Equivariant Chow Polynomials of Matroids

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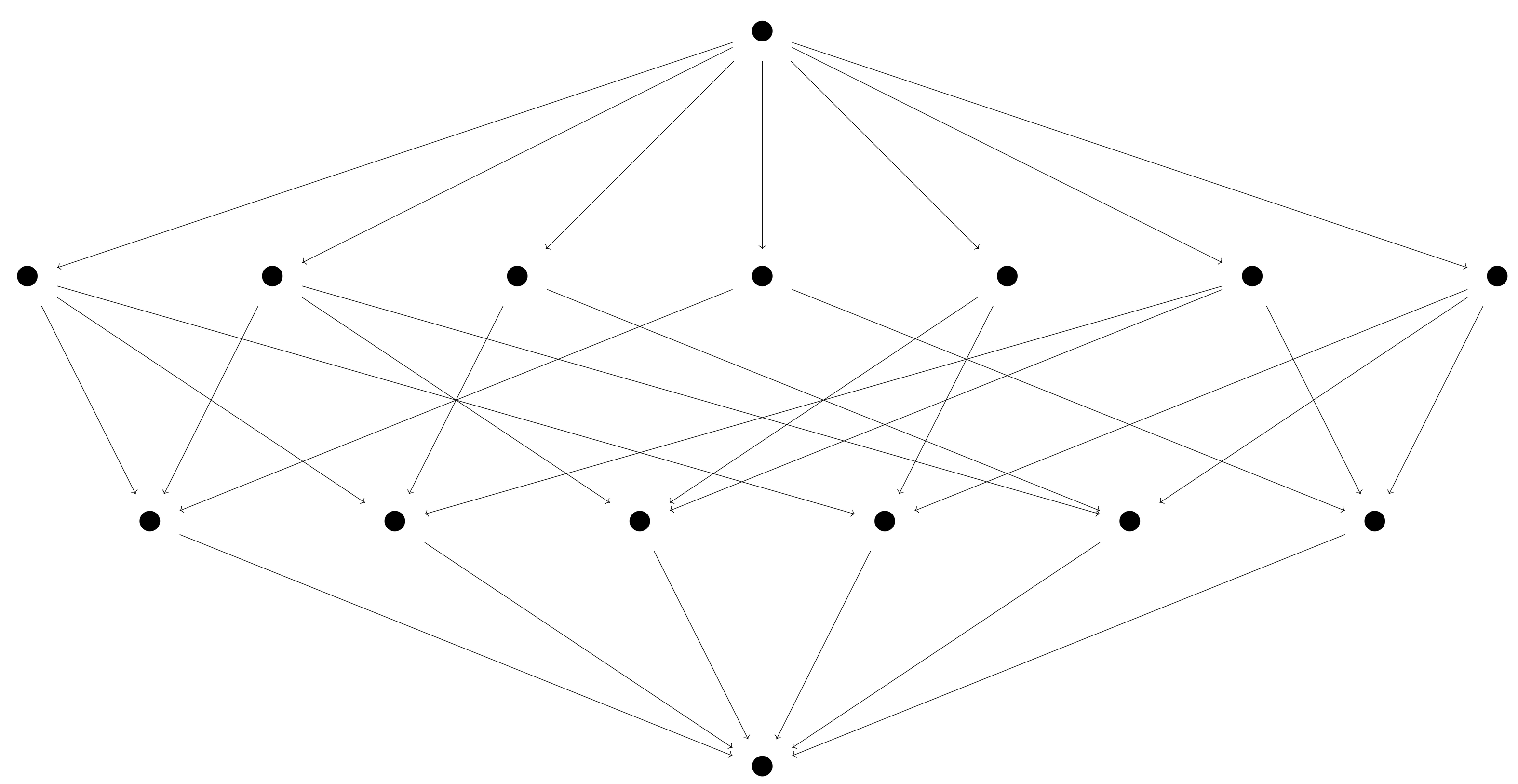
Goal

Define the equivariant Chow polynomial $\underline{\mathrm{H}}_{\mathsf{M}}^{G}(x) \in \mathrm{VRep}_G[x]$ of a matroid M :

Overview



The Chow Ring

For a matroid M with flats $F_1, \ldots, F_m$, **the Chow ring** $\underline{\mathrm{CH}}_{\mathsf{M}}$ can be defined as a graded $\mathbb{Z}$-module generated by the following **FY-monomials**:

$$x_{F_1}^{m_1} x_{F_2}^{m_2} \cdots x_{F_k}^{m_k} \mid \varnothing \subset F_1 \subset \cdots \subset F_k,\ 0 \leq m_i \leq \mathrm{rk}(F_i) - \mathrm{rk}(F_{i-1}) - 1.$$

The restriction on the exponents m_i of x_{F_i} ensures that there are exactly $\mathrm{rk}(M)$ graded pieces. The **(non equivariant) Chow polynomial** $\underline{\mathrm{H}}_{\mathsf{M}}$ is defined as:

$$\underline{\mathrm{H}}_{\mathsf{M}}(x) = a_0 + a_1 x + \cdots a_{\mathrm{rk}(M)-1} x^{\mathrm{rk}(M)-1}$$

where a_i is the rank of degree i piece in $\underline{\mathrm{CH}}_{\mathsf{M}}$.

For the braid matroid K_4 depicted above, the Chow polynomial is $1 + 8x + x^2$.

Theorem [Adiprasito-Huh-Katz]

The sequence $(a_0, a_1, \ldots, a_{\mathrm{rk}(M)-1})$ is log-concave.

Equivariant Chow Polynomial

For a matroid M with an action of a group G, there is an induced action on the Chow ring of M. It can be shown that G acts on each graded piece of $\underline{\mathrm{CH}}_{\mathsf{M}}$ separately by permuting the FY-monomials of that degree. The **equivariant Chow polynomial** $\underline{\mathrm{H}}_{\mathsf{M}}^{G}(x) \in \mathrm{VRep}_G[x]$ is defined as:

$$\underline{\mathrm{H}}_{\mathsf{M}}^{G}(x) = P(A_0) + P(A_1)x + \cdots P(A_{\mathrm{rk}(M)-1}) x^{\mathrm{rk}(M)-1}$$

where $P(A_i)$ denotes the permutation representation of G on the set A_i of degree i FY-monomials.

Example of bijection

Theorem [Angarone-Nathanson-Reiner]

Definition of τ_j

Define $\tau_j : \mathbf{P}(\mathbf{w}) \to \mathbf{P}(\mathbf{w} + \mathbf{e}_1 + \ldots + \mathbf{e}_j)$ by:

- Add 1 as far down the j^{th} chute as possible, drawing an impassable vertical line there.
- Repeat for chutes $j-1, \ldots, 1$ not crossing lines.

Example of CFT

Complete CFT example

Main conjecture (proof in progress)

The bijection T determines $\mathbb{T}$ on simple perverse sheaves; that is, $\mathbb{T}(\mathrm{IC}(\mathcal{O}_\lambda)) = \mathrm{IC}(\mathcal{O}_{\mathsf{T}(\lambda)})$.