Problem 01:

Prove that, $f(n) = n^2+3n-2$ is $O(n^2)$; determine c, n_0?

Solution:

To do so,
we must determine positive constants c, n_0, such that,

$$f(n) \leq c(g(n))$$

$$\Rightarrow n^2+3n-2 \leq c.n^2$$

for all $n \geq n_0$; dividing by n^2 yields,

$$1+\frac{3}{n}-\frac{2}{n^2} \leq c$$

by chosing any constant,
$c \geq 2$ and any value $n \geq 1$; we can make inequality of the equation.

Thus, by choosing $c=2$, $n_0=1$; we can verify that, $f(n) = n^2+3n-2 = O(n^2)$

[Proved]

Problem 02:

Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

Solution:

When, $f(n) = 2^{n+1} = 2^n . 2 = O(2^n)$

To do so, we must determine positive constants c, n_0; such that,

$$f(n) \leq c(g(n))$$

$$\Rightarrow 2^n . 2 \leq c.(2^n)$$

for all $n \geq n_0$, dividing by 2^n yields,

$$2 \leq c$$

So, $c \geq 2$ and any value $n \geq 1$, we can make inequality of the equation.

Thus, by choosing $c = 2$, $n_0 = 1$, we can varify that $2^{n+1} = O(2^n)$.

So, It is possible that, $2^{n+1} = O(2^n)$.

When, $f(n) = 2^{2n} = (2^n)^2$

Suppose for contradiction that c and n_0 exist such that, $(2^n)^2 \leq c . 2^n$

for all $n_0 \geq n_0$, then dividing by, 2^n,

$$2^n \leq c$$

which cannot hold for arbitrary large n.

So, $2^{2n} \neq O(2^n)$. ~~Show~~