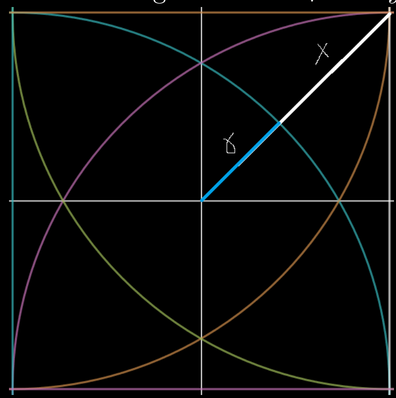
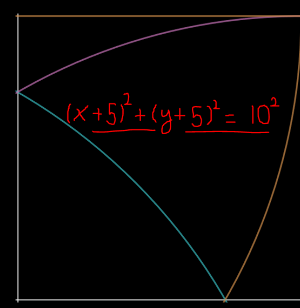
1. The given diagram consists of a square of length 10 cm and arcs of four circle of radius 10 cm. Find the area of the largest circle which can be fit into the shaded region.

Solution: Since the diagonal of the square is $10\sqrt{2}$ and the radius of each circle is 10, we find that the length $2r = 10\sqrt{2} - 2y = 10\sqrt{2} - 2(10\sqrt{2} - 10) \implies r = 10 - 5\sqrt{2}$ is symmetrical,



However, since it's not obvious that the circle we have found above is inside the shaded region, we will use Lagrange's multipliers method to minimize the distance from the center of the square to one of the arcs. This will show that the circle is inscribed inside the shaded region. Here, assume that the center of the square coincides with the origin in $\mathbb{R}^2$. We'll only consider the first quadrant and hence the arc that surrounds the shaded region in this quadrant will be an arc of the circle $(x+5)^2 + (y+5)^2 = 10^2$ Now, we only need to minimize the function $d(x, y) = x^2 + y^2$ with the above parameter.
Using Lagrange's multipliers, we have



$$\nabla \cdot d = \lambda \nabla \cdot g$$

where $g(x, y) = (x+5)^2 + (y+5)^2 - 10^2$. This gives

$$(2x, 2y) = \lambda(2x + 10, 2y + 10)$$

So, $x = y = 5\lambda/(1-\lambda)$. Substituting these for g gives

$$2\left(\frac{5\lambda}{1-\lambda} + 5\right)^2 = 100$$

$$\text{or} \quad 2\lambda^2 - 4\lambda + 1 = 0 \implies \lambda =$$