Differential Topology

Nutan Nepal

August 18, 2021

1 Manifolds and Smooth Maps

Notes 1.1. *Definitions*

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1. A function $f : X \to \mathbf{R}^m$ ($X \subset \mathbf{R}^n$) is <u>smooth</u> if around each $x \in X$ there is an open set $U \subset \mathbf{R}^n$ and a smooth map $F : U \to \mathbf{R}^m$ such that F equals f on $U \cap X$.

2. A smooth map $f : X \to Y$ of subsets of Euclidean spaces is called <u>diffeomorphism</u> if it is bijective and if the inverse map $f^{-1} : Y \to X$ is smooth.

3. A set $X \subset \mathbf{R}^n$ is a k-dimensional manifold if every $x \in X$ posseses a neighborhood V which is diffeomorphic to an open set $U \subset \mathbf{R}^k$. The diffeomorphism $\varphi : U \to V$ is called a <u>parametrization</u> of the neighborhood V and the inverse diffeomorphism $\varphi^{-1} : V \to U$ is called a <u>coordinate system</u> on V.

4. (Problem 3)

    i) For every $x \in X$, there exists an open set $U \subset \mathbf{R}^n$ and a smooth map $F : U \to Y$ such that $F|_{U \cap X} = f$.

    ii) For every $f(x) \in Y$, there exists an open set $V \subset \mathbf{R}^m$ and a smooth map $G : V \to Z$ such that $G|_{V \cap Y} = g$.