Assignment 2

Group member & Matrics No.:

1. Sam Wei Leng A24CS0295
2. Nurul Nasrahtul Balqis Binti Mohamad Fazli A24CS0177
3. Nicole Lee A24CS0287
4. Crystal Yap Wen Jing A24CS0240

Question 1: Recurrence Relation

1a. i) $a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1$, $a_1 = 6$

$a_2 = 6a_1 - 9a_0$
$= 6(6) - 9$
$= 27$

$a_3 = 6a_2 - 9a_1$
$= 6(27) - 9(6)$
$= 108$

$a_4 = 6a_3 - 9a_2$
$= 6(108) - 9(27)$
$= 405$

$a_5 = 6a_4 - 9a_3$
$= 6(405) - 9(108)$
$= 1458$

Ans: 1, 6, 27, 108, 405, 1458, ...

ii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, $a_0 = 2$, $a_1 = 5$, $a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0$
$= 6(15) - 11(5) + 6(2)$
$= 47$

$a_4 = 6a_3 - 11a_2 + 6a_1$
$= 6(47) - 11(15) + 6(5)$
$= 147$

$a_5 = 6a_4 - 11a_3 + 6a_2$
$= 6(147) - 11(47) - 6(15)$
$= 455$

Ans: 2, 5, 15, 47, 147, 455, ...

iii) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$, $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0$
$= -3(-1) - 3(-2) + 1$
$= 10$

$a_4 = -3a_3 - 3a_2 + a_1$
$= -3(10) - 3(-1) + (-2)$
$= -29$

$a_5 = -3a_4 - 3a_3 + a_2$
$= -3(-29) - 3(10) + (-1)$
$= 56$

Ans: 1, -2, -1, 10, -29, 56, ...

2i. $a_2 = 5a_1 - 3$, $a_1 = k$
$= 5k - 3$

$a_3 = 5(5k - 3) - 3$
$= 25k - 18$

$a_4 = 5a_3 - 3$
$= 5(25k - 18) - 3$
$= 125k - 93$

ii. $a_4 = 125k - 93$

$7 = 125k - 93$

$k = \frac{7+93}{125}$

$= \frac{4}{5}$

Question 2: Basic Principle

1a. $\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$ 10 Books have to be arranged on a shelf

10 9 8 7 6 5 4 3 2 1

Ways to arrange $= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 368800$

b. Case 1: Arrangement starts with Computer Science Books (C)

C M A
C A M] 2 ways

Case 2: Arrangement starts with Art Books (A)

A C M
A M C] 2 ways

Case 3: Arrangement starts with Mathematics Books (M)

M C A
M A C] 2 ways

Total of ways to arrange books group $= 2+2+2 = 6$

Number of ways to arrange Computer Science Books $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Number of ways to arrange Mathematics Books $= 3 \times 2 \times 1 = 6$

Number of ways to arrange Art Books $= 2 \times 1 = 2$

Total number of ways to arrange these books with the books of same discipline are grouped together $= 6 \times 120 \times 6 \times 2$

$= 8640$ #

c. 10 copies of one book ($A_1, A_2, A_3, \dots A_{10} \in A$)
10 different books ($B_1, B_2, B_3, \dots$)

Case 1 : $|A| = 0$

All 10 books are selected from 10 different books $\therefore$ 1 way

Case 2 : $|A| = 1$

9 books have to be selected from 10 different books

$$\therefore \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 10 \text{ ways}$$

Case 3 : $|A| = 2$

8 books from 10 different books

$$\therefore \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 45 \text{ ways}$$

Case 4 : $|A| = 3$

3 copies & 7 different books

$$\therefore \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 120 \text{ ways.}$$

Case 5 : $|A| = 4$

4 copies & 6 different books

$$\therefore \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 210 \text{ ways}$$

Case 6 : $|A| = 5$

5 copies & 5 different books

$$\therefore \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252 \text{ ways}$$

Case 7 : $|A| = 6$

6 copies & 4 different books

$$\therefore \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210 \text{ ways}$$

Case 8 : $|A| = 7$

7 copies & 3 different books

$\therefore \dfrac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$ ways

Case 9 : $|A| = 8$

8 copies & 2 different books

$\therefore \dfrac{10 \times 9}{2 \times 1} = 45$ ways

Case 10 : $|A| = 9$

9 copies & 1 different books $\therefore \dfrac{10}{1} = 10$ ways

Case 11 : $|A| = 9$

10 copies $\therefore$ 1 way

Total number of ways $= 1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 + 1 = 1024$ ways #

2a. $200 - 4 = 196$

b. Case 1 : 1 digit

$\underline{5} = 1$ number

Case 2 : 2 digits

$\underline{1\text{-}9}\ \underline{0,5}$

$9 \times 2 = 18$ numbers

Case 3 : 3 digits

$\underline{1}\ \underline{0\text{-}9}\ \underline{0,5}$

$1 \times 10 \times 2 = 20$ numbers

$\underline{2}\ \underline{0}\ \underline{0}$

$1 \times 1 \times 1 = 1$ numbers

Total $= 1 + 18 + 20 + 1 = 40$ numbers

c) Case 1 : 1 digit

$\underline{7}$ = 1 number

Case 2 : 2 digits

$\underline{1\text{-}9}\ \underline{7}$

$9 \times 1 = 9$ numbers

$\underline{7}\ \underline{0\text{-}9}$

$1 \times 10 = 10$ numbers

Total = 19 - 1
= 18 (Due to the repetition of 77)

Case 3 : 3 digits

$\underline{1}\ \underline{0\text{-}9}\ \underline{7}$

$1 \times 10 \times 1 = 10$ numbers

$\underline{1}\ \underline{7}\ \underline{0\text{-}9}$

$1 \times 1 \times 10 = 10$ numbers

Total = 10 + 10 - 1
= 19 numbers (Due to the repetition of 177)

Total number contains digit 7 : 1 + 18 + 19 = 38

d. Case 1 : 1 digit

$\underline{1\text{-}9}$ = 9 numbers

Case 2 : 2 digits

$\underline{1}\ \underline{2\text{-}9}$

$1 \times 8 = 8$ numbers

$\underline{2}\ \underline{3\text{-}9}$

$1 \times 7 = 7$ numbers

$\underline{3}\ \underline{4\text{-}9}$

$1 \times 6 = 6$ numbers

$\underline{4}\ \underline{5\text{-}9}$

$1 \times 5 = 5$ numbers

$\underline{5}\ \underline{6\text{-}9}$

$1 \times 4 = 4$ numbers

$\underline{6}\ \underline{7\text{-}9}$

$1 \times 3 = 3$ numbers

$\underline{7}\ \underline{8\text{-}9}$

$1 \times 2 = 2$ numbers

$\underline{8}\ \underline{9}$ = 1 number

Case 3 : 3 digits

$\underline{1}\ \underline{2}\ \underline{3\text{-}9}$

$1 \times 1 \times 7 = 7$ numbers

$\underline{1}\ \underline{3}\ \underline{4\text{-}9}$

$1 \times 1 \times 6 = 6$ numbers

$\underline{1}\ \underline{4}\ \underline{5\text{-}9}$

$1 \times 1 \times 5 = 5$ numbers

$\underline{1}\ \underline{5}\ \underline{6\text{-}9}$

$1 \times 1 \times 4 = 4$ numbers

$\underline{1}\ \underline{6}\ \underline{7\text{-}9}$

$1 \times 1 \times 3 = 3$ numbers

$\underline{1}\ \underline{7}\ \underline{8\text{-}9}$

$1 \times 1 \times 2 = 2$ numbers

$\underline{1}\ \underline{8}\ \underline{9}$ = 1 number

Sum = 28

Total numbers = 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 28
= 73 numbers

Question 3: Permutation & Combination

1a. $\underline{AEC}$ $\underline{B}$ $\underline{D}$ $= 3!$
$= 3 \times 2 \times 1$
$= 6$

b. $\underline{AE/EA}$ $\underline{C}$ $\underline{B}$ $\underline{D}$
$2! \times 4! = 45$

c. _ J _ J _ J _ J _ J _ J _ J _ J _
$^9C_5 = 126$
$5! \times 8! \times 126 = 609638400$

d. $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 3628800$

2. $^{11}C_3 = C(11,3) = \frac{11!}{3!(11-3)!}$
$= 165$

3. Assumption made for this question: No repetition

Specialty Pizzas = 4

Case 1: Pizza with NO unique toppings
Only 1 type of pizza

Case 2: Pizza with ONE unique topping
$^{17}C_1 = C(17,1) = \frac{17!}{1!(17-1)!}$
$= 17$

Case 3: Pizza with TWO unique toppings
$^{17}C_2 = C(17,2) = \frac{17!}{2!(17-2)!}$
$= 136$

Case 4: Pizza with THREE unique toppings
$^{17}C_3 = C(17,3) = \frac{17!}{3!(17-3)!}$
$= 680$

Total types of pizza $= 4 + 1 + 17 + 136 + 680 = 838$

4. ${}^4C_3 = C(4,3) = \frac{4!}{3!(4-3)!}$
$= 4$

Question 4: Pigeonhole Principle (First, Second, Third Form)

1. Pigeon = Students : set X , $s_1, s_2, s_3, s_4, \ldots s_n \in X$
   Pigeonhole = Scores on the final exam : set Y , $Y = \{0,1,2,3,\ldots,100\}$
   k = number of pigeonhole
   $= 0 - 100$
   $= 101$
   $|Y| = k = 101 \quad |X| = n$

   Based on second form of pigeonhole principle, $|X| > |Y|$, so that at least 2 students will receive the same score on the final exam.
   Hence, $|X| = n > 101$, $|X| = n = 102$, which is the number of students.

2. By apply third form of pigeonhole principle,
   Set X = number of students, $X = \{s_1, s_2, s_3, s_4, \ldots s_n\}$
   Set Y = letter grade, $Y = \{A, B, C, D, F\}$

   $|X| = n \quad |Y| = k = 5$
   If at least 5 students will receive the same letter grade. Hence, $m = 5$
   $m = \frac{n}{k}$
   $5 = \frac{n}{5}$
   $n = 25$
   $\therefore$ Maximum number of students is 25 for at least 5 students to receive the same letter grade.
   Thus, $|X|$ has to be greater than 25 for at least 6 students to receive the same letter grade. So, we know that minimum $|X| = 26$.

3. Pigeon = students , $n = 35$
   Pigeonhole = letters in alphabet , $k = 26$

   $n > k$, number of students is more than number of letters in alphabet.
   $\therefore$ There will be at least two students have first names that start with the same letter. Proven.

4. Pigeon = People , $n = 13$

Pigeonhole = Combination of possible first name and last name.

3 first names and 4 last names are available. Thus, the combination $= 3 \times 4 = 12$

$\therefore\ k = 12$

Now, $n > k$, based on the first form of pigeonhole principle, there will be at least 2 persons have the same first and last names.

Shown.