

### Assignment 3

Group member:

1. Nicole Lee A24CS0287
2. Sam Wei Leng A24CS0295
3. Crystal Yap Wen Jing A24CS0240
4. Nurul Nostakhtul Balqis Binti Mohamad Fazli A24CS0177

### Q1: Discrete Probability Theory

1. (a)  $\frac{{}^5C_4}{{}^{25}C_4} = 3.9526 \times 10^{-4}$

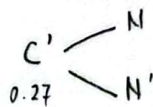
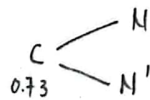
(b)  $\frac{{}^5C_2 \times {}^{20}C_2}{{}^{25}C_4} = 0.1502$

2.  $P(C \cap N) = 0.37$

$$P(N|C) = \frac{P(C \cap N)}{P(C)}$$

$$= \frac{0.37}{0.73}$$

$$= 0.5068$$



3. (a)  $A = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

(b)  $B = \{(3,6), (4,5), (5,4), (6,3)\}$

$$|S| = 36$$

$$P(B) = \frac{|B|}{|S|}$$

$$= \frac{4}{36}$$

$$= 0.1111$$

$$(c) \quad C = \{ (1,6), (2,5), (2,6), (3,4), (3,5), (4,3), (4,4), (5,2), (5,3), (6,1), (6,2) \}$$

$$P(C) = \frac{|C|}{|S|}$$

$$= \frac{11}{36}$$

$$= 0.3056$$

## Q2 : Bayes' Theorem

1. A = computer from Acme

D = computer from DotCom

N = computer from Nuclear

B = defective

$$(a) \quad P(A) = \frac{55}{100} = 0.55$$

$$P(D) = \frac{10}{100} = 0.10$$

$$P(N) = \frac{35}{100} = 0.35$$

(b)  $P(B)$

$$= P(B \cap A) + P(B \cap D) + P(B \cap N)$$

$$= P(B|A)P(A) + P(B|D)P(D) + P(B|N)P(N)$$

$$= 0.01(0.55) + 0.03(0.10) + 0.03(0.35)$$

$$= 0.019$$

2. Let C = have covid-19 active infection

Let P = positive on RT-PCR test

$$P(C) = 0.15 \quad P(P|C) = 0.95 \quad P(P|C') = 0.02$$

$$(a) \quad P(C|P) = \frac{P(C \cap P)}{P(P)}$$

$$= \frac{P(P|C)P(C)}{P(P|C)P(C) + P(P|C')P(C')}$$

$$= \frac{P(P|C)P(C)}{P(P|C)P(C) + P(P|C')P(C')}$$

$$= \frac{0.95(0.15)}{0.95(0.15) + 0.02(0.85)}$$

$$= \frac{0.1425}{0.1425 + 0.017}$$

$$= 0.8934$$

$$P(C') = 1 - 0.15$$

$$= 0.85$$

$$(b) \quad P(C'|P) = \frac{P(C' \cap P)}{P(P)}$$

$$= \frac{P(P|C')P(C')}{P(P)}$$

$$= \frac{0.02(0.85)}{0.1595}$$

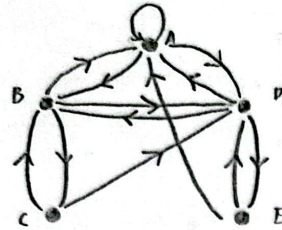
$$= 0.1066$$

$$P(P) = 0.1425 + 0.017$$

$$= 0.1595$$

### Q3: Graph Definition and Notation

1. (a)



$$(b) \deg(A) = 7$$

$$\deg(B) = 6$$

$$\deg(C) = 3$$

$$\deg(D) = 7$$

$$\deg(E) = 3$$

2. (a) Vertex is a fundamental unit of graph, representing a point or node where edges meet.

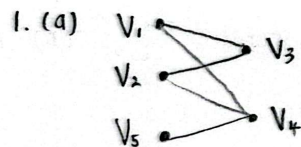
(b) An edge is a line or connection between two vertices in a graph.

(c) A loop is an edge that connects a vertex to itself.

(d) Parallel edges are two or more edges that connect the same pair of vertices.

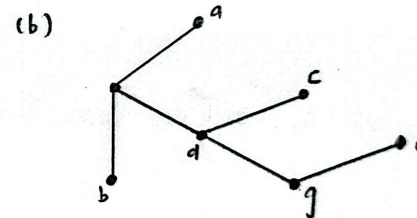
(e) Degree of vertex is the number of edges incident to that vertex.

### Q4: Representation of Graphs



$$S_1 = \{V_1, V_2, V_5\}$$

$$S_2 = \{V_3, V_4\}$$



$$\begin{aligned} \text{total costs} &= 5 + 6 + 5 + 7 + 8 + 20 \\ &= 51 \end{aligned}$$

(b) undirected graph, simple graph, connected graph

## Q5: Isomorphism of Graph

1. no. of vertices :

$$G_1 = 7$$

$$G_2 = 7$$

no. of edges :

$$G_1 = 7$$

$$G_2 = 7$$

no. of degree for each vertex :

$$G_1 = \text{all vertices have degree of } 2$$

$$G_2 = \text{all vertices have degree of } 2$$

Incident function  $f: G_1 \rightarrow G_2$ , where  $G_1 = \{a, b, c, d, e, f, g\}$  and  $G_2 = \{1, 2, 3, 4, 5, 6, 7\}$

$$f(a)=1, f(b)=3, f(c)=5, f(d)=7, f(e)=2, f(f)=4, f(g)=6$$

Adjacency matrices for both graph:

$$A_{G_1} = \begin{matrix} & \begin{matrix} a & b & c & d & e & f & g \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \\ f \\ g \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$A_{G_2} = \begin{matrix} & \begin{matrix} 1 & 3 & 5 & 7 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$\therefore$  Both  $A_{G_1}$  and  $A_{G_2}$  are the same. Therefore, they are isomorphic

2. (a) no. of vertices :

$$G_1 = 6$$

$$G_2 = 6$$

no. of edges :

$$G_1 = 9$$

$$G_2 = 9$$

no. of degree for each vertex :

$$G_1 = \text{all vertices has degree of } 3$$

$$G_2 = \text{all vertices has degree of } 3$$

$\therefore f: G_1 \rightarrow G_2$  is undefined. Hence,  $G_1$  and  $G_2$  is not isomorphic.

(b) no. of vertices :

$$G_1 = 6$$

$$G_2 = 6$$

no. of edges :

$$G_1 = 11$$

$$G_2 = 11$$

no. of degree for each vertex :

$$G_1 = 4 \text{ vertices has degree of } 4 \text{ and}$$

$$2 \text{ vertices has degree of } 3$$

$$G_2 = 4 \text{ vertices has degree of } 4 \text{ and}$$

$$2 \text{ vertices has degree of } 3$$

$\therefore f: G_1 \rightarrow G_2$  is undefined. Hence,  $G_1$  and  $G_2$  is not isomorphic

3.

