tilde{\psi} - \bar{\tilde{\psi}} i\gamma^\mu \partial^\nu \tilde{\psi} + \partial_\alpha \left(\bar{\tilde{\psi}} \gamma^\mu J^{\nu\alpha} \tilde{\psi} \right) \right] . \quad (9)$$

Note that the final term involving the spin generator can be written as,

$$\gamma^{(\mu} J^{\nu)\alpha} = \frac{i}{2} \eta^{\alpha(\mu} \gamma^{\nu)} - \frac{i}{2} \eta^{\mu\nu} \gamma^\alpha , \quad (10)$$

where parenthesized indices are symmetrized.

Because the coincidence limit of the propagator (8) vanishes in dimensional regularization the middle diagram of Figure 1 vanishes and we do not need the 4-point interaction. Because acting $i\gamma^\mu \partial_\mu$ on the fermion propagator gives a delta function we require only two of the 3-point vertices,

$$V_1 \equiv h_{\mu\nu} \times \frac{\kappa}{2} \bar{\psi} \gamma^\mu \partial^\nu \tilde{\psi} \quad , \quad V_2 \equiv h_{\mu\nu} \times -\frac{\kappa}{4} \partial^\mu [\bar{\psi} \gamma^\nu \tilde{\psi}] . \quad (11)$$

The primitive contribution to the graviton self-energy is the sum of products of one of these vertices at x^μ and another at x'^μ . The first such product is,

$$\begin{aligned} -i[\mu\nu \Sigma_{11}^{\rho\sigma}](x; x') &= \frac{\kappa^2}{4} \left\langle \Omega_0 \left| T \left[\bar{\psi}(x) \gamma^{(\mu} \partial^{\nu)} \tilde{\psi}(x) \times \bar{\psi}(x') \gamma^{(\rho} \partial^{\sigma)} \tilde{\psi}(x') \right] \right| \Omega_0 \right\rangle \\ &= -\frac{\kappa^2 \Gamma^2(\frac{D}{2})}{16\pi^D} \text{Tr} \left[\gamma^{(\rho} \partial^{\sigma)} \left(\frac{\gamma^\beta \Delta x_\beta}{\Delta x^D} \right) \gamma^{(\mu} \partial^{\nu)} \left(\frac{\gamma^\alpha \Delta x_\alpha}{\Delta x^D} \right) \right] , \end{aligned} \quad (12)$$

$$\begin{aligned} &= -\frac{\kappa^2 \Gamma^2(\frac{D}{2})}{4\pi^D} \left\{ -\frac{\eta^{\mu\nu} \eta^{\rho\sigma}}{\Delta x^{2D}} + \frac{D[\eta^{\mu\nu} \Delta x^\rho \Delta x^\sigma + \Delta x^\mu \Delta x^\nu \eta^{\rho\sigma}]}{\Delta x^{2D+2}} \right. \\ &\quad \left. + \frac{D^2 \Delta x^{(\mu} \eta^{\nu)} (\rho \Delta x^\sigma)}{\Delta x^{2D+2}} - \frac{2D^2 \Delta x^\mu \Delta x^\nu \Delta x^\rho \Delta x^\sigma}{\Delta x^{2D+4}} \right\} . \end{aligned} \quad (13)$$

The other three contributions are,

$$-i[\mu\nu \Sigma_{12}^{\rho\sigma}](x; x') = -\frac{\kappa^2 \Gamma^2(\frac{D}{2})}{8\pi^D} \partial^{(\rho} \left\{ \frac{\Delta x^\sigma) \eta^{\mu\nu} + D \eta^\sigma (\mu \Delta x^\nu)}{\Delta x^{2D}} - \frac{2D \Delta x^\sigma) \Delta x^\mu \Delta x^\nu}{\Delta x^{2D+2}} \right\} , \quad (14)$$

$$-i[\mu\nu \Sigma_{21}^{\rho\sigma}](x; x') = -\frac{\kappa^2 \Gamma^2(\frac{D}{2})}{8\pi^D} \partial^{(\mu} \left\{ \frac{\Delta x^\nu) \eta^{\rho\sigma} + D \eta^\nu (\rho \Delta x^\sigma)}{\Delta x^{2D}} - \frac{2D \Delta x^\nu) \Delta x^\rho \Delta x^\sigma}{\Delta x^{2D+2}} \right\} , \quad (15)$$

$$-i[\mu\nu \Sigma_{22}^{\rho\sigma}](x; x') = -\frac{\kappa^2 \Gamma^2(\frac{D}{2})}{16\pi^D} \partial^{(\mu} \partial^{(\rho} \left\{ \frac{\eta^{\nu}) \sigma))}{\Delta x^{2D-2}} - \frac{2\Delta x^\nu) \Delta x^\sigma))}{\Delta x^{2D}} \right\} . \quad (16)$$

The next step is to extract a total of six derivatives from each term using identities of the form,

$$\frac{1}{\Delta x^{2D}} = \frac{\partial^6}{8D(D-1)(D-2)^2(D-3)(D-4)} \left(\frac{1}{\Delta x^{2D-6}} \right) , \quad (17)$$

$$\frac{\Delta x^\mu \Delta x^\nu}{\Delta x^{2D+2}} = \left[\frac{\eta^{\mu\nu} \partial^6}{16D^2(D-1)(D-2)^2(D-3)(D-4)} + \frac{\partial^\mu \partial^\nu \partial^4}{16D(D-1)(D-2)^2(D-3)(D-4)} \right] \frac{1}{\Delta x^{2D-6}} . \quad (18)$$

When this is done and all four terms summed, the result can be expressed using the transverse projection operator,

$$\Pi^{\mu\nu} \equiv \partial^\mu \partial^\nu - \eta^{\mu\nu} \partial^2 . \quad (19)$$

In these terms the primitive contribution is,

$$-i[\mu\nu\Sigma_{\text{prim}}^{\rho\sigma}] = -\frac{\kappa^2\Gamma^2(\frac{D}{2})}{64\pi^D} \left[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1} \right] \frac{\partial^2}{(D+1)(D-2)^2(D-3)(D-4)} \left(\frac{1}{\Delta x^{2D-6}} \right). \quad (20)$$

Note that expression (20) is transverse and traceless for all D .

We must now localize the divergences and subtract them using the counterterms (7). Localization is accomplished by adding zero in the form of the massless scalar propagator equation [19],

$$\frac{\partial^2}{D-4} \left[\frac{1}{\Delta x^{2D-6}} \right] = \frac{\partial^2}{D-4} \left[\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right] + \frac{\mu^{D-4}}{D-4} \frac{4\pi^{\frac{D}{2}} i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)}, \quad (21)$$

$$= \frac{\mu^{D-4}}{D-4} \frac{4\pi^{\frac{D}{2}} i\delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} - \frac{\partial^2}{2} \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + O(D-4). \quad (22)$$

Because the primitive result (20) is transverse and traceless, the Eddington (R^2) counterterm has coefficient zero, $c_1 = 0$. The variation of the Weyl ($C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}$) counterterm is,

$$\frac{\delta^2 i\Delta S}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')} \Big|_{h_{\alpha\beta}=0} = 2c_2\kappa^2 C^{\alpha\beta\gamma\delta\mu\nu} \left[a^{D-4} C_{\alpha\beta\gamma\delta}^{\rho\sigma} i\delta^D(x-x') \right]. \quad (23)$$

Here the 2nd order tensor differential operator $C_{\alpha\beta\gamma\delta}^{\mu\nu}$ is obtained by expanding the Weyl tensor of the conformally transformed metric $\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$,

$$\tilde{C}_{\alpha\beta\gamma\delta} \equiv C_{\alpha\beta\gamma\delta}^{\mu\nu} \times \kappa h_{\mu\nu} + O(\kappa^2 h^2). \quad (24)$$

It is manifestly traceless and transverse, and its explicit form can be found in [20,21]. Of great significance is the simplification which occurs when acted on a delta function,

$$C^{\alpha\beta\gamma\delta\mu\nu} C_{\alpha\beta\gamma\delta}^{\rho\sigma} \delta^D(x-x') = (\frac{D-3}{D-2}) \left[\Pi^{\mu(\rho}\Pi^{\sigma)\nu} - \frac{\Pi^{\mu\nu}\Pi^{\rho\sigma}}{D-1} \right] \delta^D(x-x'). \quad (25)$$

It follows that the divergences will cancel if we choose,

$$c_2 = \frac{\mu^{D-4}}{64\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2})}{(D+1)(D-3)^2(D-4)}. \quad (26)$$

The final, renormalized and unregulated result is,

$$-i[\mu\nu\Sigma_{\text{ren}}^{\rho\sigma}] = \frac{\kappa^2}{2^8 \cdot 5 \cdot \pi^4} C^{\alpha\beta\gamma\delta\mu\nu} \times C'_{\alpha\beta\gamma\delta}^{\rho\sigma} \left\{ 4\pi^2 \ln(aa') i\delta^4(x-x') + \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \right\}. \quad (27)$$

3 The Gravitational Response to Fermions

The graviton self-energy can be used to quantum-correct the linearized Einstein equation,

$$\mathcal{L}^{\mu\nu\rho\sigma}\kappa h_{\rho\sigma}(x) - \int d^4x' [\mu\nu\Sigma^{\rho\sigma}](x; x')\kappa h_{\rho\sigma}(x') = \frac{\kappa^2}{2}T^{\mu\nu}(x). \quad (28)$$

With the stress tensor $T^{\mu\nu}$ set to zero one can study plane wave gravitons,

$$\kappa h_{\mu\nu} = \epsilon_{\mu\nu} e^{i\vec{k}\cdot\vec{x}} u(t, k), \quad (29)$$

where the transverse-traceless and purely spatial polarization tensor $\epsilon_{\mu\nu}$ is the same as on flat space background and the tree order mode function is,

$$u^{\text{tree}} = \frac{H}{\sqrt{2k^3}} [1 - \frac{ik}{aH}] \exp[\frac{ik}{Ha}] \quad (30)$$

The gravitational response to a point mass requires two potentials,

$$T^{\mu\nu}(x) = -\delta_0^\mu \delta_0^\nu M a \delta^3(\vec{x}), \quad ds^2 = -[1-2\Psi]dt^2 + a^2[1-2\Phi]d\vec{x}\cdot d\vec{x}. \quad (31)$$

Employing the in-out self-energy (27) in this equation would be inappropriate because it is neither real nor causal. Both of these difficulties can be avoided by employing the in-in, or Schwinger-Keldysh, formalism [22–24]. The rules for converting an in-out self-energy of the form (20) to in-in form are straightforward [25] and the result is,

$$[\mu\nu\Sigma_{\text{SK}}^{\rho\sigma}](x; x') = -\frac{\kappa^2}{2^9 \cdot 5 \cdot \pi^3} \mathcal{C}^{\alpha\beta\gamma\delta\mu\nu} \times \mathcal{C}'_{\alpha\beta\gamma\delta}{}^{\rho\sigma} \left[8\pi \ln(aa') \delta^4(x-x') + f_B(x-x') \right]. \quad (32)$$

The function $f_B(x-x')$ is [26],

$$f_B(x-x') \equiv \partial^4 \left\{ \theta(\Delta\eta - \Delta r) \left(\ln[\mu^2(\Delta\eta^2 - \Delta r^2)] - 1 \right) \right\}, \quad (33)$$

where $\Delta\eta \equiv \eta - \eta'$ and $\Delta r \equiv \|\vec{x} - \vec{x}'\|$.

Our result (32) is valid for an arbitrary cosmological background (1). To facilitate actually solving equation (28) we will specialize to de Sitter, whose scale factor is $a = e^{Ht} = -(H\eta)^{-1}$, with Hubble constant H . On this background the Lichnerowicz operator is,

$$\begin{aligned} \mathcal{L}^{\mu\nu\rho\sigma} h_{\rho\sigma} &= \frac{1}{2}a^2 \left[\partial^2 h^{\mu\nu} - \eta^{\mu\nu} \partial^2 h + \eta^{\mu\nu} \partial^\rho \partial^\sigma h_{\rho\sigma} + \partial^\mu \partial^\nu h - 2\partial^\rho \partial^{(\mu} h^{\nu)}_\rho \right] \\ &+ a^3 H \left[\eta^{\mu\nu} \partial_0 h - \partial_0 h^{\mu\nu} - 2\eta^{\mu\nu} \partial^\rho h_{\rho 0} + 2\partial^{(\mu} h^{\nu)}_0 \right] + 3a^4 H^2 \eta^{\mu\nu} h_{00}. \end{aligned} \quad (34)$$

Because the massless Dirac contribution (32) is $\frac{1}{2}$ times the contribution from electromagnetism [6], we can read off the 1-loop solutions for Dirac from those for Maxwell. Hence the 1-loop correction to the electric components of the Weyl tensor for plane wave gravitational radiation is [8],

$$C_{0ij}(x) \longrightarrow C_{0ij}^{\text{tree}}(x) \left\{ 1 + \frac{\kappa^2 H^2}{80\pi^2} \ln(a) + \dots \right\}. \quad (35)$$

The 1-loop corrections to the Newtonian potential and the gravitational slip are [6],

$$\Psi(t, r) = \frac{GM}{ar} \left\{ 1 + \frac{\kappa^2}{120\pi^2 a^2 r^2} + \frac{\kappa^2 H^2}{80\pi^2} \ln(aHr) + \dots \right\}, \quad (36)$$

$$\Psi(t, r) + \Phi(t, r) = \frac{GM}{ar} \left\{ 0 + \frac{\kappa^2}{240\pi^2 a^2 r^2} - \frac{\kappa^2 H^2}{80\pi^2} + \dots \right\}. \quad (37)$$

The fractional $\kappa^2/a^2 r^2$ corrections are de Sitter descendants of old effects on flat space background [9]; the terms proportional to $\kappa^2 H^2$ are new.

During a prolonged period of de Sitter inflation, the secular 1-loop corrections in (35) and (36) must eventually overwhelm the tree order result. At this point perturbation theory breaks down and one must invoke a nonperturbative resummation scheme to work out what happens. The renormalized self-energy (27) shows the close connection between the renormalization scale μ and the cosmological scale factor,

$$4\pi^2 \ln(aa') i\delta^4(x-x') + \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] = 4\pi^2 \ln(\mu^2 aa') i\delta^4(x-x') + \partial^2 \left[\frac{\ln(\Delta x^2)}{\Delta x^2} \right]. \quad (38)$$

This suggests that the secular logarithms can be resummed using a variant of the renormalization group. Because the same two counterterms (7) suffice to renormalize any single matter loop correction to gravity, a general analysis is possible. In a study of the massless, minimally coupled scalar loop correction to gravity it was shown that a particular combination can be considered as a field strength renormalization for the graviton [27],

$$\delta Z = D \left[2(D-1)c_1 - c_2 \right] \kappa^2 H^2. \quad (39)$$

The same combination works as well for electromagnetism [8]. Dirac fermions have $c_1 = 0$ and c_2 given by expression (26), which implies the gamma function,

$$\gamma \equiv \frac{\partial \ln(1+\delta Z)}{\partial \ln(\mu^2)} = -\frac{\kappa^2 H^2}{160\pi^2}. \quad (40)$$

One can consider both the Weyl tensor (35) and the Newtonian potential (36) to be 2-point Green's functions. Hence the Callan-Symanzik equation implies,

$$\left[\frac{\partial}{\partial \ln(\mu)} + \beta_G \frac{\partial}{\partial G} + 2\gamma \right] G^{(2)} = 0 . \quad (41)$$

The beta function vanishes to the order we are working, and the connection (38) between μ and a means that we can replace the derivative with respect to $\ln(\mu)$ in equation (41) by a derivative with respect to $\ln(a)$. At this point one sees that the renormalization group explains the secular factors in expressions (35) and (36) and even permits their resummation,

$$C_{0ij}(x) \longrightarrow C_{0ij}^{\text{tree}}(x) \times [a(t)]^{\frac{\kappa^2 H^2}{80\pi^2}} , \quad (42)$$

$$\Psi(t, r) \longrightarrow \frac{GM}{a(t)r} \times [a(t)Hr]^{\frac{\kappa^2 H^2}{80\pi^2}} . \quad (43)$$

4 Conclusions

We have used dimensional regularization and BPHZ renormalization to evaluate the 1-loop contribution of massless Dirac fermions to the graviton self-energy (27) on an arbitrary cosmological background (1). Because the massless Dirac Lagrangian (3) is conformally invariant for any dimension D , the primitive contribution (20) is a factor of two times the 2-component result long ago obtained on flat space background [9]. The reason we were able to derive a result for general scale factor $a(t)$ is because the scale factor enters only through the counterterms (7).

We used the Schwinger-Keldysh version of our result (32) to quantum-correct the linearized Einstein equation (28). That was solved on de Sitter background to work out 1-loop corrections to the electric components of the Weyl tensor for plane wave gravitational radiation (35). We also obtained results for the Newtonian potential (36) and the gravitational slip (37). And we were able to resum the results using a variant of the renormalization group (42-43).

We could not assume that the primitive contribution from photons would be the same as in flat space because electromagnetism is only conformally invariant for $D = 4$ dimensions. This shows up in the photon propagator on de Sitter background depending on the scale factor and the Hubble constant for general D [28]. However, the Lagrangian for a massless, conformally

coupled scalar is conformally invariant for general D ,

$$\mathcal{L}_{\text{MCC}} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{1}{8}(\frac{D-2}{D-1})\phi^2R\sqrt{-g}. \quad (44)$$

This means that the primitive contribution to the graviton self-energy from a loop of these scalars must agree with the result obtained decades previously [29]. Like our Dirac case, renormalization can be carried out for an arbitrary scale factor. The Schwinger-Keldysh result is $\frac{1}{12}$ times that of electromagnetism [30],

$$[\mu^\nu\Sigma_{\text{MCC}}^{\rho\sigma}](x;x') = -\frac{\kappa^2}{2^{10}\cdot 3\cdot 5\cdot \pi^3} \mathcal{C}^{\alpha\beta\gamma\delta\mu\nu} \times \mathcal{C}'_{\alpha\beta\gamma\delta}{}^{\rho\sigma} \left[8\pi \ln(aa')\delta^4(x-x') + f_B(x-x') \right]. \quad (45)$$

In the same way we can solve the linearized Einstein equation (28) and resum the results using the renormalization group,

$$C_{0i0j} = C_{0i0j}^{\text{tree}} \left\{ 1 + \frac{\kappa^2 H^2}{480\pi^2} \ln(a) + \dots \right\} \longrightarrow C_{0i0j}^{\text{tree}} \times [a(t)]^{\frac{\kappa^2 H^2}{480\pi^2}}, \quad (46)$$

$$\Psi = \frac{GM}{ar} \left\{ 1 + \frac{\kappa^2}{720\pi^2 a^2 r^2} + \frac{\kappa^2 H^2}{480\pi^2} \ln(aHr) + \dots \right\} \longrightarrow \frac{GM}{ar} \times [a(t)Hr]^{\frac{\kappa^2 H^2}{480\pi^2}}. \quad (47)$$

Another interesting spin-off from our work concerns the issue of generalizing de Sitter results to a general cosmology. Because the Dirac and scalar self-energies (32) and (45) are valid for an arbitrary scale factor $a(t)$, we can use them to solve the linearized Einstein equation (28) for a general background. All that is necessary is to work out the Lichnerowicz operator for a general cosmological background and then solve the resulting equation. Even if this cannot be done exactly, it can certainly be done numerically.

Acknowledgements

This work was partially supported by NSF grant PHY-2207514 and by the Institute for Fundamental Theory at the University of Florida.

References

- [1] A. A. Starobinsky, JETP Lett. **30**, 682-685 (1979)
- [2] S. P. Miao and R. P. Woodard, Phys. Rev. D **74**, 024021 (2006)
doi:10.1103/PhysRevD.74.024021 [arXiv:gr-qc/0603135 [gr-qc]].

- [3] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, Class. Quant. Grav. **31**, 175002 (2014) doi:10.1088/0264-9381/31/17/175002 [arXiv:1308.3453 [gr-qc]].
- [4] C. L. Wang and R. P. Woodard, Phys. Rev. D **91**, no.12, 124054 (2015) doi:10.1103/PhysRevD.91.124054 [arXiv:1408.1448 [gr-qc]].
- [5] D. Glavan, S. P. Miao, T. Prokopec and R. P. Woodard, JHEP **03**, 088 (2022) doi:10.1007/JHEP03(2022)088 [arXiv:2112.00959 [gr-qc]].
- [6] C. L. Wang and R. P. Woodard, Phys. Rev. D **92**, 084008 (2015) doi:10.1103/PhysRevD.92.084008 [arXiv:1508.01564 [gr-qc]].
- [7] S. P. Miao, N. C. Tsamis and R. P. Woodard, JHEP **07**, 099 (2024) doi:10.1007/JHEP07(2024)099 [arXiv:2405.00116 [gr-qc]].
- [8] A. J. Foraci and R. P. Woodard, [arXiv:2412.11022 [gr-qc]].
- [9] D. M. Capper and M. J. Duff, Nucl. Phys. B **82**, 147-154 (1974) doi:10.1016/0550-3213(74)90582-3
- [10] R. P. Woodard, Phys. Lett. B **148**, 440-444 (1984) doi:10.1016/0370-2693(84)90734-2
- [11] S. Deser and P. van Nieuwenhuizen, Phys. Rev. D **10**, 401 (1974) doi:10.1103/PhysRevD.10.401
- [12] S. Deser and P. van Nieuwenhuizen, Phys. Rev. Lett. **32**, 245-247 (1974) doi:10.1103/PhysRevLett.32.245
- [13] S. Deser and P. van Nieuwenhuizen, Phys. Rev. D **10**, 411 (1974) doi:10.1103/PhysRevD.10.411
- [14] N. N. Bogoliubov and O. S. Parasiuk, Acta Math. **97**, 227-266 (1957) doi:10.1007/BF02392399
- [15] K. Hepp, Commun. Math. Phys. **2**, 301-326 (1966) doi:10.1007/BF01773358
- [16] W. Zimmermann, Commun. Math. Phys. **11**, 1-8 (1968) doi:10.1007/BF01654298

- [17] W. Zimmermann, Commun. Math. Phys. **15**, 208-234 (1969) doi:10.1007/BF01645676
- [18] G. 't Hooft and M. J. G. Veltman, Ann. Inst. H. Poincare A Phys. Theor. **20**, 69-94 (1974)
- [19] V. K. Onemli and R. P. Woodard, Class. Quant. Grav. **19**, 4607 (2002) doi:10.1088/0264-9381/19/17/311 [arXiv:gr-qc/0204065 [gr-qc]].
- [20] S. Park and R. P. Woodard, Phys. Rev. D **83**, 084049 (2011) doi:10.1103/PhysRevD.83.084049 [arXiv:1101.5804 [gr-qc]].
- [21] K. E. Leonard, S. Park, T. Prokopec and R. P. Woodard, Phys. Rev. D **90**, no.2, 024032 (2014) doi:10.1103/PhysRevD.90.024032 [arXiv:1403.0896 [gr-qc]].
- [22] K. c. Chou, Z. b. Su, B. l. Hao and L. Yu, Phys. Rept. **118**, 1-131 (1985) doi:10.1016/0370-1573(85)90136-X
- [23] R. D. Jordan, Phys. Rev. D **33**, 444-454 (1986) doi:10.1103/PhysRevD.33.444
- [24] E. Calzetta and B. L. Hu, Phys. Rev. D **35**, 495 (1987) doi:10.1103/PhysRevD.35.495
- [25] L. H. Ford and R. P. Woodard, Class. Quant. Grav. **22**, 1637-1647 (2005) doi:10.1088/0264-9381/22/9/011 [arXiv:gr-qc/0411003 [gr-qc]].
- [26] S. P. Miao, N. C. Tsamis, R. P. Woodard and B. Yesilyurt, [arXiv:2407.07864 [gr-qc]].
- [27] S. P. Miao, N. C. Tsamis and R. P. Woodard, Class. Quant. Grav. **41**, no.21, 215007 (2024) doi:10.1088/1361-6382/ad7dc8 [arXiv:2405.01024 [gr-qc]].
- [28] N. C. Tsamis and R. P. Woodard, J. Math. Phys. **48**, 052306 (2007) doi:10.1063/1.2738361 [arXiv:gr-qc/0608069 [gr-qc]].
- [29] D. M. Capper, Nuovo Cim. A **25**, 29 (1975) doi:10.1007/BF02735608
- [30] M. J. Duff and J. T. Liu, Phys. Rev. Lett. **85**, 2052-2055 (2000) doi:10.1088/0264-9381/18/16/310 [arXiv:hep-th/0003237 [hep-th]].