6DOF AUV Trajectory Optimization using MPC

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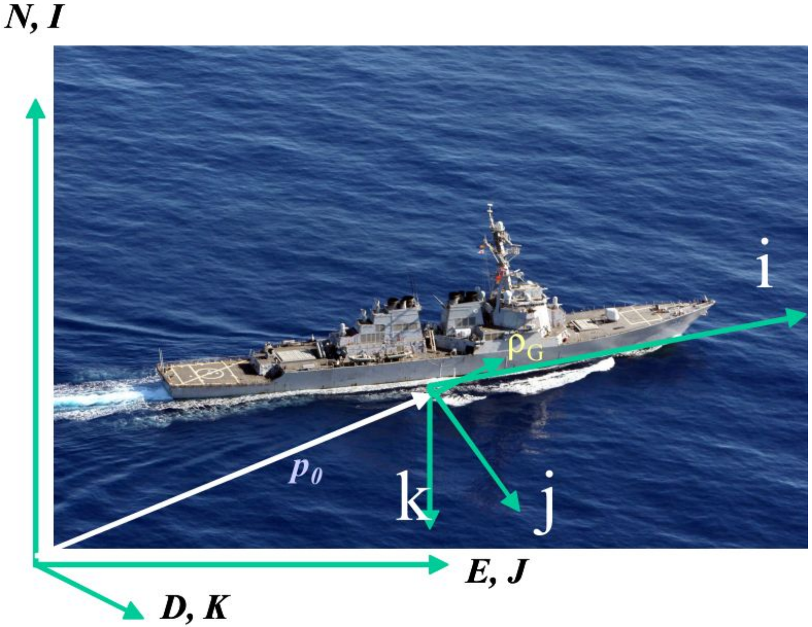


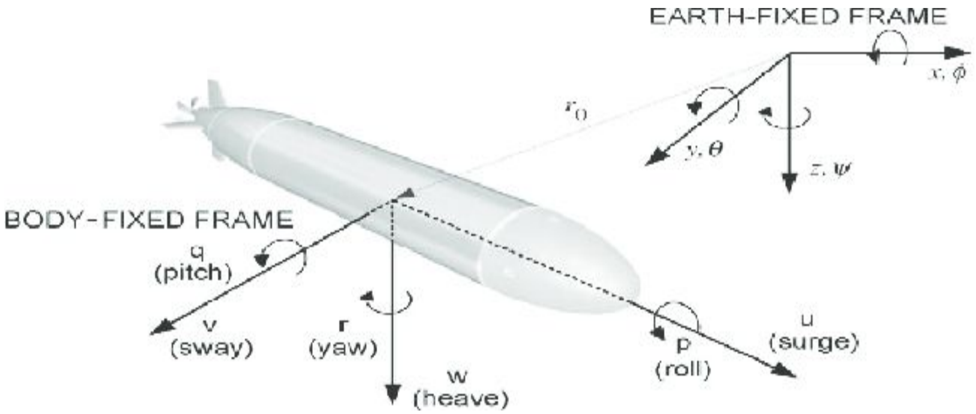


Figure 1(left): Earth fixed and body fixed reference frame of a surface vehicle. Same coordinate system has been used for underwater vehicle study. Source: Healey course [2]

Figure 2(top): Six degrees of freedom of an AUV. Credits: [ANFN controller based on differential evolution for Autonomous Underwater Vehicles]()

6DOF Kinematic equations of motions

$$\boldsymbol{T}(\varphi,\theta,\psi)=\begin{bmatrix}\cos\psi\cos\theta, & \sin\psi\cos\theta, & -\sin\theta\\ \cos\psi\sin\theta\sin\varphi-\sin\psi\cos\varphi, & \sin\psi\sin\theta\sin\varphi+\cos\psi\cos\varphi, & \cos\theta\sin\varphi\\ \cos\psi\sin\theta\cos\varphi+\sin\psi\sin\varphi, & \sin\psi\sin\theta\cos\varphi-\cos\psi\sin\varphi, & \cos\theta\cos\varphi\end{bmatrix}$$

Euler rotation matrix

$$\begin{bmatrix}u\\ v\\ w\end{bmatrix}_b=[\boldsymbol{T}(\varphi,\theta,\psi)]_n^b\bullet\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix}_n$$

Translational velocity transformation

$$\begin{bmatrix}p\\ q\\ r\end{bmatrix}=\begin{bmatrix}1 & 0 & -\sin\theta\\ 0 & \cos\varphi & \sin\varphi\cos\theta\\ 0 & -\sin\varphi & \cos\varphi\cos\theta\end{bmatrix}\begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}$$

Angular velocity transformation

$$\boldsymbol{F}=m\left\{\dot{\boldsymbol{v}}+\dot{\omega}\times\rho_G+\omega\times\omega\times\rho_G+\omega\times\boldsymbol{v}\right\}$$

Translational EOM

$$\boldsymbol{M}_o=\boldsymbol{I}_o\dot{\omega}+\omega\times\left(\boldsymbol{I}_o\omega\right)+m\left\{\rho_G\times\dot{\boldsymbol{v}}+\rho_G\times\omega\times\boldsymbol{v}\right\}$$

Rotational EOM

Buoyancy and gravity effects-

The net vertical force components along the body axes, f_g-

$$\boldsymbol{f}_g=(W-B)\begin{bmatrix}-\sin\theta\\ \cos\theta\sin\varphi\\ \cos\theta\cos\varphi\end{bmatrix}$$

The effects of the weight and buoyant forces as moments about the body fixed frame origin-

$$\boldsymbol{m}_g=\mathrm{W}\,\rho_G\times\begin{bmatrix}-\sin\theta\\ \cos\theta\sin\varphi\\ \cos\theta\cos\varphi\end{bmatrix}-\mathrm{B}\,\rho_B\times\begin{bmatrix}-\sin\theta\\ \cos\theta\sin\varphi\\ \cos\theta\cos\varphi\end{bmatrix}$$

The total force and moment vector along vertical axis can be written in body coordinates as

$$\boldsymbol{K}_g(z)=\begin{bmatrix}\boldsymbol{f}_g\\ \boldsymbol{m}_g\end{bmatrix}$$

6DOF Kinematic equations of motions

Separating out the terms for inertial and non-inertial forces we get the vehicle state equations as

$$\boldsymbol{M\dot{x}+C(x)+K_g(z)=F_{app}(\dot{x},x,t)}$$

where, $\boldsymbol{x}=[u,v,w,p,q,r]'$

and $F(t)=[X_{app},(t)\,Y_{app},(t),Z_{app}(t),K_{app}(t),M_{app}(t),N_{app}(t)]'$

M is a 6x6 mass matrix-

$$\boldsymbol{M}=\begin{bmatrix}\begin{bmatrix}m&0&0\\0&m&0\\0&0&m\end{bmatrix} & m\begin{bmatrix}0&z_G&-y_G\\-z_G&0&x_G\\y_G&-x_G&0\end{bmatrix}\\ m\begin{bmatrix}0&-z_G&y_G\\z_G&0&-x_G\\-y_G&x_G&0\end{bmatrix} & \begin{bmatrix}I_{xx}&I_{xy}&I_{xz}\\I_{yx}&I_{yy}&I_{yz}\\I_{zx}&I_{zy}&I_{zz}\end{bmatrix}\end{bmatrix}$$

The remaining terms on the left hand side arising from centripetal and coriolis accelerations

$$\boldsymbol{C}(\boldsymbol{x})=\begin{bmatrix}m(\omega\times\omega\times\rho_G+\omega\times\boldsymbol{v})\\ \omega\times(\boldsymbol{I}_o\omega)+m\{\rho_G\times\omega\times\boldsymbol{v}\}\end{bmatrix}$$

Ocean Current calculations

In order to account for the ocean currents, we adopt an additional coordinate frame that moves parallel to the N-E-D reference frame with the local water particles, and has some relative velocity vector with respect to the global reference frame.

Transformation to body fixed reference frame-

$$[u_c,v_c,w_c]'_b=[\boldsymbol{T(a)}]^b_n[U_{cx},U_{cy},U_{cz}]_n'$$

Inertial velocity of center of gravity of vehicle-

$$\boldsymbol{\dot{p}_G=[T(a)]^b_n U_c+v_r}+\varpi\times\rho_G$$

The only term that changes is the non-inertial force component so that the coriolis additional force and moment terms induced by current becomes

$$\boldsymbol{C(x_r)}=\begin{bmatrix}m(\boldsymbol{S}^2(\omega)\rho_G+\boldsymbol{S}(\omega)\boldsymbol{v}_r)\\ \boldsymbol{S}(\omega)\boldsymbol{I}_o\omega+m\boldsymbol{S}(\rho_G)\boldsymbol{S}(\omega)\boldsymbol{v}_r\end{bmatrix}+\begin{bmatrix}m\boldsymbol{S}(\omega)[\boldsymbol{T(a)}]^b_n\boldsymbol{U}_c\\ m\boldsymbol{S}(\rho_G)\boldsymbol{S}(\omega)[\boldsymbol{T(a)}]^b_n\boldsymbol{U}_c\end{bmatrix}$$

Body-Fluid Interaction

Four ideal cases. In real fluids, the hydrodynamic force on a submerged vehicle depends on-

1. the relative velocity between the water particles and the vehicle (the drag force); Term comes from ideal case (4)
2. the relative acceleration between the water particles and the vehicle (the inertia and 'added mass' terms), and
3. the modifications to the 'buoyant' forces resulting from water particle acceleration.

$$[m + C_a \rho(\pi c^2)]\frac{du}{dt} = (1 + C_a)\rho(\pi c^2)\frac{dU_f}{dt} + 0.5\rho C_d D(U_f - u)\left|(U_f - u)\right|$$

C_a is added mass coefficient; $\rho(\pi c^2)$ is the mass of displaced fluid; D is the projected area per unit length of the body exposed to the flow, for flow at high Reynolds numbers, C_d is commonly taken to be 2.0 and the modified square law mathematically preserves the correct sign

This is the longitudinal motion equation for an underwater vehicle in the u-direction in actual ocean environment with no other motion involved.

Model Predictive Control (MPC)

Kinematic model of a point-based AUV [1]

$$\dot{x}(t) = u_1(t) + 1.1\, e^{-\left(\frac{y}{0.3}\right)^2} \cos(2\pi\tau),$$

$$\dot{y}(t) = u_2(t),$$

$$\dot{\tau}(t) = 1$$

Cost function for the MPC

$$\min_{u1,u2} J = \int_t^{t+\tau_h} [u_1^2 + u_2^2 + (x - x_d)^2 + (y - y_d)^2]\, dt,$$

Subject to:

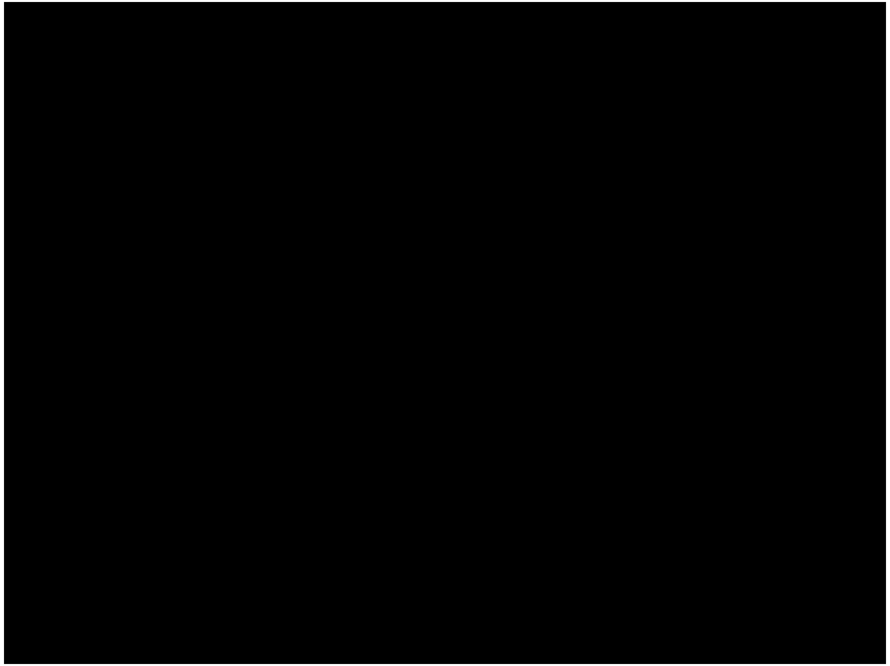
$$\dot{X} = f(X, U, t),$$

$$U \in [U^-, U^+]$$

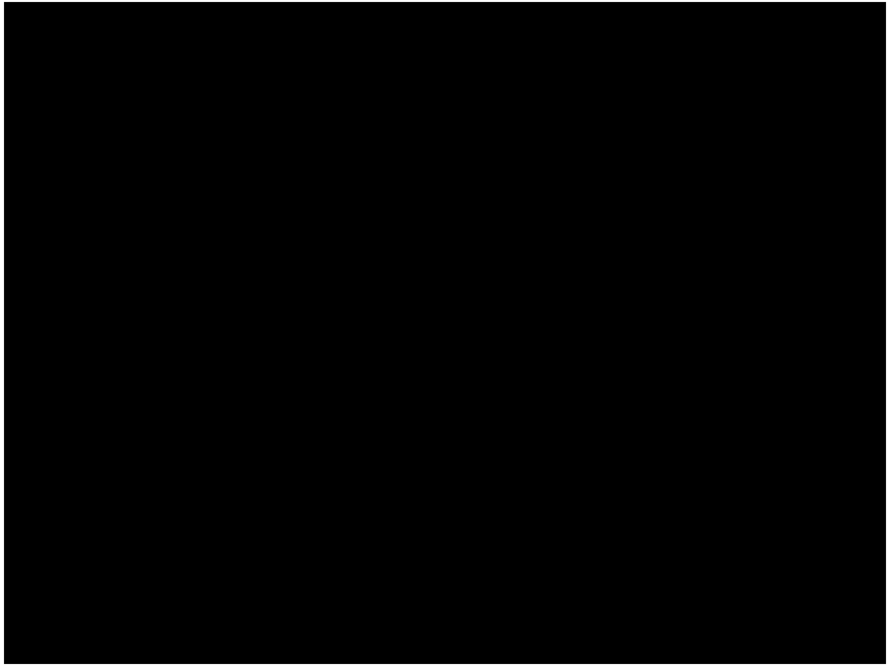
where $X = \{x, y, \tau\}, U = \{u_1, u_2\}, \tau_h$ is the prediction horizon, and x_d, y_d are the destination coordinates.

Simulation results for point mass Robot

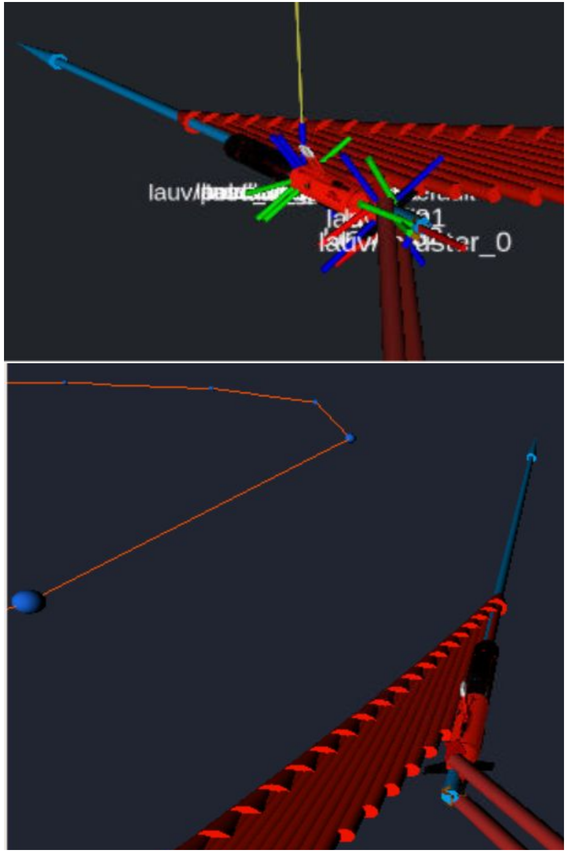
Time-varying flow field

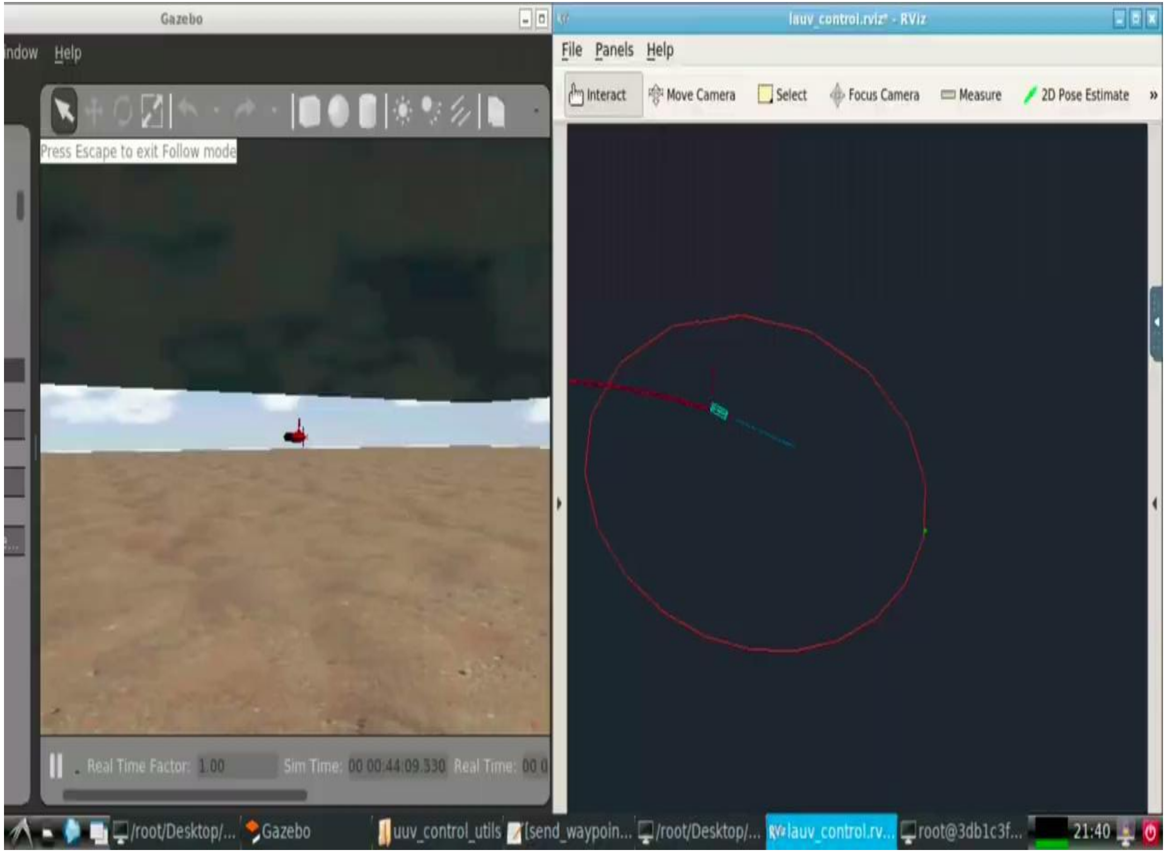


Cross field flow with obstacle avoidance



UUV Simulator





Simulation of a time-varying ocean environment and go-to-goal scheme for LAUV model[4].

References

1. Miguel Aguiar, João Borges de Sousa, João Miguel Dias, Jorge Estrela da Silva, Renato Mendes and Américo S. Ribeiro, ”Trajectory Optimization for Underwater Vehicles in Time-Varying Ocean Flows”, IEEE/OES Autonomous Underwater Vehicle Workshop (AUV), 2018 link
2. Course notes on dynamics and control of marine robotic vehicles by A. J. Healey, Department of Mechanical and Astronautical Engineering Naval Postgraduate School, Monterey, California
3. Workshop on Optimization based Solutions for Control and State Estimation in Dynamical Systems (Implementation to Mobile Robots) link
4. Sousa, Alexandre, et al. "LAUV: The man-portable autonomous underwater vehicle." IFAC Proceedings Volumes 45.5 (2012): 268-274. Link
5. Guidance and control of Ocean vehicles by Thor I. Fossen