Thermal RG Flow of AS Quantum Gravity

E. Nyergesy,[1] I. G. Márián,[1, 2] E. Meskhi,[3] Y. Turovtsi-Shiutev,[4] and I. Nándori[1, 5, 2]

[1] University of Debrecen, Institute of Physics, P.O.Box 105, H-4010 Debrecen, Hungary
[2] HUN-REN Atomki, P.O.Box 51, H-4001 Debrecen, Hungary
[3] University of Bonn, Department of Physics and Astronomy, Nusallee 12, 53115 Bonn, Germany
[4] Uzhhorod National University, 14, Universytetska str., Uzhhorod, 88000, Ukraine
[5] University of Miskolc, Institute of Physics and Electrical Engineering, H-3515, Miskolc, Hungary
(Dated: April 1, 2025)

We perform the thermal Renormalization Group (RG) study of the Asymptotically Safe (AS) quantum gravity in the Einstein-Hilbert truncation by relating the temperature parameter to the running RG scale as $T \equiv k_T = \tau k$ (in natural units) in order to determine its thermal evolution in terms of the dimensionless temperature τ which is associated with the temperature of the expanding Universe. Thus, k_T and k are understood as running cutoffs for thermal and quantum fluctuations, respectively. Quantum effects are taken into account by moving along the thermal RG trajectory with fixed value of τ producing the quantum effective action at a given dimensionless temperature. The τ-evolution of the dimensionless Newton coupling $g(\tau)$ and the dimensionless cosmological constant $\lambda(\tau)$ results in a vanishing g-coordinate of the Reuter (i.e., non-Gaussian UV) fixed point in the high temperature limit ($\tau \to \infty$) which means that only the symmetric phase of AS gravity survives at $\tau = \infty$. Thus, in case of large temperatures the cosmological constant takes on a negative value in the limit $k \to 0$ which was also initially predicted by certain string theories, however, in our approach this is not in disagreement with observations, since during the thermal evolution of the Universe a phase transition occurs and the cosmological constant runs to the expected positive value at low temperatures.

I. INTRODUCTION

The main point of this work is to apply a modified thermal Renormalization Group (RG) approach recently introduced in [1] on the Asymptotically Safe (AS) quantum gravity, see e.g., [2–12] by using the Einstein Hilbert truncation in $d = 4$ dimensions. Since RG method, in particular the Functional RG (FRG) approach [13] has been used in cosmology very frequently [14–20], the modified thermal RG approach [1] could naturally find application in quantum cosmology, too. It solves an inherent riddle in the application of RG and finite temperature quantum field theory techniques. From one side one knows i) that one has to extend cosmological models to finite temperatures in which different energy and time scales are connected by the RG flow – from the other side ii) one of the first extension of the FRG method to finite temperatures [24] and also all further thermal FRG investigations based on this assumption, have problems in determining non-trivial fixed points of the RG flow which are crucial to consider phase transitions.

Rationale of the point i) is that although particle physics and cosmology are intimately connected, in accelerators a limited number of colliding particles are taken into account, but the early Universe must be seen as a hot dense plasma. Thus, the zero temperature quantum field theory which is the usual theoretical framework for particle processes must be extended to finite temperatures for cosmological applications.

Regarding point ii), thermal field theory is a very well-developed framework for finite temperature applications [21–23]. However, there are open question on its connection to renormalization and the RG method. By using a

natural unit in which $c = \hbar = k_B = 1$, the temperature parameter has to be related to a momentum scale. In the perturbative RG approach the temperature T is set to be equal to the running RG momentum scale μ, i.e., $\mu = 2\pi T$ while in non-perturbative approach, it is linked to a fixed momentum Λ, i.e., $T = \tau \Lambda$ where τ is a variable dimensionless parameter, see i.e., one of the very first publication on thermal FRG [24] which served as a reference for thermal FRG applications. The latter choice is motivated by the fact that the RG scale-dependence is introduced artificially in the Wilsonian approach [25] and the quantized theory must be obtained in the physical limit where the RG scale is sent to zero, $k \to 0$, thus it seems reasonable to connect the temperature to a fixed momentum. Indeed, this type of non-perturbative thermal FRG equation is well-known and used in a huge number of works with various applications. Some other thermal FRG approaches, see e.g., [26–29], were using a dimensionless temperature, $\tau_k = T/k$ which was introduced as a technical tool with the important constraint: the dimensionful temperature parameter T is kept constant over the RG flow, so, τ_k has a trivial scale dependence. For example in [27] at page 42, in Eq.(132) one finds $\partial_t \tau = -\tau$ where $t = \ln(k/\Lambda)$ and in [29] at the top of the page 12 one can read $\partial_t T/k = -T/k$ which result in the trivial RG scale dependence for τ_k. Another example is found in [26] at page 4 above equation (21), $\tilde{T}(\Lambda^\star) = 1$ where the intermediate momentum scale $\Lambda^\star$ is fixed by the running dimensionless temperature $\tilde{T}$ which is identical to τ. Finally, we refer to [28] at page 7 right below equation (50), where one finds $\tilde{T}_k = T/(c_k k)$. Thus, even in these works the temperature parameter was practically linked to a fixed UV momentum cutoff Λ, i.e., $T = \tau \Lambda$.

However, it has a serious drawback: one finds an explicit RG scale dependence in the dimensionless RG flow equations and it makes no room for non-trivial fixed points (only pseudo fixed points can be identified).

In Ref. [1] we proposed to reconcile points i) and ii) by relating the temperature parameter of the finite-temperature formalism to the running RG scale

$$T \equiv k_T = \tau k \tag{1}$$

where k_T and k serve as the running cutoff for thermal and quantum fluctuations, respectively. In this approach, the dimensionless variable factor τ is kept constant over the RG flow and it plays the role of the temperature of the cosmic plasma. By this choice we showed that the dimensionless RG flow equations have no explicit RG scale-dependence [1]. Thus, one can find non-trivial fixed points and the usual RG flow diagram method can be used to study critical behaviour [30]. We also showed how this modified thermal RG equation was used to "solve the triviality" of the ϕ^4 scalar field theory, i.e., the Higgs-like inflationary model. The "triviality" means that in $d = 4$ dimensions the ϕ^4 model has two phases but the classical and quantum analysis gives the same separatrix which is a vertical line at the vanishing mass. The thermal RG flow equations suggested by us modifies the flow diagram and with non-vanishing value for τ, the RG trajectory which separates the phases is no longer a vertical line which "solves the triviality" of the model. However, the infinite temperature limit cannot be taken unambiguously without taking into account quantum gravity effects which become relevant at the Planck scale. This analysis was missing in [1].

In this work, our goal is to understand how does the inclusion of gravity affects the proposed scheme. Thus, we apply the thermal FRG approach with the relation (1) on the AS quantum gravity with the Einstein-Hilbert truncation and determine its thermal RG flow in terms of τ which is considered as the temperature of the cosmic plasma.

II. AS QUANTUM GRAVITY AT ZERO TEMPERATURE

As a first step let us summarise the cornerstones of AS quantum gravity at zero temperature starting from its simplest realization which is the Einstein-Hilbert truncation of the effective average action

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^4x \sqrt{g} \left(2\Lambda_k - R\right), \tag{2}$$

where g is the determinant of the metric tensor, R is the Ricci scalar and the scale-dependent parameters are the cosmological constant Λ_k and the Newton coupling G_k.

The FRG study of (2) is given in terms of dimensionless couplings, $\lambda_k \equiv \Lambda_k k^{-2}$, $g_k \equiv G_k k^2$ with the help of the β-functions, see for example [17]

$$k\partial_k \lambda_k = \beta_\lambda(\lambda_k, g_k), \qquad k\partial_k g_k = \beta_g(\lambda_k, g_k)$$
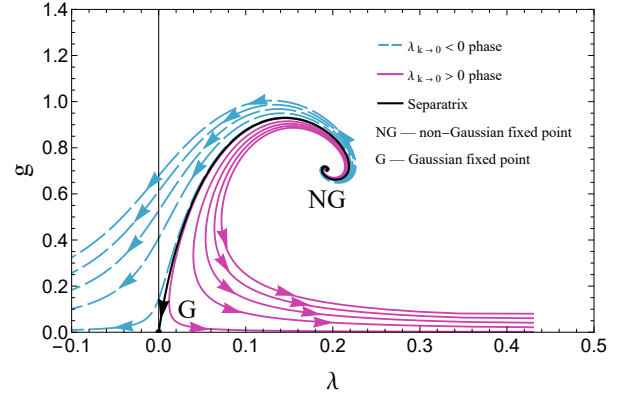

FIG. 1. Zero-temperature RG flow diagram of AS quantum gravity obtained by the FRG equations with the Litim regulator using the Einstein-Hilbert truncation (2).

which are calculated by the Litim regulator [38]

$$\beta_g = (2 + \eta_N)g_k,$$

$$\beta_\lambda = (\eta_N - 2)\lambda_k + \frac{g_k}{12\pi} \left[\frac{30}{1 - 2\lambda_k} - 24 - \frac{5}{1 - 2\lambda_k}\eta_N \right],$$

where the anomalous dimension of the operator $\int d^dx\sqrt{g}R$, i.e., the Newton constant, $\eta_N = G_k^{-1}k\partial_k G_k$ is given by

$$\eta_N = \frac{g_k \, B_1}{1 - g_k \, B_2}, \tag{3}$$

where

$$B_1 = \frac{1}{3\pi} \left[\frac{5}{1 - 2\lambda_k} - \frac{9}{(1 - 2\lambda_k)^2} - 7 \right],$$

$$B_2 = -\frac{1}{12\pi} \left[\frac{5}{1 - 2\lambda_k} - \frac{6}{(1 - 2\lambda_k)^2} \right]. \tag{4}$$

Based on these β-functions, the RG flow diagram is plotted on Fig. 1. The model has two phases separated by the black RG trajectory which starts from the non-Gaussian (NG) to the Gaussian (G) fixed point. The asymptotic safety is guaranteed by the NG also called the Reuter fixed point [2]. The separatrix can be interpreted similarly to that of the phase-transition line in a ϕ^4 theory. In that case one can distinguish two different phases based on the sign of the mass term; a symmetric phase with $m^2 > 0$ and a symmetry broken one where $m^2 < 0$ applies. In the Einstein-Hilbert action Λ is positive, however the RG equations are well defined in the case of $S \to -S$ as well [2], this is used to trade the "wrong sign" of the kinetic term for an upside down potential [31]. This transformation results in a negative Λ_k term in the scale dependent effective action, therefore the phase which results in a negative cosmological contant in the IR limit, i.e., $\lambda_{k\to0} < 0$ can be seen as the symmetric one, while the phase with $\lambda_{k\to0} > 0$ can be interpreted as the symmetry broken. In this work we apply this terminology

which was also used in Ref. [32] to differentiate between the two phases, where the cosmological constant takes on a different sign. Thus, the symmetric (i.e., $\lambda_{k\to 0} < 0$) phase is indicated by dashed blue RG trajectories while the broken (i.e., $\lambda_{k\to 0} > 0$) phase is given by pink solid lines. A similar situation can be seen for conformally reduced AS gravity [31, 33–37].

In order to study the model at finite temperatures, the $\boldsymbol{\beta}$-functions [3],

$$k\partial_k \lambda_k = \boldsymbol{\beta}_\lambda(\lambda_k, g_k), \qquad k\partial_k g_k = \boldsymbol{\beta}_g(\lambda_k, g_k)$$

must be given by their most general form including the so called threshold functions [3],

$$\boldsymbol{\beta}_g(\lambda_k, g_k) = (d - 2 + \eta_N)\, g_k \tag{5}$$

$$\boldsymbol{\beta}_\lambda(\lambda_k, g_k) = -(2 - \eta_N)\,\lambda_k + \frac{1}{2}\,(4\pi)^{1-d/2}\, g_k \tag{6}$$
$$\big[2\,d(d+1)\,\Phi^1_{d/2}(-2\lambda_k) - 8\,d\,\Phi^1_{d/2}(0)$$
$$- d(d+1)\,\eta_N\,\widetilde{\Phi}^1_{d/2}(-2\lambda_k)\big]$$

where the anomalous dimension η_N is given by [3]

$$\eta_N(g_k, \lambda_k) = \frac{g_k\, B_1(\lambda_k)}{1 - g_k\, B_2(\lambda_k)}, \tag{7}$$

and the functions $B_1(\lambda_k)$ and $B_2(\lambda_k)$ have the following definition

$$B_1(\lambda_k) = \frac{1}{3}\,(4\pi)^{1-d/2}\bigg[d(d+1)\,\Phi^1_{d/2-1}(-2\lambda_k) \tag{8}$$

$$- 6d(d-1)\,\Phi^2_{d/2}(-2\lambda_k) - 4d\,\Phi^1_{d/2-1}(0) - 24\Phi^2_{d/2}(0)\bigg]$$

$$B_2(\lambda_k) = -\frac{1}{6}\,(4\pi)^{1-d/2}\bigg[d(d+1)\,\widetilde{\Phi}^1_{d/2-1}(-2\lambda_k)$$

$$- 6d(d-1)\widetilde{\Phi}^2_{d/2}(-2\lambda_k)\bigg], \tag{9}$$

where we introduced the (dimensionless) regulator $R^{(0)}$, i.e., the shape function which appears in threshold functions Φ^p_n and $\widetilde{\Phi}^p_n$ as [3]

$$\Phi^p_n(w) = \frac{1}{\Gamma(n)}\int_0^\infty dz\, z^{n-1}\frac{R^{(0)}(z) - z\,R^{(0)\prime}(z)}{[z + R^{(0)}(z) + w]^p} \tag{10}$$

$$\widetilde{\Phi}^p_n(w) = \frac{1}{\Gamma(n)}\int_0^\infty dz\, z^{n-1}\frac{R^{(0)}(z)}{[z + R^{(0)}(z) + w]^p}, \tag{11}$$

with $p = 1, 2$. For a simple consistency check, on Fig. 2 we plotted the RG flow diagram by using these general $\boldsymbol{\beta}$-functions with the Litim regulator [38] and we obtained results identical to Fig. 1.

III. THERMAL RG STUDY OF AS QUANTUM GRAVITY

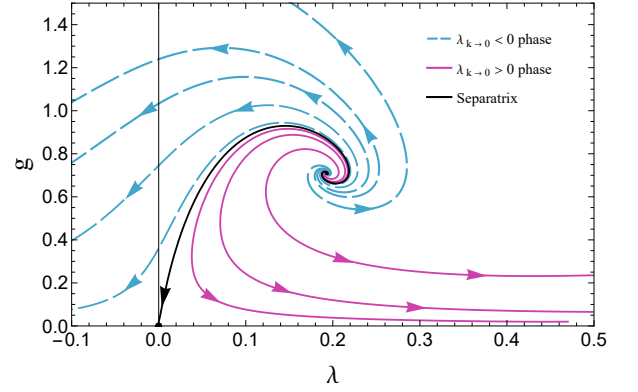In order to extend the zero-temperature RG study of the AS quantum gravity for finite temperatures one

FIG. 2. Identical to Fig. 1 but obtained by the numerical solution of the general $\boldsymbol{\beta}$-functions (5) and (6) with the Litim regulator.

has to modify the threshold functions (10) and (11) but nothing else. This modification is required because the upper bound of the imaginary time $\tilde{t}$ integral $\int d^d x \to \int_0^\beta d\tilde{t} \int d^{d-1}x$ becomes finite where $\beta = 1/T$ and T is the temperature parameter. As a result, the momentum integral with respect to the imaginary time is replaced by a summation

$$\int d^d p \to T \sum_{\omega_m} \int d^{d-1}p \tag{12}$$

over the Matsubara frequencies; for bosonic degrees of freedom $\omega_m = 2\pi m T$, where $m \in \mathbb{Z}$. In order to apply (12) to the zero-temperature threshold functions (10) and (11), one should transform them back to momentum integrals, implement the Matsubara summation on one of them, and then perform again the solid angle integrals. This implies the following changes to (10) and (11)

$$\Phi^p_n(w, \tau) = \frac{2\tau\sqrt{\pi}}{\Gamma\left(n - \frac{1}{2}\right)}\sum_{m=-\infty}^{\infty}\int_0^\infty dy\, y^{n-\frac{3}{2}}$$
$$\frac{R^{(0)}(y) - y\,R^{(0)\prime}(y)}{[y + (2m\pi\tau)^2 + R^{(0)}(y) + w]^p}, \tag{13}$$

$$\widetilde{\Phi}^p_n(w, \tau) = \frac{2\tau\sqrt{\pi}}{\Gamma\left(n - \frac{1}{2}\right)}\sum_{m=-\infty}^{\infty}\int_0^\infty dy\, y^{n-\frac{3}{2}}$$
$$\frac{R^{(0)}(y)}{[y + (2m\pi\tau)^2 + R^{(0)}(y) + w]^p}, \tag{14}$$

where we used the notation $z \equiv y$ and introduced the dimensionless temperature (1), i.e., $\tau = T/k \equiv k_T/k$. It is important to note that τ is kept constant over the RG flow.

Thermal RG flow diagrams of AS quantum gravity for various values of the dimensionless temperature τ are plotted on Fig. 3. In the zero-temperature case at $\lambda = 1/2$ singularities are present in the beta functions, causing the trajectories to terminate. This pole persists
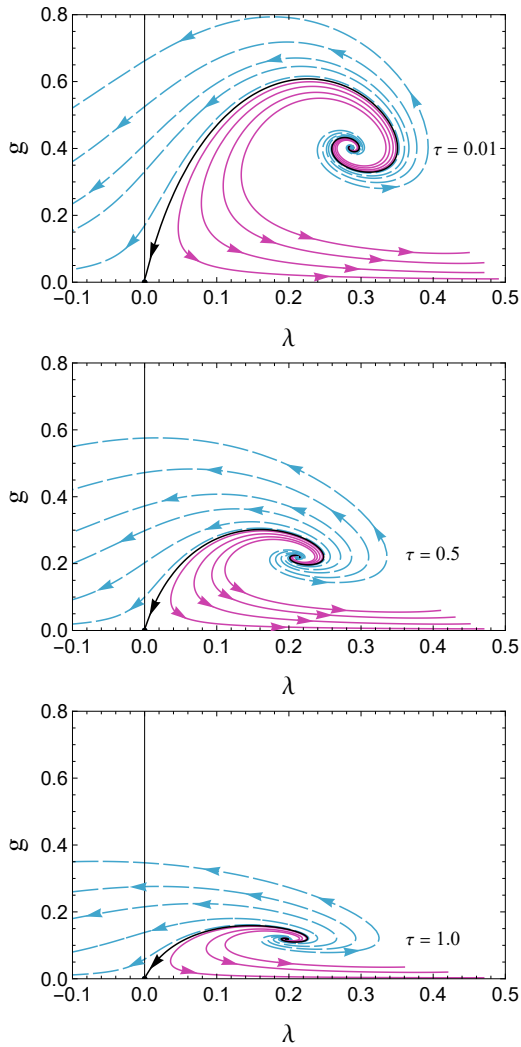
FIG. 3. Thermal RG flow diagrams of AS quantum gravity for various values of the dimensionless temperature τ. In the high temperature limit, i.e., $\tau \to \infty$, the g-coordinate of the Reuter fixed point tends to zero.

FIG. 4. QPT-CPT diagram of AS quantum gravity in terms of the dimensionless temperature τ and the g-coordinate of the Reuter fixed point. The black line, i.e., the function $g^{\star}(\tau_c)$, is the critical line which separates the symmetric and the broken phases of the model. For a given (but fixed) $g^{\star}$-value, for $\tau > \tau_c$ or $\tau < \tau_c$ model is in the symmetric or its broken phase. The inset shows how the positions of the Reuter fixed point $(g^{\star}, \lambda^{\star})$ changes by τ.

for finite temperatures as well, one can see on Fig. 3 the trajectories stop at a certain point in the $\lambda > 0$ phase. The subfigures suggest, that in the limit of high temperature, i.e., for $\tau \to \infty$, the g-coordinate of the Reuter fixed point, which we denote by $g^{\star}$, vanishes. This is confirmed by the inset of Fig. 4 where the positions of the Reuter fixed point is plotted for various values of τ.

Before we analyse the consequences of the large temperature behaviour let us first comment the zero-temperature limit. It is clear from Fig. 3 and Fig. 4 that the position of the Reuter fixed point in the limit $\tau \to 0$ *does not* coincide to its zero-temperature value, see Fig. 1 or Fig. 2 (keep in mind that Fig. 1 and Fig. 2 are identical). This is a consequence of the use frequency-dependent or independent regulator function in the calculation of threshold functions, see for example the discus-
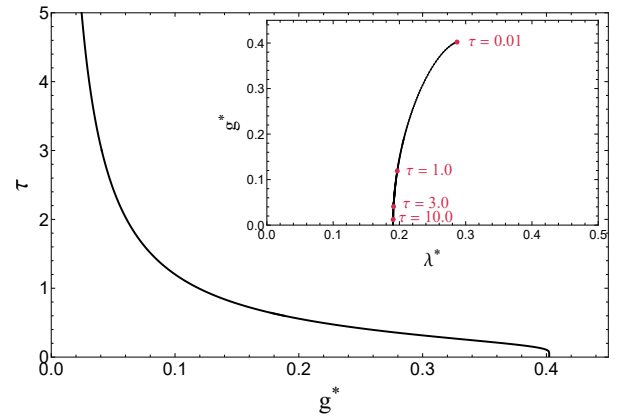
sion in [30]. The finite-temperature formalism requires a frequency independent, i.e., cylindrically symmetric [39, 40] regulator which does not regulate the Matsubara sum. This is not true for the zero-temperature case where momentum integrals (including the imaginary time direction) are performed by the regulator function i.e., with a frequency-dependent, i.e., spherically symmetric regulator. This is the reason why, the zero-temperature limit of truncated thermal FRG equations may differ from their counterparts at zero temperature.

For intermediate temperatures, i.e., $0 < \tau < \infty$, one can observe a thermal or classical phase transition (CPT) in the following way. Let us choose, arbitrary (but positive) initial values for the couplings in the vicinity of the Gaussian fixed point as a starting point. Then, one can always define a particular (dimensionless) temperature, τ_c, which results in a separatrix, see e.g., black lines of subfigures in Fig. 3, which goes through this particular initial point. For large temperatures, $\tau > \tau_c$, the RG trajectory starting form this chosen initial point always ends up in the symmetric phase. For small temperatures, $\tau < \tau_c$, the RG trajectory from the initial point runs into the broken phase. Thus, the model undergoes a thermal phase transition at $\tau = \tau_c$.

For a fixed (dimensionless) temperature τ, one can observe a quantum phase transition (QPT) depending on the choice of the initial values of the couplings, i.e., starting point chosen in the vicinity of the Gaussian fixed point. If this starting point is chosen to be above the separatrix (the black trajectory connecting the G and NG fixed points), the RG trajectory from this starting point ends up in the symmetric phase, and if the starting point is below the separatrix, its RG trajectory runs to the broken phase of the model. For different separatri-
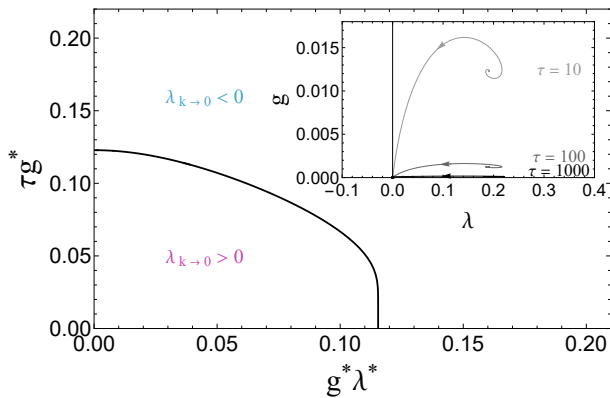
FIG. 5. QPT-CPT diagram in terms of $\tau g^\star$ and $g^\star \lambda^\star$ where $g^\star$ and $\lambda^\star$ are the coordinates of the Reuter fixed point. The black critical line which separates the phases terminates at finite value in the limit of $g^\star \lambda^\star \to 0$ which represents an important difference compared to Fig. 4 where the critical line runs to infinity. The inset shows how the separatrix looks like for large values of τ.

ces at various large τ see the inset on Fig. 5. Although, the slope of the separatrix serves as the real "quantum parameter" which controls the QPT, a good approximation for this could be the g-coordinate of the Reuter fixed point which we denote by $g^\star$.

One can construct a function, $g^\star(\tau_c)$ which represents a critical line in the $\tau - g$ plane and separates the symmetric (above this line) and broken (below this line) phases of the model. This critical line is plotted on Fig. 4 which represents the so called QPT-CPT diagram of the AS quantum gravity. It is illustrative to compare it to the QPT-CPT diagram of the ϕ^4 model in lower dimensions, see Fig. (5) of [30]. One finds many similarities. For example, for $\tau \to 0$ critical lines always end up in the quantum critical point of the model. Let us note that the QPT of the ϕ^4 scalar field theory has also been investigated in connection to the Naturalness/Hierarchy problem [42–45]. For example, in [44, 45] its was shown that the hierarchy problem as well as the metastability of the electroweak vacuum can be understood as the Higgs potential being near-critical, i.e., close to a QPT. Another similarity is that for vanishing "quantum parameter", the critical (dimensionless) temperature τ_c tends to infinity, however, as it is discussed in [30] this limit is non-analytic: at zero "quantum parameter" the critical temperature must be zero.

As discussed before, for $\tau \to \infty$ the g^* disappears, i.e., $g^* \to 0$, therefore we investigated their product (τg^*) and plotted it with respect to $g^*\lambda^*$, which serves as the "quantum parameter" of the model. The latter choice is motivated by the fact that $G_k \Lambda_k = g_k \lambda_k$ is a dimensionless combination of the couplings. It is worth noting, however, that the change in λ^* during the thermal evolution is relatively small compared to that in g^*, at large τ one can view λ^* as "constant". This implies, that e.g.,

the slope of the separatrix (g^*/λ^*) or g^* quantum parameters produce qualitatively the same QPT-CPT diagram as of Fig. 5. We have found that as $g^*\lambda^*$ quantum parameter disappears, the τg^* dimensionless parameter tends to a constant value. Thus, Fig. 5 gives a more direct analogy to the known QPT-CPT diagrams of various statistical physical models (e.g., Ising model).

Finally let us come back to the discussion of the large temperature limit. As it is clearly shown by the inset of Fig. 4 (and also suggested by the subfigures of Fig. 3) for $\tau \to \infty$ the λ-coordinate of the Reuter fixed point goes to a positive value and its g-coordinate vanishes, i.e., $g^\star(\tau_c \to \infty) = 0$. This means that for very large temperatures, larger than for example the Planck temperature where quantum gravity effects must be taken into account, according to our framework, only the symmetric phase of the AS quantum gravity survives which is the most important finding of this work.

IV. CONCLUSIONS

In this work, we applied a modified thermal FRG approach recently introduced in [1] on the AS quantum gravity by using the Einstein Hilbert truncation in $d = 4$ dimensions. The essence of this modified method was the use of a constant dimensionless temperature, $\tau = T/k$ where T was assumed to run with the RG scale k. We explored the possibility of this choice on the phase structure of AS quantum gravity. The focus was on the large temperature limit, i.e, $\tau \to \infty$, in which quantum gravity effects must be taken into account.

Our most important finding was that for very large temperatures, only the symmetric phase of the AS quantum gravity survived because the g-coordinate of the Reuter fixed point vanished. On the one hand it is a very natural expectation, that for very large temperatures the model is in its symmetric phase. On the other hand it is not in the usual/standard picture of quantum gravity which is linked to behaviour of the model around UV fixed points. This is because at zero temperature, it is very natural to identify the running scale parameter k with physical properties of the system and look for a particular RG trajectory picked up by Nature [46–48] which starts from the vicinity of the Gaussian fixed point at very high energies and tends to low energies [49–51]. However, at finite temperatures, it is more natural to associate physical properties of the model to its temperature (τ) and the RG scale k is considered only as a tool to take into account quantum effects which are fully incorporated in the limit $k \to 0$ only. In other words, we do not consider the k-flow as an approximation to catch the main physics properties of the system at a given range of energy.

One can notice that in case of large temperatures the cosmological constant takes on a negative value in the limit $k \to 0$. This was also initially predicted by certain string theories, which contradicts current cosmological

observations prompting attempts to resolve the discrepancy [52]. However, these solutions might describe unstable worlds [53]. In our framework, it seems natural for high-energy (high-temperature) models to yield a negative cosmological constant. This is not in disagreement with observations, since during the thermal evolution of the Universe a phase transition occurs and the cosmological constant runs to the expected positive value at low temperatures.

Finally, let us draw the attention of the reader that all computations of the present work are performed in the Euclidean spacetime. There is an increasing interest in the literature to discuss whether an analytic continuation to Minkowski spacetime is possible in the framework of the Wilsonian RG method, see e.g., [41, 54–57]. However, we refer to lattice calculations performed in the Euclidean spacetime [58], as a strong support for the use of Euclidean instead of the Minkowski spacetime.

ACKNOWLEDGEMENTS

[1] I.G. Márián, A. Trombettoni, I. Nándori, Phys. Lett. B **858**, 139051 (2024).
[2] M. Reuter, Phys. Rev. D **57**, 971 (1998).
[3] M. Reuter, F. Saueressig, Phys. Rev. D **65**, 065016 (2002).
[4] O. Lauscher, M. Reuter, Phys. Rev. D **66**, 025026 (2002).
[5] A. Platania, *Special issue "Coarse Graining in Quantum Gravity: Bridging the Gap between Microscopic Models and Spacetime-Physics"*, arXiv:2003.13656 [gr-qc].
[6] Percacci, R. *An Introduction to Covariant Quantum Gravity and Asymptotic Safety, 100 Years of General Relativity*; World Scientific: Singapore, Volume 3 (2017).
[7] A. Eichhorn, Front. Astron. Space Sci. **5**, 47 (2019).
[8] M. Reuter, F. Saueressig, *Quantum Gravity and the Functional Renormalization Group: The Road towards Asymptotic Safety* (Cambridge, Cambridge University Press, 2019).
[9] A. Bonanno, A. Eichhorn, H. Gies, H. J.M. Pawlowski, R. Percacci, M. Reuter, F. Saueressig, G.P. Vacca, Front. Phys. **8**, 269 (2020).
[10] J.M. Pawlowski, M. Reichert, Front. Phys. **8**, 551848 (2021).
[11] A. Eichhorn, M. Schiffer, *Invited chapter for the "Handbook of Quantum Gravity"*, arXiv:2212.07456 [hep-th].
[12] F. Saueressig, arXiv:2302.14152.
[13] C. Wetterich, Phys. Lett. B **301**, 90 (1993); T. R. Morris, Int. J. Mod. Phys. A **9**, 2411 (1994).
[14] A. Babic, B. Guberina, R. Horvat, H. Stefancic, Phys. Rev. D **71**, 124041 (2005).
[15] M. Hindmarsh, I. D. Saltas, Phys. Rev. D **86**, 064029 (2012).
[16] E. J. Copeland, C. Rahmede, I. D. Saltas, Phys. Rev. D **91**, 103530 (2015).
[17] A. Bonanno, F. Saueressig, C. R. Phys. **18**, 254 (2017).
[18] G. Gubitosia, R. Ooijera, C. Ripkena, F. Saueressig, JCAP **1805**, 004 (2018).
[19] A. Platania, F. Saueressig, Found. Phys. **48**, 1291 (2018).
[20] A. Platania, Front. Phys. **8**, 188 (2020).
[21] J. W. Negele, H. Orland, *Quantum many-particle systems* (Reading, Perseus, 1998).
[22] S. Sachdev, *Quantum phase transitions* (Cambridge, Cambridge University Press, 2011).
[23] J. I. Kapusta, *Finite-temperature field theory: principles and applications* (Cambridge, Cambridge University Press, 2023).
[24] N. Tetradis and C. Wetterich, Nucl. Phys. B **398**, 659 (1993).
[25] K. G. Wilson, Phys. Rev. B **4**, 3174 (1971); K. G. Wilson, Phys. Rev. B **4**, 3184 (1971).
[26] P. Jakubczyk, J. Bauer, W. Metzner, Phys. Rev. B **82**, 045103 (2010).
[27] J. Braun. J. Phys. G **39**, 033001 (2012).
[28] A. Rancon, O. Kodio, N. Dupuis, P. Lecheminant, Phys. Rev. E **88**, 012113 (2013).
[29] J. Braun, M. Leonhardt, and M. Pospiech, Phys. Rev. D **96**, 076003 (2017).
[30] I.G. Márián, A. Trombettoni, I. Nándori, arXiv:2407.20704 [hep-th].
[31] M. Reuter, H. Weyer, Phys. Rev. D **79**, 105005 (2009).
[32] F. Gégény, K. Sailer and S. Nagy, Class. Quantum Grav., **40** 045004, (2023).
[33] P.F. Machado, R. Percacci, Phys. Rev. D **80**, 024020 (2009).
[34] A. Bonanno, F. Guarnieri, Phys. Rev. D **86**, 105027 (2012).
[35] B. Knorr, Class. Quantum Gravity **38**, 065003 (2021).
[36] M. Reuter, H. Weyer, Phys. Rev. D **80**, 025001 (2009).
[37] A. Bonanno, M. Conti, S. Cacciatori, Phys. Rev. D **108**, 026008 (2023).
[38] D. F. Litim, Phys. Lett. B **486**, 92 (2000).
[39] I. Steib, S. Nagy, J. Polonyi, Int. J. Mod. Phys. A **36**, 2150031 (2021).
[40] F. Gégény, S. Nagy, Int. J. Mod. Phys. A **36**, 2250061 (2022).
[41] I. Steib, S. Nagy, J. Polonyi, Int. J. Mod. Phys. A **36**, 2150031 (2021); F. Gégény, S. Nagy, Int. J. Mod. Phys. A **36**, 2250061 (2022).
[42] C. Branchina, V. Branchina, F. Contino, N. Darvishi, Phys. Rev. D **106**, 065007 (2022).
[43] C. Branchina, V. Branchina, F. Contino, Phys. Rev. D **107**, 096012 (2023).
[44] T. Steingasser, arXiv:2405.02415 [hep-ph].
[45] T. Steingasser, David I. Kaiser, Phys. Rev. D **108**, 095035 (2023).
[46] M. Reuter, H. Weyer, Phys. Rev. D **79**, 105005 (2009).
[47] G. Gubitosi, R. Ooijer, C. Ripken, F. Saueressig, JCAP **12**, 004 (2018).
[48] G. Gubitosi, C. Ripken, F. Saueressig, Found. Phys. **49**, 972 (2019).
[49] S. Nagy, J. Krizsan, K. Sailer, JHEP **1207**, 102 (2012)

[50] N. Christiansen, D.F. Litim, J.M. Pawlowski, A. Rodigast,Phys. Lett. B **728**, 114 (2014).

[51] S. Nagy, Ann. Phys. **350**, 310 (2014).

[52] S. Kachru, R. Kallosh, A. Linde, S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).

[53] T. Hertog, G. T. Horowitz, K. Maeda, JHEP **0305**, 060 (2003).

[54] E. Manrique, S. Rechenberger, F. Saueressig, Phys. Rev. Lett. **106**, 251302, (2011).

[55] A. Bonanno, T. Denz, J. M. Pawlowski, M. Reichert, SciPost Phys. **12**, 001 (2022).

[56] J. Fehre, D. F. Litim, J. M. Pawlowski, M. Reichert, Phys. Rev. Lett. **130**, 081501, (2023).

[57] J. M. Pawlowski, M. Reichert, *Quantum Gravity from Dynamical Metric Fluctuations.* In: Bambi, C., Modesto, L., Shapiro, I. (eds) Handbook of Quantum Gravity. Springer, Singapore. (2024). https://doi.org/10.1007/978-981-19-3079-9_17-1

[58] J. Laiho, D. Coumbe, Phys. Rev. Lett. **107**, 161301 (2011).