

Conjecture 1. Let M be the parallel connection of two cycles $C_n = U(n-1, n)$ and $C_m = U(m-1, m)$, where $n \geq 3$ and $m \geq 3$. Let e be the shared edge between C_n and C_m . Then, we have

$$Q_M = Q_{M \setminus e} + (t+1) \cdot Q_{M/e} - t \cdot (Q_{C_{n-1}} \cdot \text{lt}(C_{m-1}) + \text{lt}(C_{n-1}) \cdot Q_{C_{m-1}}) \quad (1)$$

where, $\text{lt}(M)$ is 0 if the rank of M is even, and leading term of Q_M otherwise.

Conjecture 2. Let M be a matroid and e be a non coloop of M . Then, we have

$$Q_M = Q_{M \setminus e} + (t+1) \cdot Q_{M/e} + \sum_{C \in \mathcal{C}} t \cdot Q_{M/C} \cdot \text{lt}(M|_C/e). \quad (2)$$

where, $\text{lt}(M)$ is 0 if the rank of M is even, and leading term of Q_M otherwise.

Theorem 3. Let M be a matroid with ground set E and let $e \in E$. Then, we have

$$P_M = P_{M \setminus e} - tP_{M/e} + \sum_{F \in S} \tau(M/F \cup e) t^{crk(F)/2} P_{M|F}. \quad (3)$$

Here, the sum is taken over the set S of all subsets F of $E \setminus e$ such that F and $F \cup e$ are both flats of M (any such F is automatically also a flat of $M \setminus e$), and $\tau(M)$ is the coefficient of $t^{(rk(M)-1)/2}$ in $P_M(t)$ if $rk(M)$ is odd, and zero otherwise.

Theorem 4. Let M be the parallel connection of two cycles $C_n = U(n-1, n)$ and $C_m = U(m-1, m)$, where $n \geq 3$ and $m \geq 3$. Let e be the shared edge between C_n and C_m . Then, we have

$$P_M(t) = P_{M \setminus e} - tP_{C_{n-1}}P_{C_{m-1}}. \quad (4)$$

Conjecture 5. Let W be a finite Coxeter group with S as the set of simple reflections. Let $w \in W$ and $s \in S$. Let $w = s_1 s_2 \cdots s_k$ be a reduced expression for w . Then, for any $i \in \{1, 2, \dots, k\}$, we have

$$P_w(q) = P_{s_i w}(q) + qP_{s_i w}(q)P_{s_{i+1}s_{i+2}\cdots s_k}(q). \quad (5)$$