Sequences

You can read a gentle introduction to Sequences in [Common Number Patterns](#).

What is a Sequence?

A Sequence is a list of things (usually numbers) that are in order.

("term", "element" or "member" mean the same thing)

Infinite or Finite

When the sequence goes on forever it is called an **infinite sequence**, otherwise it is a **finite sequence**

Examples:

{1, 2, 3, 4, ...} is a very simple sequence (and it is an **infinite sequence**)

{20, 25, 30, 35, ...} is also an infinite sequence

{1, 3, 5, 7} is the sequence of the first 4 odd numbers (and is a **finite sequence**)

{4, 3, 2, 1} is 4 to 1 **backwards**

{1, 2, 4, 8, 16, 32, ...} is an infinite sequence where every term doubles

{a, b, c, d, e} is the sequence of the first 5 letters **alphabetically**

{f, r, e, d} is the sequence of letters in the name "fred"

{0, 1, 0, 1, 0, 1, ...} is the sequence of **alternating** 0s and 1s (yes they are in order, it is an alternating order in this case)

In Order

When we say the terms are "in order", we are free to define **what order that is**! They could go forwards, backwards ... or they could alternate ... or any type of order we want!

Like a Set

A Sequence is like a Set, except:

- the terms are **in order** (with Sets the order does not matter)
- the same value can appear many times (only once in Sets)

Example: {0, 1, 0, 1, 0, 1, ...} is the **sequence** of alternating 0s and 1s.

The **set** is just {0,1}

Notation

Sequences also use the same **notation** as sets:
list each element, separated by a comma,
and then put curly brackets around the whole thing.

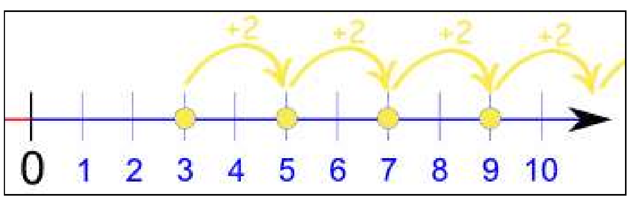
{3, 5, 7, ...}

The curly brackets { } are sometimes called "set brackets" or "braces".

A Rule

A Sequence usually has a **Rule**, which is a way to find the value of each term.

Example: the sequence {3, 5, 7, 9, ...} starts at 3 and jumps 2 every time:



As a Formula

Saying "*starts at 3 and jumps 2 every time*" is fine, but it doesn't help us calculate the:

- 10^{th} term,
- 100^{th} term, or
- n^{th} term, where ***n*** could be any term number we want.

So, we want a formula with "n" in it (where n is any term number).

So, What Can A Rule For {3, 5, 7, 9, ...} Be?

Firstly, we can see the sequence goes up 2 every time, so we can **guess** that a Rule is something like "2 times n" (where "n" is the term number). Let's test it out:

Test Rule: 2n

n	Term	Test Rule
1	3	2**n** = 2×**1** = **2**
2	5	2**n** = 2×**2** = **4**
3	7	2**n** = 2×**3** = **6**

That **nearly** worked ... but it is **too low** by 1 every time, so let us try changing it to:

Test Rule: 2n+1

n	Term	Test Rule
1	3	2**n**+1 = 2×**1** + 1 = 3
2	5	2**n**+1 = 2×**2** + 1 = 5
3	7	2**n**+1 = 2×**3** + 1 = 7

That Works!

So instead of saying "starts at 3 and jumps 2 every time" we write this:

2n+1

Now we can calculate, for example, the **100th term**:

2 × 100 + 1 = **201**

Many Rules

But mathematics is so powerful we can find **more than one Rule** that works for any sequence.

Example: the sequence {3, 5, 7, 9, ...}

We have just shown a Rule for {3, 5, 7, 9, ...} is: **2n+1**

And so we get: {3, 5, 7, 9, 11, 13, ...}

But can we find another rule?

How about **"odd numbers without a 1 in them"**:

And we get: {3, 5, 7, 9, 23, 25, ...}

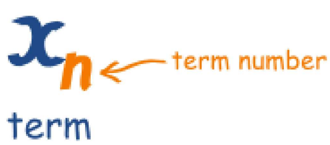
A completely different sequence!

And we could find more rules that match **{3, 5, 7, 9, ...}**. Really we could.

So it is best to say "A Rule" rather then "The Rule" (unless we know it is the right Rule).

Notation

To make it easier to use rules, we often use this special style:



- $\mathbf{x_n}$ is the term
- **n** is the term number

Example: to mention the "5th term" we write: x_5

So a rule for **{3, 5, 7, 9, ...}** can be written as an equation like this:

$$x_n = 2n+1$$

And to calculate the 10th term we can write:

$$x_{10} = 2\mathbf{n}+1 = 2\times\mathbf{10}+1 = 21$$

Can you calculate x_{50} (the 50th term) doing this?

Here is another example:

Example: Calculate the first 4 terms of this sequence:

$$\{a_n\} = \{ (-1/n)^n \}$$

Calculations:

- $a_1 = (-1/1)^1 = -1$
- $a_2 = (-1/2)^2 = 1/4$
- $a_3 = (-1/3)^3 = -1/27$
- $a_4 = (-1/4)^4 = 1/256$

Answer:

$$\{a_n\} = \{ -1, 1/4, -1/27, 1/256, ... \}$$

Special Sequences

Now let's look at some special sequences, and their rules.

Arithmetic Sequences

In an Arithmetic Sequence **the difference between one term and the next is a constant**.

In other words, we just add some value each time ... on to infinity.

Example:

1, 4, 7, 10, 13, 16, 19, 22, 25, ...

This sequence has a difference of 3 between each number.
Its Rule is $\mathbf{x_n = 3n-2}$

In General we can write an arithmetic sequence like this:

$$\{a, a+d, a+2d, a+3d, \dots \}$$

where:

- **a** is the first term, and
- **d** is the difference between the terms (called the **"common difference"**)

And we can make the rule:

$$x_n = a + d(n-1)$$

(We use "n-1" because **d** is not used in the 1st term).

Geometric Sequences

In a Geometric Sequence each term is found by **multiplying** the previous term by a **constant**.

Example:

2, 4, 8, 16, 32, 64, 128, 256, ...

This sequence has a factor of 2 between each number.

Its Rule is $\mathbf{x_n = 2^n}$

In General we can write a geometric sequence like this:

$$\{a, ar, ar^2, ar^3, \dots \}$$

where:

- **a** is the first term, and
- **r** is the factor between the terms (called the **"common ratio"**)

Note: **r** should not be 0.

- When **r=0**, we get the sequence {a,0,0,...} which is not geometric

And the rule is:

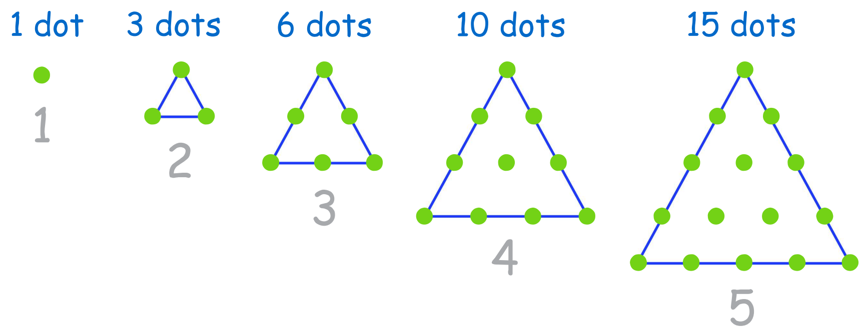
$$x_n = ar^{(n-1)}$$

(We use "n-1" because ar^0 is the 1st term)

Triangular Numbers

1, 3, 6, 10, 15, 21, 28, 36, 45, ...

The Triangular Number Sequence is generated from a pattern of dots which form a triangle:



By adding another row of dots and counting all the dots we can find the next number of the sequence.

But it is easier to use this Rule:

$$x_n = n(n+1)/2$$

Example:

- the 5th Triangular Number is $x_5 = 5(5+1)/2 = \mathbf{15}$,
- and the sixth is $x_6 = 6(6+1)/2 = \mathbf{21}$

Square Numbers

1, 4, 9, 16, 25, 36, 49, 64, 81, ...

The next number is made by squaring where it is in the pattern.

Rule is $\mathbf{x_n = n^2}$

Cube Numbers

1, 8, 27, 64, 125, 216, 343, 512, 729, ...

The next number is made by cubing where it is in the pattern.

Rule is $\mathbf{x_n = n^3}$

Fibonacci Sequence

This is the Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The next number is found by **adding the two numbers before it** together:

- The 2 is found by adding the two numbers before it (1+1)
- The 21 is found by adding the two numbers before it (8+13)
- etc...

Rule is $\mathbf{x_n = x_{n-1} + x_{n-2}}$

That rule is interesting because it depends on the values of the previous two terms.

Rules like that are called **recursive** formulas.

The Fibonacci Sequence is numbered **from 0 onwards** like this:

n =	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*...*
x_n =	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	...

Example: term "6" is calculated like this:

$$\mathbf{x_6 = x_{6-1} + x_{6-2} = x_5 + x_4 = 5 + 3 = 8}$$

Series and Partial Sums

Now you know about sequences, the next thing to learn about is how to sum them up. Read our page on Partial Sums.

When we **sum** up just **part** of a sequence it is called a Partial Sum.

But a **sum** of an **infinite** sequence it is called a "Series" (it sounds like another name for sequence, but it is actually a sum). See Infinite Series.

Example: Odd numbers

Sequence: **{1, 3, 5, 7, ...}**

Series: **1 + 3 + 5 + 7 + ...**

Partial Sum of first 3 terms: **1 + 3 + 5**

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