# 1

Write out the terms of the expression and then evaluate.

1.
$$\sum_{j=-1}^{2} \left((3j)^2 - 5j\right)$$

**Solution:**

$$\sum_{j=-1}^{2} \left((3j)^2 - 5j\right) = \left((-3)^2 + 5\right) + \left((0)^2 + 0\right) + \left((3)^2 - 5\right) + \left((3 * 2)^2 - 5 * 2\right)$$
$$= 14 + 0 + 4 + 26 = 44$$

2.
$$\prod_{k=2}^{7} \left(1 - \frac{1}{k^2}\right)$$

**Solution:**

$$\prod_{k=2}^{7} \left(1 - \frac{1}{k^2}\right) = \left(1 - 1/2^2\right) \cdot \left(1 - 1/3^2\right) \cdot \left(1 - 1/4^2\right) \cdot \left(1 - 1/5^2\right) \cdot \left(1 - 1/6^2\right) \cdot \left(1 - 1/7^2\right)$$
$$= \frac{3}{4} \cdot \frac{8}{9} \cdot \frac{15}{16} \cdot \frac{24}{25} \cdot \frac{35}{36} \cdot \frac{48}{49} = \frac{16}{21}$$

# 2

1. .

**Solution:** The $n^{th}$ pentagonal number $p_n$ is given by

$$p_n = p_{n-1} + 3n + 1$$

where the base case is given by $p_0 = 1$.

2. .

**Solution:** We see that $p_0 = 1$ and using the recursive definition, the few other terms are given by:

$$\begin{aligned}
p_1 &= 1 + 3*1 + 1 = 2 + 3*1 \\
p_2 &= (1 + 3*1 + 1) + 3*2 + 1 = 3 + 3*3 \\
p_3 &= (3 + 3*3) + 3*3 + 1 = 4 + 3*6 \\
&\vdots \\
p_n &= (n+1) + 3*t_n
\end{aligned}$$

where $t_n$ is the nth triangular number.

So, we have

$$p_n = \sum_{i=0}^{n} 1 + 3i$$

3. .

**Solution:** We get,

$$H_k : p_k = \sum_{i=0}^{k} 1 + 3i = (k+1) + \frac{3k(k+1)}{2} = \frac{(k+1)(3k+2)}{2} \tag{1}$$

Here, we see that the base case $p_0 = 2/2 = 1$, which is true. Assuming that the statement $H_k$ is true for some $k$, we have

$$\begin{aligned}
p_{k+1} &= p_k + 3(k+1) + 1 \\
&= \frac{(k+1)(3k+2)}{2} + 3k + 4 \\
&= \frac{3k^2 + 5k + 2 + 6k + 8}{2} \\
&= \frac{3k^2 + 11k + 10}{2} \\
&= \frac{(k+2)(3k+5)}{2}
\end{aligned}$$

Then, since $H_k \implies H_{k+1}$, the statement is true for all integers $k \geq 0$.