6DOF Autonomous Underwater Vehicle trajectory optimization using Model Predictive Control

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Abstract

This study aims to address the problem of predicting optimal trajectory for a 6 degrees of freedom (DOF) Autonomous Underwater Vehicle (AUV) in time-varying ocean currents in presence of continuous ship traffic. The physics of a rigid body has been studied and a complete dynamical system of equations has been prepared which takes into account a wide range of inertial and gravitational forces acting real-time on the submerged underwater vehicle. This model includes the interactions of the rigid body with the complex ocean environment that changes from time to time. The control inputs will be incorporated into UUV simulator in an exact ocean environment and the simulation results will be displayed.

1 Introduction

This study aims to understand the control of autonomous vehicles for underwater service and model the same with the help of a simulator. A Newton-Euler approach for the derivation of the 6 DOF equations of motion is carried out. Unmanned Underwater Vehicles (UUVs) derive their propulsive and maneuvering forces from interaction with the ocean water, so hydro static and hydrodynamic forces play a major role in deriving the trajectories. In this model, the equations for these forces have been evaluated for drag and lift from body and control surfaces. The control model is required to traverse the best possible trajectory in order to reach the target way point in minimum time elapsed. Model Predictive Control have been used to find the optimal trajectory of the underwater vehicle in constrained systems to accomplish this task. Further, an actual ocean environment have been simulated with the help of UUV simulator and an AUV model known as Light Autonomous Underwater Vehicle (**LAUV**)2, developed by the Laboratório de Sistemas e Tecnologia Subaquática (LSTS) from Porto University and OceanScan-MST has been deployed into it[5].

2 Theory

2.1 Dynamics of Rigid Body Vehicles

There are certain assumption that have been considered for better understanding of the problem statement-

1. The vehicle behaves as a rigid body.
2. The Earth's rotation is negligible as far as acceleration components of vehicle's center of mass is concerned.
3. The primary forces acting on vehicle is of **inertial** and **gravitational** origins.

The forces being considered here are hydrostatic, propulsion, thruster and hydrodynamic forces from lift and drag.

2.1.1 Coordinate frame and notation

For maneuvering and control studies of underwater vehicles two reference frames are most commonly used: body-fixed reference frame with its origin at the center of buoyancy (different from center of mass) of the vehicle and earth-fixed reference frame, also known as North-East-Down (NED) frame which remains unchanged throughout the mission.

Thus, in global navigation frame, a vehicle's position, $\mathbf{p_0}$, can be written as

$$\mathbf{p_0} = \left[X_0\mathbf{I} + Y_0\mathbf{J} + Z_0\mathbf{K}\right] = \left[X_0, Y_0, Z_0\right]_n \tag{1}$$

Upon making any rotation, the new position components $\mathbf{p_1} = [x_1, y_1, z_1]_1$ can be expressed in matrix form by the rotation matrix operation below-

$$\mathbf{p_1} = \left[\boldsymbol{T}(\varphi, \theta, \psi)\right]_n^b\right] \mathbf{p_0} \tag{2}$$

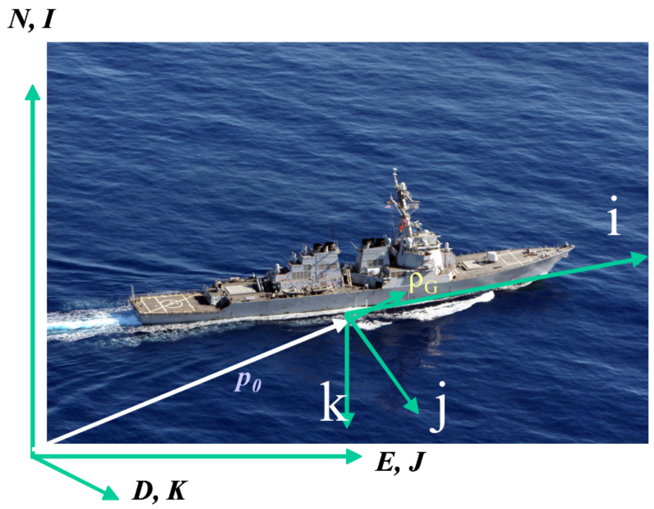


Figure 1: Earth fixed reference frame and body fixed reference frame of a surface vehicle, Ship. Same will be the case for underwater vehicle.

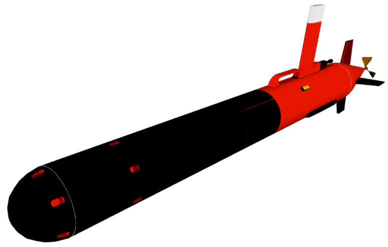


Figure 2: Light Autonomous Underwater Vehicle (LAUV) model in Gazebo, developed by the Laboratório de Sistemas e Tecnologia Subaquática (LSTS) from Porto University and OceanScan-MST has been deployed into it[5].

the rotation matrix $\left[\boldsymbol{T}(\varphi,\theta,\psi)\right]_n^b$ indicates a transformation from the n frame (Earth fixed) to the b frame (body fixed). The angles φ,θ,ψ are known as *Euler Angles* and the complete matrix of rotation can be calculated by performing a series of rotations known as *Euler Rotations* as described in [2].

$$\boldsymbol{T}(\varphi,\theta,\psi)=\begin{bmatrix} \cos\psi\cos\theta, & \sin\psi\cos\theta, & -\sin\theta \\ \cos\psi\sin\theta\sin\varphi-\sin\psi\cos\varphi, & \sin\psi\sin\theta\sin\varphi+\cos\psi\cos\varphi, & \cos\theta\sin\varphi \\ \cos\psi\sin\theta\cos\varphi+\sin\psi\sin\varphi, & \sin\psi\sin\theta\cos\varphi-\cos\psi\sin\varphi, & \cos\theta\cos\varphi \end{bmatrix} \tag{3}$$

2.1.2 Kinematics

Upon performing similar computations for translational and rotational (attitude) velocities, their values in terms of global quantities are given by the forward transformation defined in the previous section to be,

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}_b = \left[\boldsymbol{T}(\varphi,\theta,\psi)\right]_n^b \bullet \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix}_n \tag{4}$$

where the components u, v, w, are called the **surge, sway** and **heave** velocity components in the body fixed frame respectively.
Extending it for acceleration in body fixed frame and global frame we get a system of equations-

TABLE 2.1

SURGE EQUATION OF MOTION

$$m\left[\dot{u}-vr+wq-x_G\left(q^2+r^2\right)+y_G(pq-\dot{r})+z_\sigma(pr+\dot{q})\right]+(W-B)\sin\theta=X_f$$

SWAY EQUATION OF MOTION

$$m\left[\dot{v}+ur-wp+x_G(pq+\dot{r})-y_G\left(p^2+r^2\right)+z_G(qr-\dot{p})\right]-(W-B)\cos\theta\sin\varphi=Y_f$$

HEAVE EQUATION OF MOTION

$$m\left[\dot{w}-uq+vp+x_G(pr-\dot{q})+y_G(qr+\dot{p})-z_G\left(p^2+q^2\right)\right]-(W-B)\cos\theta\cos\varphi=Z_f$$

ROLL EQUATION OF MOTION

$$\begin{aligned} &I_x\dot{p}+(I_z-I_y)\,qr+I_{xy}(pr-\dot{q})-I_{yz}\left(q^2-r^2\right)-I_x(pq+\dot{r})+m\left[y_d(\dot{w}-uq+vp)\right.\\ &\left.-z_G(\dot{v}+ur-wp)\right]-(y_GW-y_BB)\cos\theta\cos\varphi+(z_GW-z_BB)\cos\theta\sin\varphi=K_f \end{aligned}$$

PITCH EQUATION OF MOTION

$$\begin{aligned} &I_y\dot{q}+(I_x-I_z)\,pr-I_{xy}(qr+\dot{p})+I_{yz}(pq-\dot{r})+I_x\left(p^2-r^2\right)-m\left[x_d(\dot{w}-vq+vp)\right.\\ &\left.-z_G(\dot{u}-vr+wq)\right]+(x_GW-x_BB)\cos\theta\cos\varphi+(z_GW-z_BB)\sin\theta=M_f \end{aligned}$$

YAW EQUATION OF MOTION

$$I_y\dot{r} + (I_y - I_x)\,pq - I_{xy}\left(p^2 - q^2\right) - I_{yz}(pr + \dot{q}) + I_x(qr - \dot{p}) + m\left[x_G(\dot{v} + u = r - wp)\right.$$
$$\left.-y_G(\dot{u} - vr + wq)\right] - (x_G W - x_B B)\cos\theta\sin\varphi - (y_G W - y_B B)\sin\theta = N_f$$

2.2 Body-Fluid Interaction

In the case of an underwater vehicle moving in an uncontrolled ocean environment, four separate cases arise-

1. Cases where the hydrodynamic forces from fluid in motion act on stationary bodies and arise from fluid inertia;
2. Inertial forces acting on bodies in motion in stationary fluid;
3. Inertial forces for cases where both fluid and body are in motion;
4. Fluid loading on bodies in steady motion and drag and lift considerations.

2.2.1 Inertial Forces On A Stationary Body In Unsteady Flow

A force model for this case is,

$$X_f = \rho\pi c^2[1 + C_a]\frac{dU_f}{dt} \tag{5}$$

It acts in the direction of the positive fluid acceleration. The quantity, $\rho(\pi c^2)$, is the mass of displaced fluid. C_a is called "added mass coefficient" and the quantity $C_a\rho(\pi c^2)$ is the "added mass" affecting the loading on the body.

2.2.2 Forces On A Moving Body In Stationary Flow

In this case, since the fluid is stationary, the "horizontal buoyancy effect" arising from horizontal fluid acceleration is not present. If a cylinder inside such a fluid were accelerating with some time dependent velocity and acceleration within a stationary fluid medium, the fluid reaction force on the body would arise from the added mass being forced to accelerate with the body. This force can be expressed in terms of the body acceleration as

$$X_f = -C_a\rho(\pi c^2)\frac{du_r}{dt} \tag{6}$$

2.2.3 Forces On Moving Bodies In Unsteady Flow

When both fluid and body are moving with separate motion, the relative acceleration is used and the extra horizontal buoyant force that exists because of the accelerating external fluid is taken into account. It follows that in this case

$$X_f = [\rho(^2) + C_a\rho(\pi c^2)]\frac{dU_\mathrm{f}}{dt} - C_a\rho(\pi c^2)\frac{du}{dt} \tag{7}$$

2.2.4 Forces On Underwater Bodies From Drag And Lift

In the case of real fluids, there is a drag force that depends on the relative flow velocity between the fluid particles and the body. If the body is cylindrical and stationary, the relative velocity is U_f , and the drag force is represented by,

$$X_f = 0.5\rho C_d D(U_f)\mid U_f\mid + C_I\rho\pi c^2\frac{dU_f}{dt} \tag{8}$$

The modified square law mathematically preserves the correct sign of X_f. D is the projected area per unit length of the body exposed to the flow.

2.2.5 Extension to six degrees of freedom

Completing the analysis for all six degrees of freedom for a symmetric body, there is in general a six by six added mass matrix which may be shown under the ideal conditions stated, to be symmetric and positive definite with only 21 not 36 independent coefficients. Thus the final evaluations needed then become,

$$\begin{aligned} \boldsymbol{F}_f &= -\begin{bmatrix} X_{\dot{u}}\dot{u}_r + Z_{\dot{w}}w_r q - Y_{\dot{v}}v_r r \\ Y_{\dot{v}}\dot{v}_r + X_{\dot{u}}u_r r - Z_{\dot{w}}w_r p \\ Z_{\dot{w}}\dot{w}_r + Y_{\dot{v}}v_r p - X_{\dot{u}}u_r q \end{bmatrix}; Z_{\dot{w}} = Y_{\dot{v}} \\ \boldsymbol{M}_f &= -\begin{bmatrix} 0 \\ M_{\dot{q}}\dot{q} - rp\,(N_{\dot{r}}) + w_r u_r\,(X_{\dot{u}} - Z_{\dot{w}}) \\ N_{\dot{r}}\dot{r} + pq\,(M_{\dot{q}}) + u_r v_r\,(Y_{\dot{v}} - X_{\dot{u}}) \end{bmatrix}; N_{\dot{r}} = M_{\dot{q}} \end{aligned} \tag{9}$$

This set of equations will be the input to MPC discussed in next section.

2.3 Model Predictive Control

Model Predictive Control (MPC)[1] reflects human behaviour whereby we select control actions which we think will lead to the best predicted outcome (or output) over some limited horizon. We constantly update our decisions as new observations become available.

Salient features(Lars Grüne, Nonlinear MPC, lecture notes (2013))-

1. Can be applied to nonlinear systems.
2. Natural consideration of both states and control constraints.
3. Approximately optimal control.
4. Requires online optimization

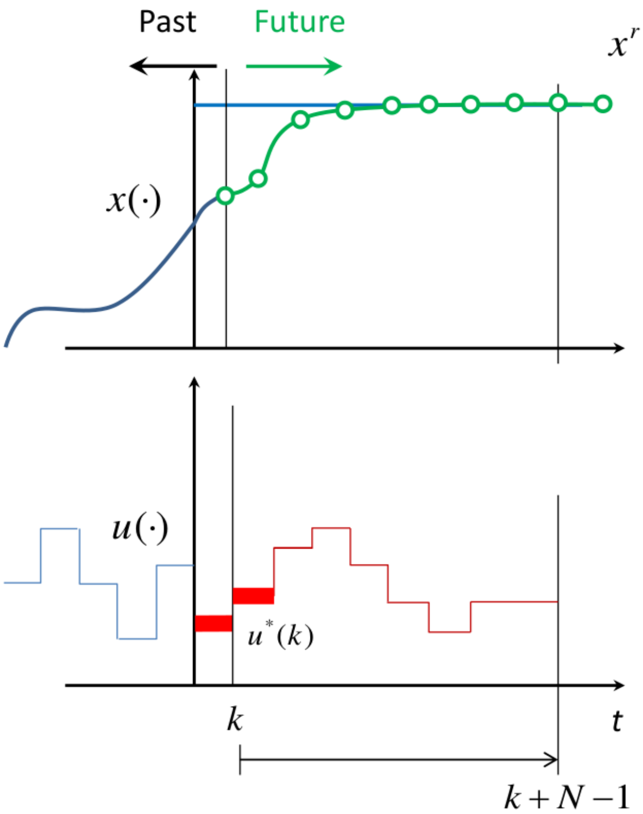
An example[4]-

Single input single output- $x(k+1) = f(x(k), u(k))$

- At decision instant k, measure the state $x(k)$.
- Based on $x(k)$, compute the (optimal) sequence of controls over a prediction horizon N: $u*(x(k)) := (u*(k), u*(k+1), \ldots u*(k+N-1))$
- Apply the control $u*(k)$ on the sampling period $[k, k+1]$.
- Repeat the same steps at the next decision instant.

MPC Strategy Summary (Mark Cannon (2016))-

1. Prediction
2. Online optimization
3. Receding horizon implementation



Prediction of an output $x(\cdot)$ based on control input $u(\cdot)$ at each timestep. Credits: [4]

MPC Mathematical Formulation
Running (stage) Costs: characterizes the control objective

$$\ell(\mathbf{x}, \mathbf{u}) = \|\mathbf{x_u} - \mathbf{x}^r\|^2_{\mathbf{Q}} + \|\mathbf{u} - \mathbf{u}^r\|^2_{\mathbf{R}}$$

Cost Function: Evaluation of the running costs along the whole prediction horizon

$$J_N(\mathbf{x}, \mathbf{u}) = \sum_{k=0}^{N-1} \ell\left(\mathbf{x_u}(k), \mathbf{u}(k)\right)$$

Optimal Control Problem (OCP): to find a minimizing control sequence

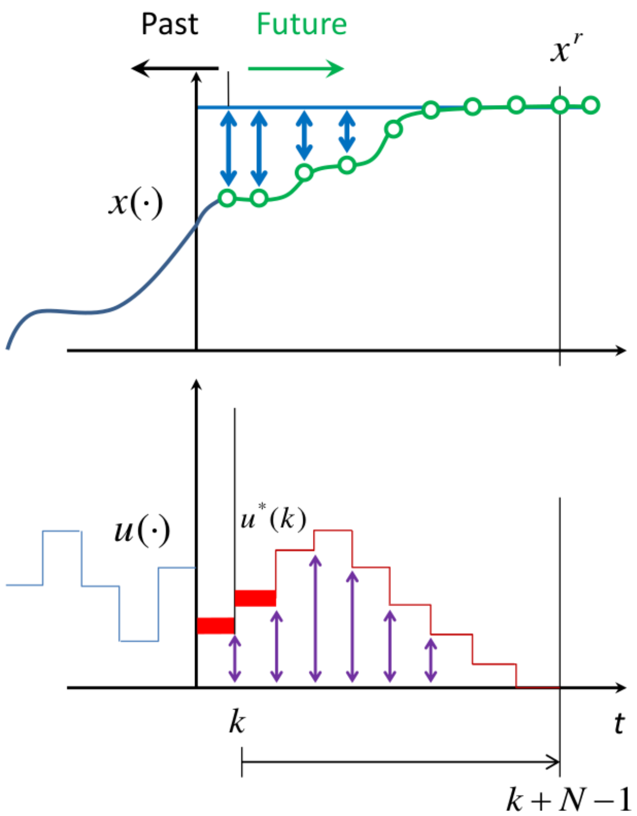
$$\underset{\mathbf{u}}{\text{minimize}} J_N\left(\mathbf{x}_0, \mathbf{u}\right) = \sum_{k=0}^{N-1} \ell\left(\mathbf{x_u}(k), \mathbf{u}(k)\right)$$

subject to : $\mathbf{x_u}(k+1) = \mathbf{f}\left(\mathbf{x_u}(k), \mathbf{u}(k)\right)$,

$$\begin{aligned} &\mathbf{x_u}(0) = \mathbf{x}_0, \\ &\mathbf{u}(k) \in U, \quad \forall k \in [0, N-1] \\ &\mathbf{x_u}(k) \in X, \quad \forall k \in [0, N] \end{aligned}$$

Value Function: minimum of the cost function

$$V_N(\mathbf{x}) = \min_{\mathbf{u}} J_N\left(\mathbf{x}_0, \mathbf{u}\right)$$



Optimization of the output $x(\cdot)$ based on control input $u(\cdot)$ at each timestep. Credits: [4]

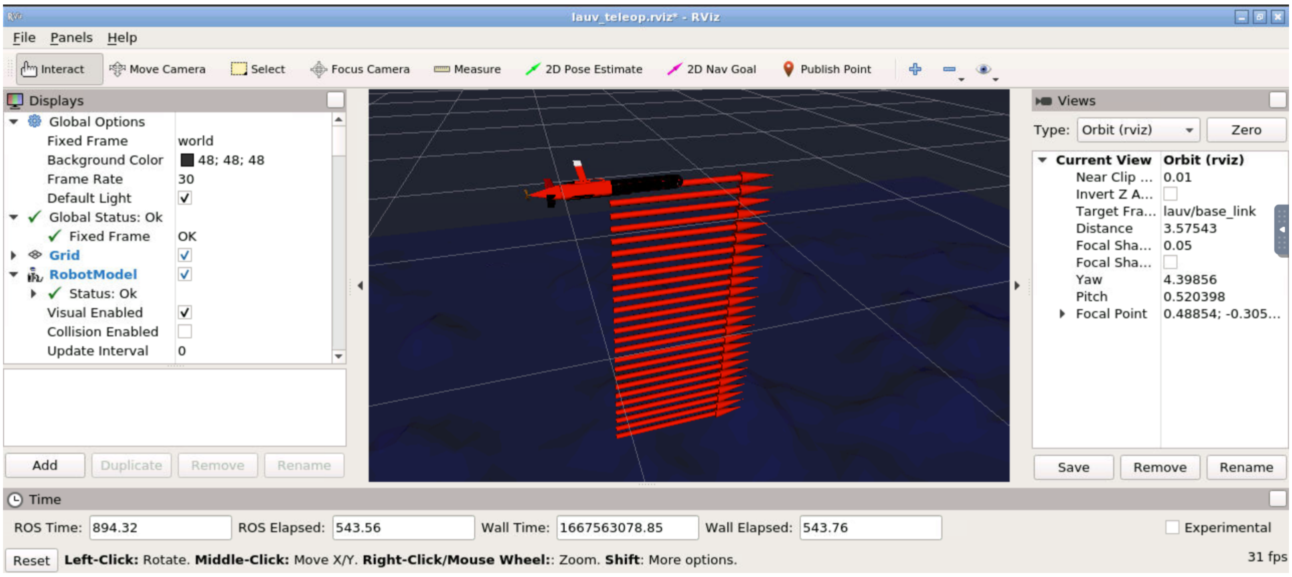


Figure 3: UUV Simulator interface showing LAUV Gazebo model deployed in an ocean environment in an ocean floor.

3 Simulation Results

UUV Simulator[3] has been used to simulate the ocean environment and control the motion of **Lauv** (see figure 2). A simulation box as shown in 3 involves the physics of an actual ocean with tides and waves. The Lauv model is deployed into it. The course of action of this vehicle will be to accurately traverse the way points assigned to it.

4 Conclusion and Future Works

In this report, the study of physics of a rigid body is carried out and a dynamic model incorporating all possible forces is prepared. The complex interactions of the rigid body with the fluid is also studied and all possible kinds of interactions were considered. In order to validate the correctness of the equations, a simulator was used that can mimic the ocean environment and the AUV was deployed into it.

Controlled maneuvering of LAUV in the simulated environment will be carried out so that it accurately traverses the way points assigned to it. The control equations derived from the above study will be incorporated into LAUV and it's motion and behaviour will be analysed. Further study would include traversal of optimal trajectory by the vehicle as predicted by the MPC model.

References

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