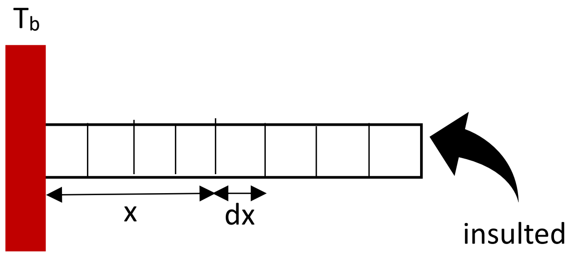
NUMERICAL ANALYSIS OF 1D CONDUCTION STEADY STATE HEAT TRANSFER

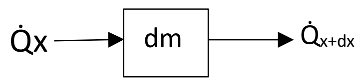
STATEMENT:

Determine the steady state temperature distribution within an Aluminium fin using the finite difference method. Assume the base temperature of the fin at 100°C and the tip of the fin is insulted. Initially fin is at temperature of 30°C.

STEP-1: Formulation of governing differential equation

Energy equation (Law of conservation of energy)





$dm = \rho dV$

$= \rho A_c dx$

$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{stored}$

or, $\dot{Q}_x - \dot{Q}_{x+dx} = dmC_p(dT/dt)$

$= \rho A_c dx C_p(dT/dt)$ __________(1)

Here,

T_b= Base temperature

dm= mass of each element

ρ= density

A_c= Area of cross-section

From Taylor Series Expansion,

$f(x+dx) = f(x) + f'(x)dx/1! + f''(x)(dx)^2/2! + f'''(x)(dx)^3/3! + ...$

$f(x) = \dot{Q}_x$ and $f(x+dx) = \dot{Q}_{x+dx}$

So, $\dot{Q}_{x+dx} = \dot{Q}_x + \dot{Q}'_x dx/1!$ _________(2)

$\dot{Q}x - (\dot{Q}_x + \dot{Q}'_x\, dx/1!) = \rho A c dx C p (dT/dt)$

or, $-\dot{Q}'x\, dx/1! = \rho A c dx C p (dT/dt)$

For steady state, $dT/dt = 0$

So, $\dot{Q}_x = 0$ _____________(3)

From fourier's law of heat conduction,

$\dot{Q}_x = KA_c(dT/dx)$ ___________(4)

or, $d\dot{Q}_x/dx=0$

or, $d(-KA_c dT/dx) = 0$

or, $d^2T/dx^2 = 0$ _________Governing equ^n for 1D steady state heat conduction

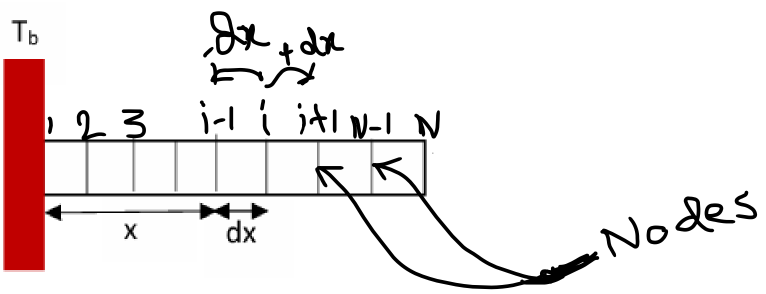
Boundary Conditions:

At x=0 , $T = T_b$

At x=L, $\dot{Q}_L = 0 \Rightarrow -KA_c(dT/dx) = 0 \Rightarrow dT/dx = 0$

STEP-2: Discretization of the domain

dx= L/(N-1)



STEP-3: Converting differential equ^n to algebraic equ^n

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$$D^2T/dx^2 = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_(5)}$$

Taylor's expansion,

$$T(x+dx) = T(x) + T'(x)dx/1! + T''(x)(dx)^2/2! + T'''(x)(dx)^3/3! + \ldots\ldots$$

$$T(x-dx) = T(x) - T'(x)dx/1! + T''(x)(dx)^2/2! - T'''(x)(dx)^3/3! + \ldots\ldots$$

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$$T(x+dx) + T(x-dx) = 2T(x) + (d^2T/dx^2)(dx)^2$$

or, $d^2T/dx^2 = ((T(x+dx) + T(x-dx) - 2T(x))/(dx)^2$ _____________(6)

So, $T(x+dx) + T(x-dx) - 2T(x) = 0$

or, $T(x) = (T(x+dx) + T(x-dx))/2$

or $T(i) = (T(i+1) + T(i-1))/2$ $\quad (2 \le i \le N-1)$

Now,

For, i=N , $dT/dx = 0$

$T(x-dx) = T(x) - (dT/dx)/(dx/1!) + (d^2T/dx^2)/(dx)^2/2! + \dots\dots$

or, $dT/dx = (T(x-dx) - T(x))/dx$

or, $T(x-dx) - T(x) = 0$

or, $T(x-dx) = T(x)$

or, $T(N-1) = T(N)$

Therefore,

For $i=1$, $T = T_b$

For $2 <= i <= (N-1)$, $T(i) = (T(i+1) + T(i-1))/2$

For $i = N$, $T(N) = T(N-1)$