

**Conjecture 1.** Let  $M$  be the parallel connection of two cycles  $C_n = U(n-1, n)$  and  $C_m = U(m-1, m)$ , where  $n \geq 3$  and  $m \geq 3$ . Let  $e$  be the shared edge between  $C_n$  and  $C_m$ . Then, we have

$$Q_M = Q_{M \setminus e} + (t+1) \cdot Q_{M/e} - t \cdot (Q_{C_{n-1}} \cdot \text{lt}(C_{m-1}) + \text{lt}(C_{n-1}) \cdot Q_{C_{m-1}}) \quad (1)$$

where,  $\text{lt}(M)$  is 0 if the rank of  $M$  is even, and leading term of  $Q_M$  otherwise.

**Conjecture 2.** Let  $M$  be a matroid and  $e$  be a non coloop of  $M$ . Then, we have

$$Q_M = Q_{M \setminus e} + (t+1) \cdot Q_{M/e} + \sum_{C \in \mathcal{C}} t \cdot Q_{M/C} \cdot \text{lt}(M|_C/e). \quad (2)$$

where,  $\text{lt}(M)$  is 0 if the rank of  $M$  is even, and leading term of  $Q_M$  otherwise.

**Theorem 3.** Let  $M$  be a matroid with ground set  $E$  and let  $e \in E$ . Then, we have

$$P_M = P_{M \setminus e} - tP_{M/e} + \sum_{F \in S} \tau(M/F \cup e) t^{\text{crk}(F)/2} P_{M|F}. \quad (3)$$

Here, the sum is taken over the set  $S$  of all subsets  $F$  of  $E \setminus e$  such that  $F$  and  $F \cup e$  are both flats of  $M$  (any such  $F$  is automatically also a flat of  $M \setminus e$ ), and  $\tau(M)$  is the coefficient of  $t^{(\text{rk}(M)-1)/2}$  in  $P_M(t)$  if  $\text{rk}(M)$  is odd, and zero otherwise.

**Theorem 4.** Let  $M$  be the parallel connection of two cycles  $C_n = U(n-1, n)$  and  $C_m = U(m-1, m)$ , where  $n \geq 3$  and  $m \geq 3$ . Let  $e$  be the shared edge between  $C_n$  and  $C_m$ . Then, we have

$$P_M(t) = P_{M \setminus e} - tP_{C_{n-1}}P_{C_{m-1}}. \quad (4)$$

**Conjecture 5.** Let  $W$  be a finite Coxeter group with  $S$  as the set of simple reflections. Let  $w \in W$  and  $s \in S$ . Let  $w = s_1 s_2 \cdots s_k$  be a reduced expression for  $w$ . Then, for any  $i \in \{1, 2, \dots, k\}$ , we have

$$P_w(q) = P_{s_i w}(q) + qP_{s_i w}(q)P_{s_{i+1}s_{i+2}\cdots s_k}(q). \quad (5)$$