Computer Algebra 522
Homework 2

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Programming Exercise 1

```
S.<x,y> = PolynomialRing(QQ,order='deglex')
def multidiv(f, listf):
    listofqs = []
    p=f
    for fs in listf:
        q, r = p.quo_rem(fs)
        listofqs.append(q)
        p=r
    return (listofqs, p)

def buchbergerAlgorithm(listf):
    listg = listf
    while True:
        gTemp = listg
        for p in gTemp:
            for q in gTemp:
                if p != q:
                    gamma = lcm(p.lt(),q.lt())
                    s = p*gamma.quo_rem(p.lt())[0] -
                        q*gamma.quo_rem(q.lt())[0]
                    r = multidiv(s, gTemp)[1]
                    if r != 0:
                        listg.append(r)
        if listg == gTemp:
            break

    return listg
```

```
f = x^3-2*x*y
g = x^2*y-2*x*y^2+x
buchbergerAlgorithm([f,g])
```

Programming Exercise 2

```
def minimalizeGB(listg):
    gTemp = listg
    for gi in gTemp:
        gi = gi*(1/gi.lc())
    for gj in gTemp:
        for gi in gTemp:
            if gi != gj and (gi.lt()).quo_rem(gj.lt())[1]==0:
                gTemp.remove(gi)

    return gTemp


def reducedGB(listg):
    gTemp = minimalizeGB(listg)
    reducedBasis = []
    for gi in gTemp:
        temp = 0
        while True:
            s = gi.lt()
            if s == 0: break
            flag = 0
            for gj in gTemp:
                if gi != gj and s.quo_rem(gj.lt())[1] == 0: flag = 1
            if not flag: temp = temp + s
            gi = gi - s
        reducedBasis.append(temp)

    return reducedBasis
```