Equivariant Chow Polynomials of Matroids

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Goal

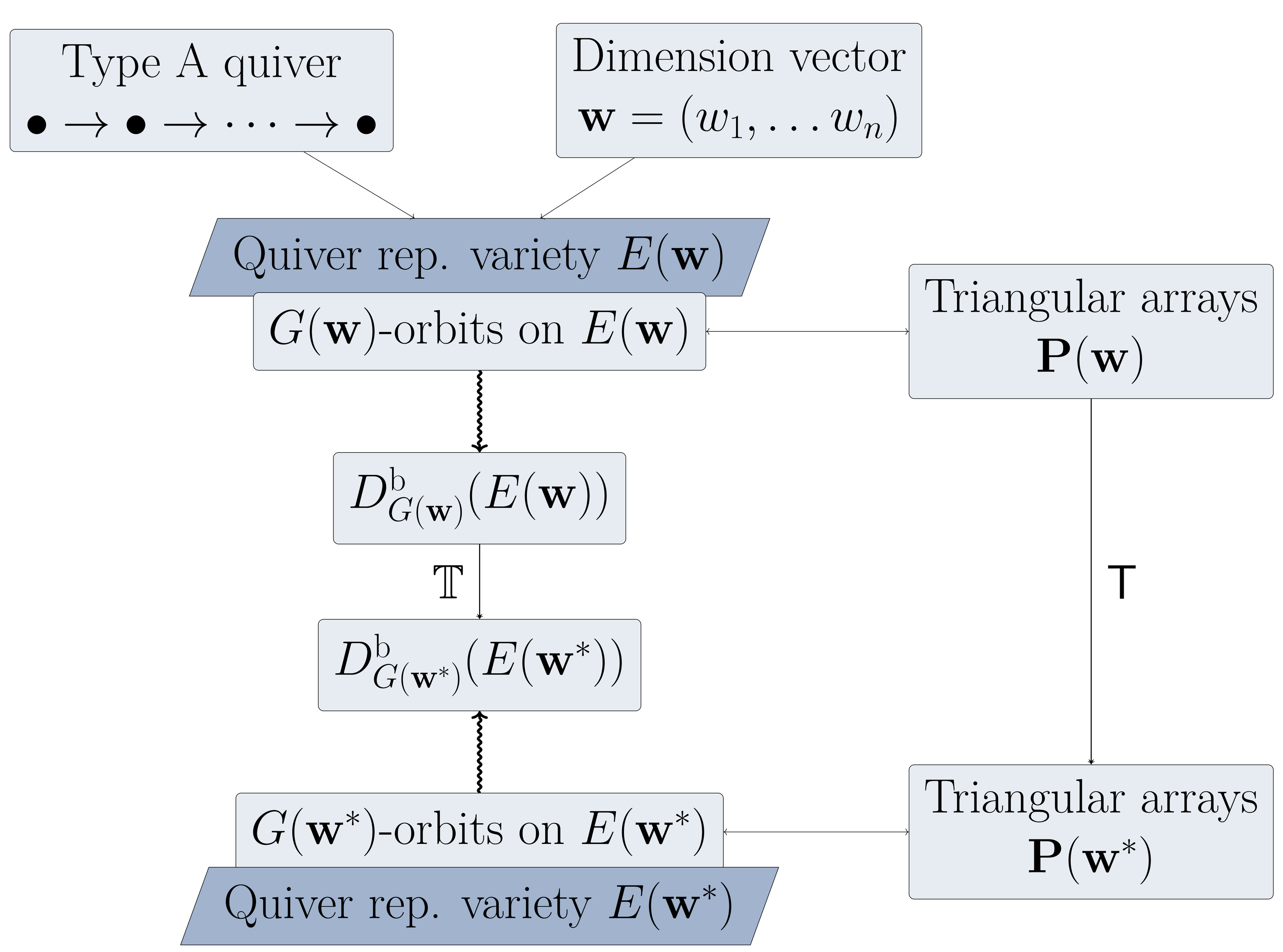
Define the equivariant Chow polynomial $\underline{\mathrm{H}}^G_{\mathsf{M}}$ of matroid M :

$$D^{\mathrm{b}}_{G(\mathbf{w})}(E(\mathbf{w})) \xrightarrow{\mathbb{T}} D^{\mathrm{b}}_{G(\mathbf{w}^*)}(E(\mathbf{w}^*))$$
$$\mathcal{F} \longmapsto q_{2!}q_1^*(\mathcal{F})[\dim E(\mathbf{w})]$$

where

$$\begin{tikzcd} & E(\mathbf{w}) \times E(\mathbf{w}^*) & \\ & Q \arrow[u, hook] \arrow[dl, "q_1"'] \arrow[dr, "q_2"] & \\ E(\mathbf{w}) & & E(\mathbf{w}^*) \end{tikzcd}$$

Overview



Quiver representations

A **quiver representation** is:

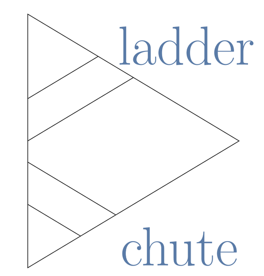
- A finite-dimensional $\mathbb{C}$-vector space for each vertex.
- A linear map for each arrow.

$$\mathbb{C} \xrightarrow{1} \mathbb{C} \xrightarrow{\begin{pmatrix} 2 \\ 5 \end{pmatrix}} \mathbb{C}^2$$

A **quiver representation variety** $E(\mathbf{w})$ is the space of all quiver representations for a fixed dimension vector $\mathbf{w}$.

$G(\mathbf{w}) = \mathbf{GL}(w_1) \times \dots \times \mathbf{GL}(w_n)$ acts on $E(\mathbf{w})$ splitting it into orbits.

The set of triangular arrays $\mathbf{P}(\mathbf{w})$



Define the set $\mathbf{P}(\mathbf{w})$ of triangular arrays of nonnegative integers such that:

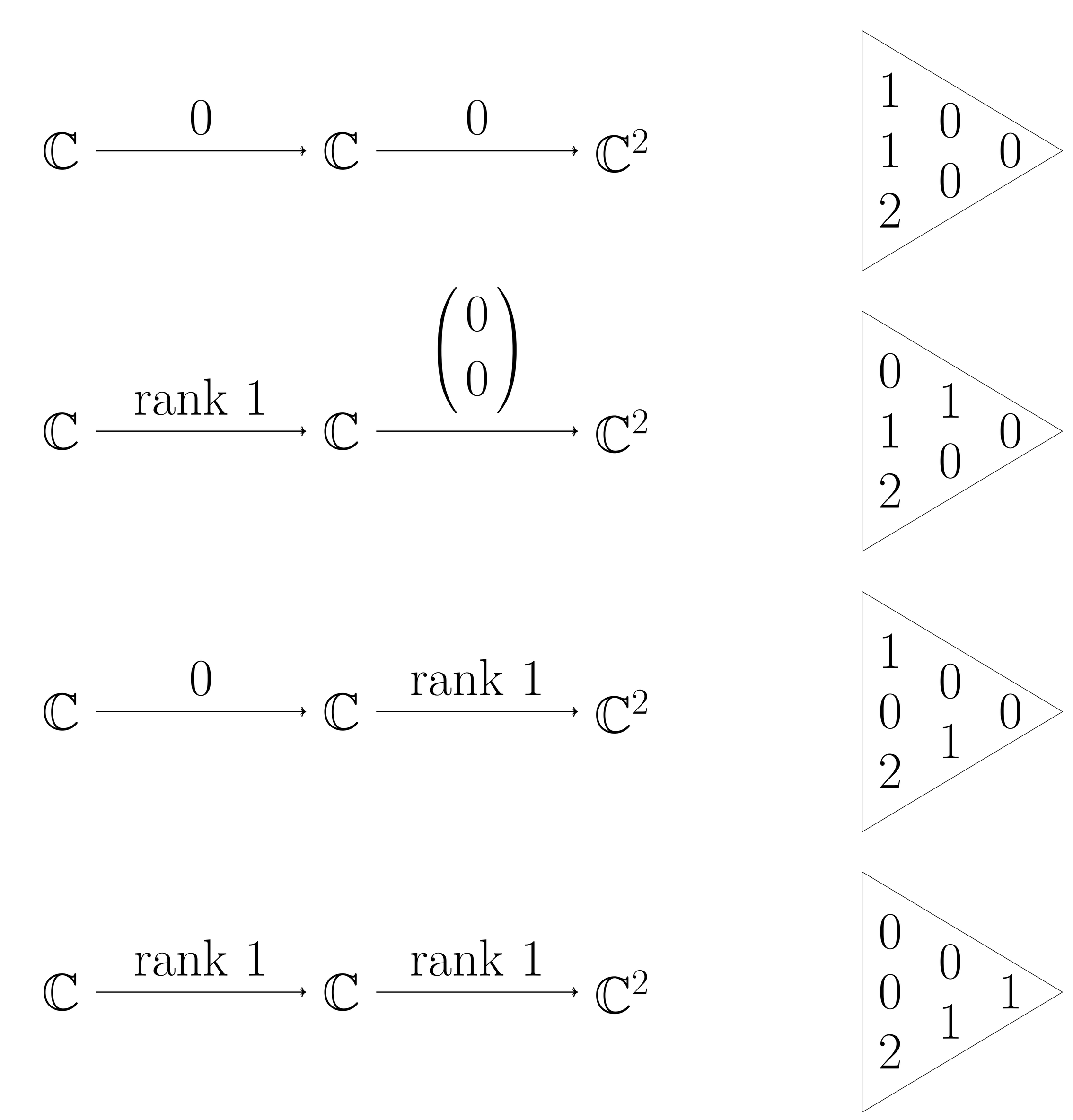
- $\forall j$, the entries in the j^{th} chute sum to w_j.
- Ladders are weakly decreasing.

Theorem [Achar–Kulkarni–M.]

There is a bijection

$$\{G(\mathbf{w})\text{-orbits in } E(\mathbf{w})\} \xleftrightarrow{1-1} \mathbf{P}(\mathbf{w}) = \{\text{certain tri. arrays}\}.$$

Example of bijection

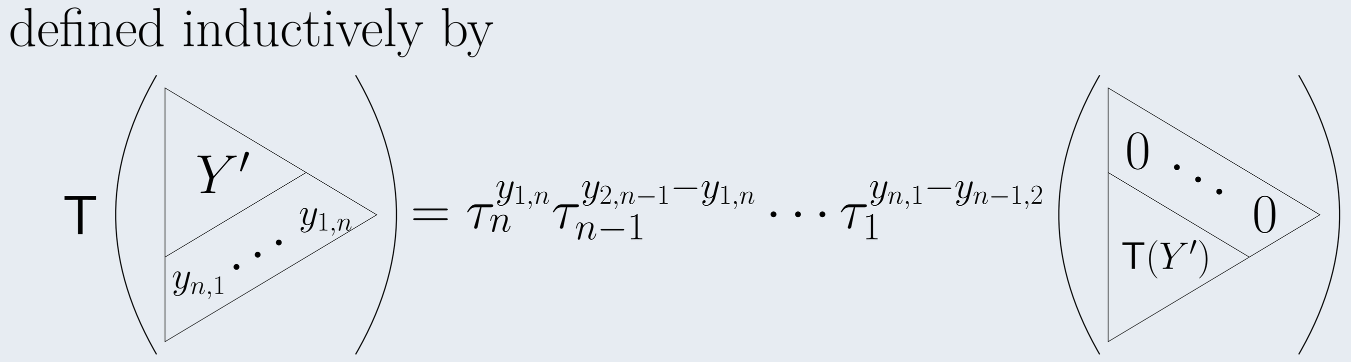


Theorem (Combinatorial Fourier transform) [Achar–Kulkarni–M.]

There is a bijection

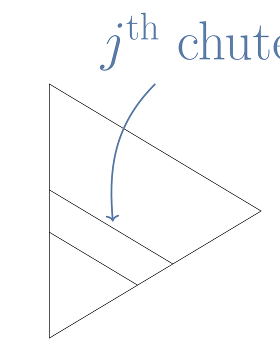
$$\mathbf{P}(\mathbf{w}) \xrightarrow{\mathsf{T}} \mathbf{P}(\mathbf{w}^*)$$

defined inductively by



where $\mathsf{T}(a) = a$.

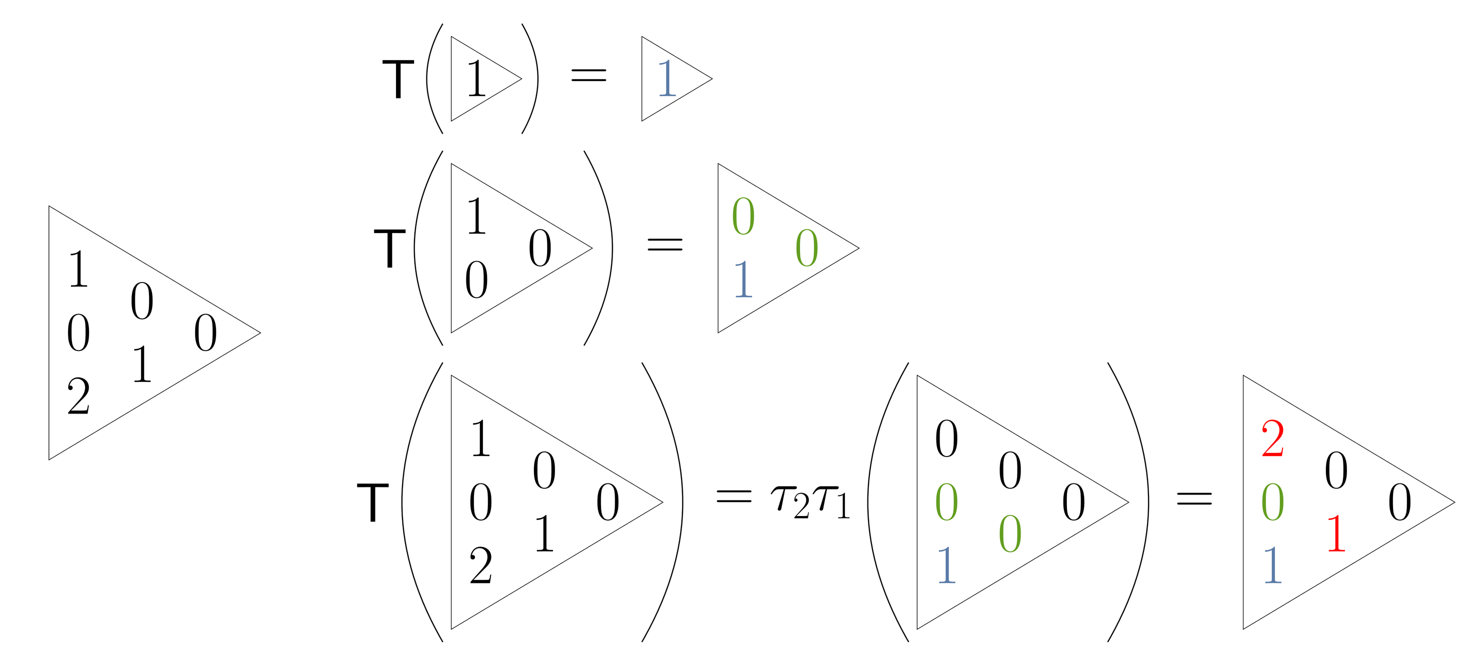
Definition of τ_j



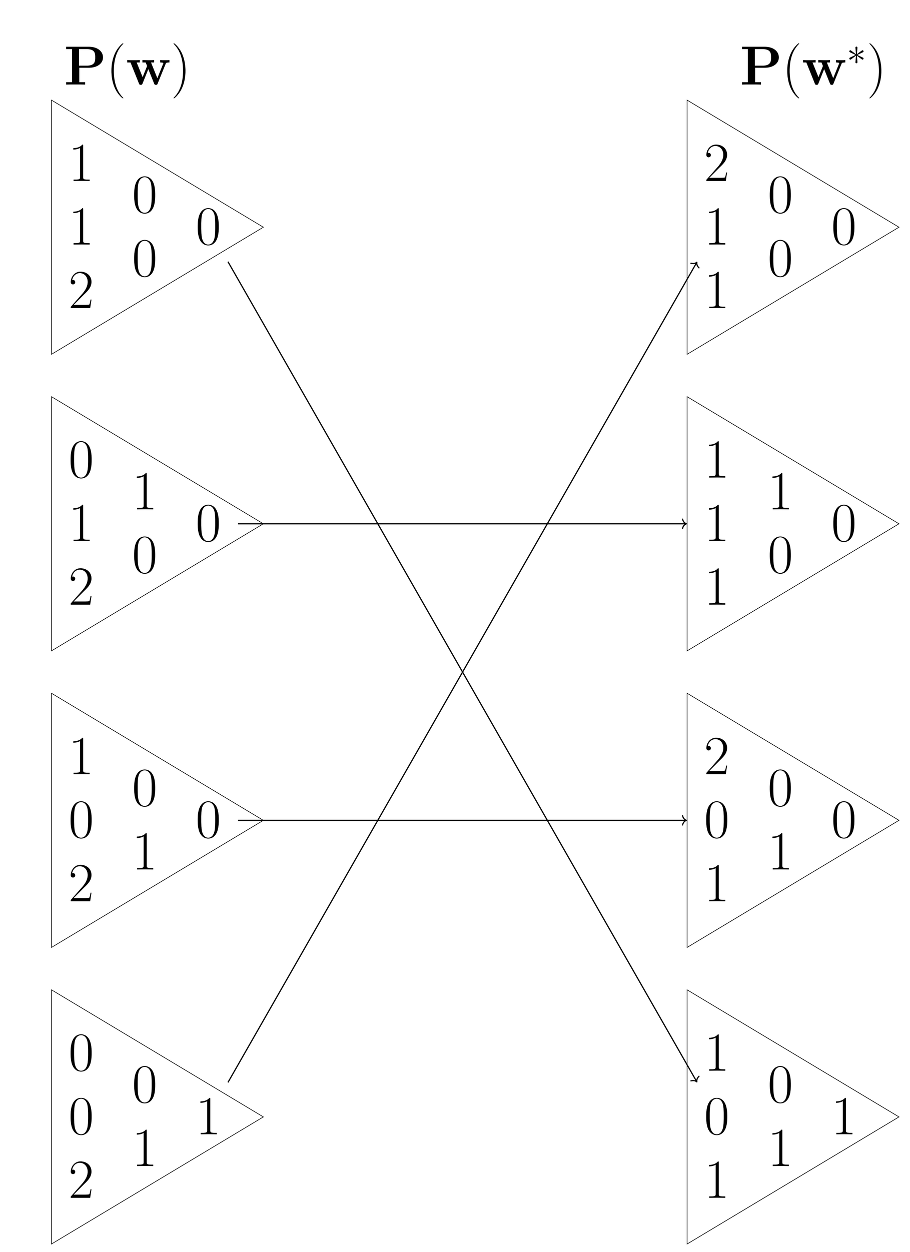
Define $\tau_j : \mathbf{P}(\mathbf{w}) \to \mathbf{P}(\mathbf{w} + \mathbf{e}_1 + \ldots + \mathbf{e}_j)$ by:

- Add 1 as far down the j^{th} chute as possible, drawing an impassable vertical line there.
- Repeat for chutes $j-1, \ldots, 1$ not crossing lines.

Example of CFT



Complete CFT example



Main conjecture (proof in progress)

The bijection T determines $\mathbb{T}$ on simple perverse sheaves; that is, $\mathbb{T}(\mathrm{IC}(\mathcal{O}_\lambda)) = \mathrm{IC}(\mathcal{O}_{\mathsf{T}(\lambda)})$.