
Conjecture 1. *Let M be the parallel connection of two cycles $C_n = U(n-1, n)$ and $C_m = U(m-1, m)$, where $n \geq 3$ and $m \geq 3$. Let e be the shared edge between C_n and C_m. Then, we have*

$$Q_M = Q_{M\setminus e} + (t+1)\cdot Q_{M/e} - t\cdot (Q_{C_{n-1}}\cdot \mathrm{lt}(C_{m-1}) + \mathrm{lt}(C_{n-1})\cdot Q_{C_{m-1}}) \tag{1}$$

where, $\mathrm{lt}(M)$ is 0 if the rank of M is even, and leading term of Q_M otherwise.

Conjecture 2. *Let M be a matroid and e be a non coloop of M. Then, we have*

$$Q_M = Q_{M\setminus e} + (t+1)\cdot Q_{M/e} + \sum_{C\in\mathcal{C}} t\cdot Q_{M/C}\cdot \mathrm{lt}\left(M|_C/e\right). \tag{2}$$

where, $\mathrm{lt}(M)$ is 0 if the rank of M is even, and leading term of Q_M otherwise.

Theorem 3. *Let M be a matroid with ground set E and let $e \in E$. Then, we have*

$$P_M = P_{M\setminus e} - tP_{M/e} + \sum_{F\in S} \tau(M/F\cup e)t^{crk(F)/2}P_{M|F}. \tag{3}$$

Here, the sum is taken over the set S of all subsets F of $E\setminus e$ such that F and $F\cup e$ are both flats of M (any such F is automatically also a flat of $M\setminus e$), and $\tau(M)$ is the coefficient of $t^{(rk(M)-1)/2}$ in $P_M(t)$ if $rk(M)$ is odd, and zero otherwise.

Theorem 4. *Let M be the parallel connection of two cycles $C_n = U(n-1, n)$ and $C_m = U(m-1, m)$, where $n \geq 3$ and $m \geq 3$. Let e be the shared edge between C_n and C_m. Then, we have*

$$P_M(t) = P_{M\setminus e} - tP_{C_{n-1}}P_{C_{m-1}}. \tag{4}$$

Conjecture 5. *Let W be a finite Coxeter group with S as the set of simple reflections. Let $w \in W$ and $s \in S$. Let $w = s_1 s_2 \cdots s_k$ be a reduced expression for w. Then, for any $i \in \{1, 2, \ldots, k\}$, we have*

$$P_w(q) = P_{s_i w}(q) + qP_{s_i w}(q)P_{s_{i+1}s_{i+2}\cdots s_k}(q). \tag{5}$$

$$x_F = \sum_{G\geq F} q^{\mathrm{rk}(G)-\mathrm{rk}(F)}\cdot P_{M_F^G}(q^{-2})\cdot \zeta_F \tag{6}$$

$$\zeta_F = \sum_{G\geq F} q^{\mathrm{rk}(G)-\mathrm{rk}(F)}\cdot \hat{Q}_{M_F^G}(q^{-2})\cdot x_F \tag{7}$$