Introduction to Manifold Theory

Homework 3

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1. Do Exercise 3.1 (show that if $U \subset \mathbb{R}^n$ and $V \subset \mathbb{R}^m$ are open, then a function $f : U \to V$ is smooth if and only if each of its component functions $f_i : U \to R$ are smooth).

2. Check that Definition 3.6 gives an equivalence relation (a binary relation that is reflexive, symmetric, and transitive) on the set of smooth atlases on a given topological manifold X.

3. Do Exercise 3.2 (show that a product of smooth atlases is a smooth atlas on the product manifold).

4. Define $f : \mathbb{R}^2 \to \mathbb{R}^3$ by

$$f(u,v) = \left(\cos(u^2 v) - e^{u-v},\ \frac{u^2-3}{u^2+v^2},\ e^{e^{uv}}\right)$$

Compute the Jacobian matrix of f.