Mathemathische Methoden
Hausaufgabenblatt 10

1) Harmonische Näherung

$$V(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$ Skalares Feld

Minimum:

$$V'(r) = 4\varepsilon\left[-\frac{\sigma^{12}\cdot 12}{r^{13}} + \frac{\sigma^{6}\cdot 6}{r^{7}}\right] \stackrel{!}{=} 0$$

$$\Rightarrow \frac{12\sigma^{12}}{r^{13}} = \frac{6\sigma^{6}}{r^{7}} \Rightarrow 2\sigma^{6} = r^{6}$$

$$\Rightarrow r = 2^{\frac{1}{6}}\sigma$$

$$4\varepsilon\Big(\underbrace{\frac{1}{4}} - \frac{1}{4}\Big) =$$

$$\underline{r} = \begin{pmatrix} 2^{\frac{1}{6}}\sigma \\ 0 \\ 0 \end{pmatrix} \qquad V(\underline{r}) = 4\varepsilon\left[\frac{\sigma^{12}}{\sigma^{12}}\frac{1}{(2^{\frac{1}{6}})^{12}} - \frac{\sigma^{6}}{\sigma^{6}(2^{\frac{1}{6}})^{6}}\right]$$

$$V'(r) = 4\varepsilon\left[\frac{6\sigma^{6}}{r^{7}} - \frac{12\sigma^{12}}{r^{13}}\right]$$

$$V'(2^{\frac{1}{6}}\sigma) = 4\varepsilon\Big[6\sigma^{6}$$

2) Taylorentwicklung einfacher Felder

↳ bis zur 2. Ordnung um $\underline{r} = 0$

$\underline{a}$ = konstant

$$\varphi(\underline{r}) = \underline{a} \cdot \underline{r} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$