

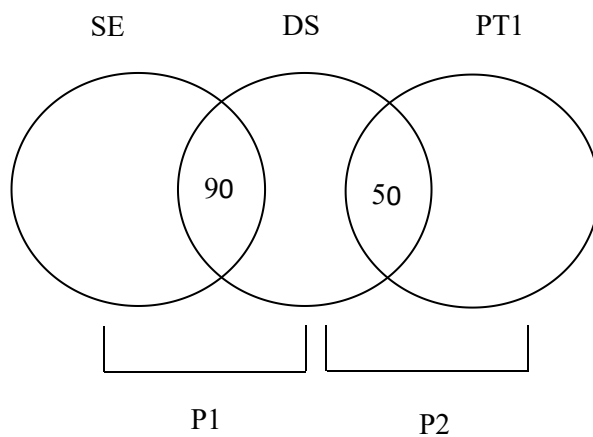
Assignment 1

Group member & Matric number:

1. Sam Wei Leng A24CS0295
2. Nurul Nasrahtul Balqis Binti Mohamad Fazli A24CS0177
3. Nicole Lee A24CS0287
4. Crystal Yap Wen Jing A24CS0240

Question 1

1a. i)



$$\text{ii) } |DS| = |U| - |SE - (SE \cap DS)| - |PT1 - (PT1 \cap DS)|$$

$$= 250 - |SE - 90| - |PT1 - 50|$$

$$= 110 - SE - PT1$$

$$\text{iii) } 90 + 50 = 140$$

$$\text{iv) } 250 - 40 = 110$$

$$\text{b. } A = \{3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

$$N = \{1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

$$B = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$C = \{5\}$$

$$\text{i) } |A| = 9, |B| = 8, |C| = 1$$

ii) $|A| = 9$

$$\begin{aligned} |P(A)| &= 2^9 \\ &= 512 - 1 \\ &= 511 \end{aligned}$$

iii) $C \times B = \{5, 2\}, \{5, 3\}, \{5, 5\}, \{5, 7\}, \{5, 11\}, \{5, 13\}, \{5, 17\}, \{5, 19\}$

Question 2

a. i) $m \wedge n$ = You play table tennis and miss the midterm test.

ii) $\neg(m \vee n) \vee o = \neg m \wedge \neg n \vee o$ [De Morgan's Law]

= You are not playing table tennis and not miss the midterm test or pass the subject.

b. $(a \rightarrow b) \equiv (\neg a \vee b)$

a	b	$a \rightarrow b$	$\neg a \vee b$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Based on the truth table above, it is shown that $(a \rightarrow b) \equiv (\neg a \vee b)$.

c. i) $x \rightarrow y$

ii) $(\neg x \vee \neg z) \rightarrow \neg y$

$$\neg(x \wedge z) \rightarrow \neg y$$

iii) $y \leftrightarrow (x \wedge z)$

Question 3

a. $P(n) = n$ divides 15, Domain of discourse = \mathbb{Z}^+

i) $P(5) = 5$ is the value of n, **True**

- ii) $\forall n P(n)$ = For all values of n, P(n) is true. **False**
 iii) $\exists n \neg P(n)$ = For some values of n, n are not divide 15. **True**

b. $P(m, n) = m \geq n$, Domain of discourse = $\mathbb{Z}^+ \times \mathbb{Z}^+$

- $\exists m \exists n P(m, n)$ = For some values of m and n, P(m, n) is true.

The statement above is **true**. Since the domain of discourse is $\mathbb{Z}^+ \times \mathbb{Z}^+$, that's means the values of m and n could be 1, 4, 9, 16,.....

Let says,

1. The value of m = 1; the value of n = 1. Hence, m = n. $P(m, n) = m \geq n$
2. The value of m = 9; the value of n = 1. Hence, m > n. $P(m, n) = m \geq n$
3. The value of m = 4; the value of n = 16. Hence, n > m. $P(m, n) \neq m \geq n$

In conclusion, there is more than 1 value of m and n are true for the P(m, n) propositional function. However, it's not all true, some of them are false.

- The negation of $\exists m \exists n P(m, n)$, means that none of the value of m and n is true for P(m, n)

$$\neg(\exists m \exists n P(m, n)) = \neg \forall m \neg \forall n P(m, n)$$

Question 4

$\exists x P(x)$

$\exists x (x < a < y)$

Domain of discourse is x and y are real numbers and a is a rational number

If $x = 0, y = 1, a = \frac{1}{2}$,

$0 < \frac{1}{2} < 1$, 0 and 1 are real numbers while $\frac{1}{2}$ is a rational number. Thus, this statement is true.

Question 5

a. If 2 divides m - n

$$(m, n) \in R, x = \{1, 2, 3, 4, 5\}, x, y \in X$$

$$R = \{ (1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), \\ (5, 3), (5, 5) \}$$

$$R^{-1} = \{ (1, 1), (3, 2), (5, 1), (2, 2), (2, 4), (1, 3), (3, 3), (5, 3), (2, 4), (4, 4), (1, 5), \\ (3, 5), (5, 5) \}$$

b. If $m + n \leq 4$,

$$x = \{1, 2, 3, 4, 5\}, x, y \in X$$

$$R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$$

$$R^{-1} = \{(1, 1), (2, 1), (3, 1), (1, 2), (2, 2), (1, 3)\}$$

c. Relation of question a and b are both symmetric.

d. i) $(x, y) \in R$

$$\text{if } x \text{ and } y = 1, R = \{(1, 1)\}$$

- symmetric, antisymmetric and transitive

ii) $(x, y) \in R$

$$\text{if } x = y^2, R = \{ (1, 1), (2, 4), (3, 9), \dots \}$$

- antisymmetric

iii) $(x, y) \in R$

$$\text{if } x = y, R = \{ (1, 1), (2, 2), (3, 3), \dots \}$$

- reflexive, symmetric, antisymmetric, transitive and partial order.

Question 6

a. i) $f(n) = n + 1$

$$f(n_1) = f(n_2)$$

$$n_1 + 1 = n_2 + 1$$

$$n_1 = n_2$$

Function is one-to-one

Let $f(n) = m$,

$$m = n + 1$$

$$n = m - 1$$

m can be any integer. Hence, the function is onto.

\therefore function $f(n) = n + 1$ is both one-to-one and onto.

ii) $f(n) = |n|$

Let $n_1 = 4, n_2 = -4$,

$$f(n_1) = f(n_2)$$

$$|n_1| = |n_2|$$

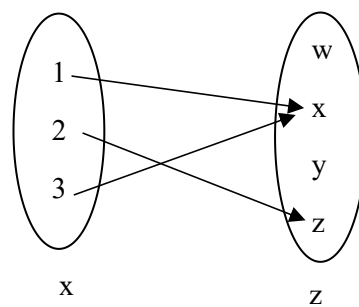
$$|4| = |-4|$$

$$4 = 4$$

Different input, same output. Function is not one-to-one. $|n|$ will only produce positive integers for $f(n)$. No negative integer $f(n)$ will map onto n . Hence, function is not onto.

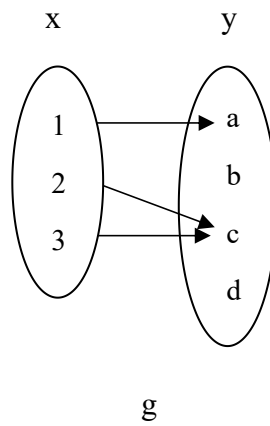
\therefore Function $f(n) = |n|$ is neither one-to-one or onto.

b.



$$m \circ n = \{(1, x), (2, z), (3, x)\}$$

c.



$$g(S) = \{g(x) \mid x \in S\} = \{g(1)\}$$

$$g(1) = a$$

$$g(S) = a$$

$$g(T) = \{g(x) \mid x \in T\} = \{g(1), g(3)\}$$

$$g(1) = a, \quad g(3) = c$$

$$g(T) = \{a, c\}$$

$$g^{-1}(U) = \{x \in X \mid g(x) \in U\}$$

$$g(1) = a \quad (1 \in g^{-1}(U))$$

$$g(2) = c \quad (2 \notin g^{-1}(U))$$

$$g(3) = c \quad (3 \notin g^{-1}(U))$$

$$g^{-1}(U) = \{1\}$$

$$g^{-1}(V) = \{x \in X \mid g(x) \in V\}$$

$$g(1) = a \quad (1 \in g^{-1}(V))$$

$$g(2) = c \quad (2 \in g^{-1}(V))$$

$$g(3) = c \quad (3 \in g^{-1}(V))$$

$$g^{-1}(V) = \{1, 2, 3\}$$