Homework 2 | Nutan Nepal
Number Theory | February 21, 2019

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1. Give 3 different fun facts about the number 2019. Note that there is no formal definition of fun fact, so be creative. For each fun fact find another number that has the same fun fact or state why one is not possible. Find an example and counterexample to the fun fact.

Solution:

(a) The lowest prime that divides that divides 2019. The only other prime is 673. The difference here is 670. The next number with such property is 4739.

(b) Current year is 2019. There's no other number that's current year.

(c) $2019^2 = 4076361$. Sum of digits of the square is 27 which equals $(2+0+1)*9$.

2. Prove that a square is either divisible by 4 or leaves a remainder of 1 when divided by 4.

Solution: For some $n \in \mathbb{Z}$, if n is in the form $2k$ for some integer k, then obviously $4|n^2$.

But then, if $n = 2k+1$ for some $k \in \mathbb{Z}$, then $n^2 = 4k^2 + 4k + 1 = 4(k^2+k)+1$. Since, n^2 can be written as $4k+1$, we get a remainder of 1 when dividing n^2 by 4.

3. Show that $4|(n^4 + 2n^3 + n^2)$ for all $n \in \mathbb{Z}$.

Solution: We have, $(n^4 + 2n^3 + n^2) = (n^2+n)^2$. For any integer n in the form $2k+1$, n^2 is odd, so $n^2 + n = 2m$ for some $m \in \mathbb{Z}$. Then surely, $4|(n^2+n)^2$.

Similarly, if n is in the form $2k$, $n^2 + n = 2m$ for some integer m, and hence 4 divides $(n^2+n)^2$. □

4. Let F_n be then nth Fibonacci number. Show that F_n is even if and only if n is a multiple of 3.

Solution:

5. It has been conjectured that every even number greater than 2 is the sum of two primes. This is known as Goldbach's conjecture, since he asked the question in 1742 in a letter to Euler.

(a) Write every even integer between 4 and 20, inclusively as the sum of two primes.

Solution:

$$4 = 2 + 2, \tag{1}$$
$$6 = 3 + 3, \tag{2}$$
$$8 = 5 + 3, \tag{3}$$
$$10 = 5 + 5, \tag{4}$$
$$12 = 5 + 7, \tag{5}$$
$$14 = 7 + 7, \tag{6}$$
$$16 = 13 + 3, \tag{7}$$
$$18 = 11 + 7, \tag{8}$$
$$20 = 17 + 3. \tag{9}$$

(b) If you wanted to show that is holds for a large number, say 1234567890, then describe a procedure to find such a sum. Yes, you may write it as a computer program if you so choose.

Solution: We start from the half of the given number and check for each odd numbers.

```
#returns the primes
def goldbachify(n):
    [x, y] = [n//2,n//2]
    if not x % 2: [x,y] = [x + 1, y - 1]
    for _ in range (1, y):
        if is_prime(x) and is_prime(y):
            return [x, y]
        [x,y]=[x+2, y-2]
    return [x,y]
#check primality
def is_prime(n):
    if not n % 2: return False
    for x in range(3, int(n**(0.5))+1, 2):
        if not n % x: return False
    return True
```

(c) Give an example of an odd integer that cannot be written as the sum of two primes.

Solution: 23 cannot be represented as sum of two prime numbers.

(d) Classify all odd numbers that cannot be expressed as the sum of two primes.

Solution: Since odd numbers are always partitioned into an odd and even, and odd number k can be written as $k = 2m + (2n + 1)$ for some natural numbers m, n. Since one of them is always even, k cannot be written as sum of two primes if $k - 2$ is not prime.