Notes - Mathematics

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August 31, 2021

1 Tensor products, direct and inverse limits

Notes 1.1. *Basics of Tensors and Homological Algebra (Matsumura Appendix A)*

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1. $\mathrm{Hom}_A(M \otimes_A N, L) \simeq \mathscr{L}(M, N; L)$. This follows from the definition of tensor products. Here, g is an A-linear map and $\varphi \in \mathscr{L}(M, N; L)$ (the set of bilinear maps).

$$\begin{tikzcd} & M \otimes_A N \arrow[d, "g"] \\ M \times N \arrow[ur, "\otimes_A"] \arrow[r, "\varphi"'] & L \end{tikzcd}$$

2. $(M \otimes_A M') \otimes_A M'' = M \otimes_A M' \otimes_A M'' = M \otimes_A (M' \otimes_A M'')$ (Associativity); $M \otimes_A N = N \otimes_A M$ (Commutativity); $M \otimes_A A = M$; $(\bigoplus_\lambda M_\lambda) \otimes_A N = \bigoplus_\lambda (M_\lambda \otimes_A N)$ (Distributivity with direct sum).

3. $(f \otimes g)(\sum_i x_i \otimes y_i) = \sum_i f(x_i) \otimes g(y_i)$.