Let M be a matroid with lattice of flats $\mathcal{L}(\mathrm{M})$. The module $\mathcal{H}(\mathrm{M})$ is the free $\mathbb{Z}[x, x^{-1}]$-module with a standard basis indexed by the flats of M. The elements of $\mathcal{H}(\mathrm{M})$ are formal sums of the form

$$\alpha = \sum_{F \in \mathcal{L}(\mathrm{M})} \alpha_F \cdot \mathrm{F}$$

where $\alpha_F \in \mathbb{Z}[x, x^{-1}]$.

There is a subgroup $\mathcal{H}_p(\mathrm{M})$ of $\mathcal{H}(\mathrm{M})$ consisting of all elements α such that for every flat $F \in \mathcal{L}(\mathrm{M})$:

1. $\alpha_F \in \mathbb{Z}[x]$
2. $\sum_{G \geq F} x^{\mathrm{rk}(F) - \mathrm{rk}(G)} \alpha_G$ satisfies the palindromic condition: $f(x) = f(x^{-1})$.

We relax the first condition and define another subgroup $\mathcal{H}'_p(\mathrm{M})$ of $\mathcal{H}(\mathrm{M})$ consisting of all elements α that satisfy the second condition.

We define a new basis of $\mathcal{H}(\mathrm{M})$ given by

$$\mathrm{c}^F = \sum_{G \leq F} \mathrm{c}^F_G \cdot \mathrm{G} := \sum_{G \leq F} x^{\mathrm{rk}(F) - \mathrm{rk}(G)} \underline{\mathrm{H}}_{\mathrm{M}^F_G}(x^{-2}) \cdot \mathrm{G}$$

where $\underline{\mathrm{H}}_{\mathrm{M}}$ is the Chow polynomial of the matroid M.

Let $\mathrm{Pal}(n)$ be the ring of Laurent polynomials that satisfy $f(x) = x^n f(x^{-1})$. We note that

$$\mathrm{c}^F_F = 1 \in \mathrm{Pal}(0), \quad \text{and} \quad \mathrm{c}^F_G \in \mathrm{Pal}(2) \text{ for all } G < F.$$

Let $\mathcal{S}(\mathrm{M})$ be the $\mathrm{Pal}(0)$-module generated by the elements c^F.

Lemma 1. $\mathrm{c}^F \in \mathcal{H}'_p(\mathrm{M})$ *for all* $F \in \mathcal{L}(\mathrm{M})$.

Lemma 2. $\mathcal{S}(\mathrm{M}) \cong \mathcal{H}'_p(\mathrm{M})$ *as* $\mathrm{Pal}(0)$*-modules.*

Lemma 3. *Any Laurent polynomial f can be written as a sum $\alpha + \beta$ where $\alpha \in \mathrm{Pal}(0)$ and $\beta \in \mathrm{Pal}(2)$.*