Simulation studies: TLP, BLP, and extensions of BLP

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Simulation studies

The data generating process for the observation Y in the regression model is

$$Y = X_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + \epsilon, \epsilon \sim (0, 1)$$

where a_1, a_2, and a_3 are real constants that vary across different simulation studies, and X_0, X_1, X_2, X_3, and ϵ are independent, standard normal random variables. The individual predictive densities have partial access of the above set of covariates. f_1 has access to only X_0 and X_1, f_2 has access to only X_0 and X_2, and f_3 has access to only X_0 and X_3. We want to combine f_1, f_2, and f_3 to predict Y. In this setup, X_0 represent shared information, while other covariates represent information unique to each individual model.

We estimate the pooling/combination formulas on a random sample $(f_{1i}, f_{2i}, f_{3i}, Y_i) : i = 1, ..., n$ of size $n = 50, 000$ and evaluate on an independent test sample of the same size.

Scenario 1: Unbiased and calibrated components

In this scenario, $a_1 = a_2 = 1$ and $a_3 = 1.1$, so that f_3 is a more concentrated, sharper density forecast than f_1 and f_2 (Gneiting and Ranjan (2013)) and they are defined as follows:

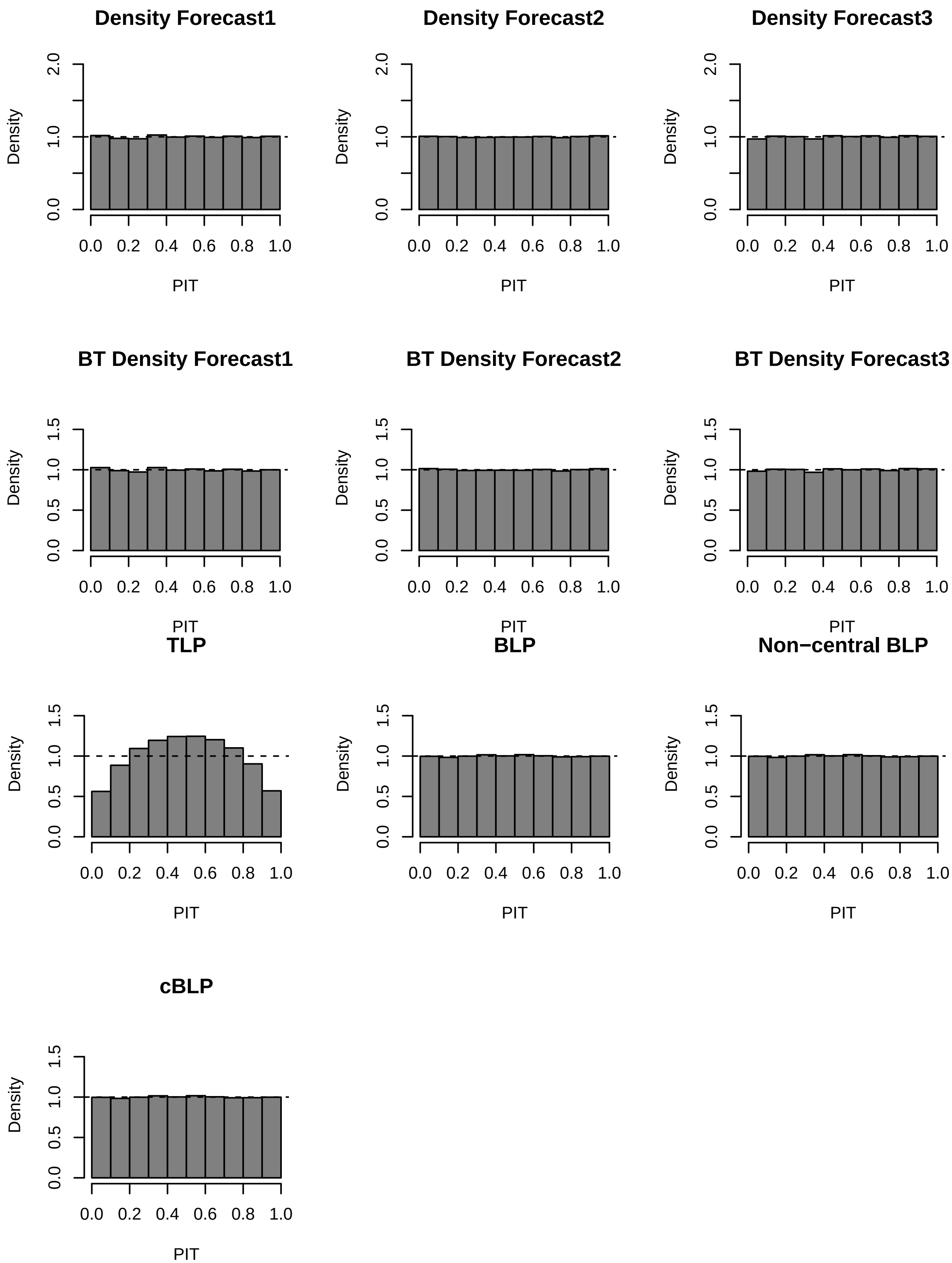
$$f_1 = \mathrm{N}(X_0 + a_1 X_1, 1 + a_2^2 + a_3^2)$$
$$f_2 = \mathrm{N}(X_0 + a_2 X_2, 1 + a_1^2 + a_3^2)$$
$$f_3 = \mathrm{N}(X_0 + a_3 X_3, 1 + a_1^2 + a_2^2)$$

Table 1: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp
TLP	0.267	0.259	0.474	NA	NA	NA
BLP	0.298	0.292	0.410	1.464	1.459	NA
nBLP	0.298	0.292	0.410	1.462	1.461	0.009
cBLP	0.298	0.293	0.409	1.457	1.457	NA

	Training	Test
f1	-2.001	-2.004
f2	-2.001	-2.008
f3	-1.965	-1.966
TLP	-1.908	-1.910
BLP	-1.865	-1.870
nBLP	-1.865	-1.870
cBLP	-1.865	-1.870

	Training	Test
f1	0.083	0.083
f2	0.083	0.084
f3	0.083	0.083
TLP	0.064	0.064
BLP	0.083	0.083
nBLP	0.083	0.083
cBLP	0.083	0.083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Density Forecast2
Density Forecast3
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BT Density Forecast3
TLP
BLP
Non−central BLP
cBLP
Density
PIT
0.0 0.2 0.4 0.6 0.8 1.0
0.0 1.0 2.0
0.0 0.5 1.0 1.5

Scenario 2 : Underdispersed components

We subtract constants from the variances of component density forecasts as follows:

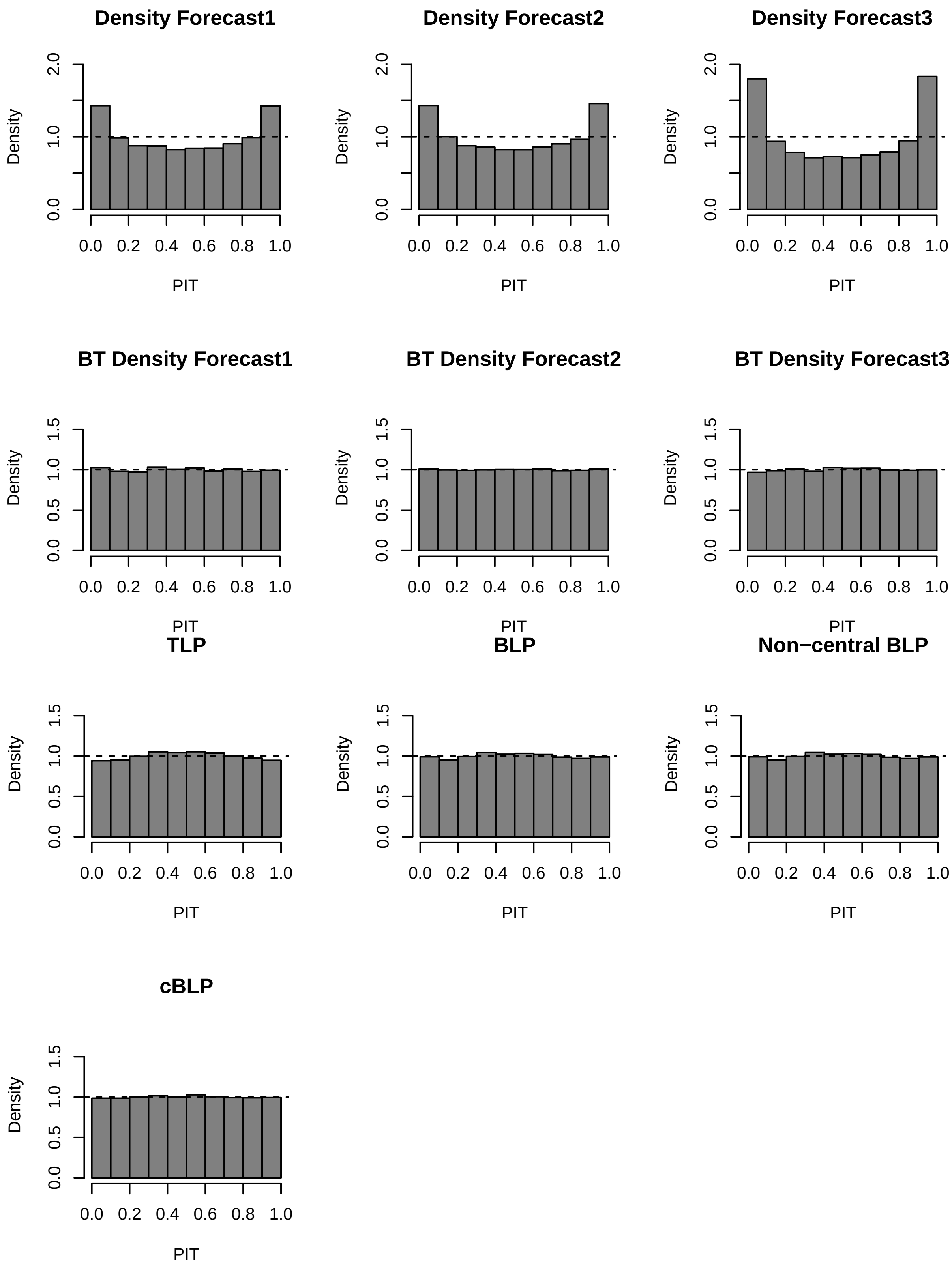
$$
\begin{aligned}
f_1 &= \mathrm{N}(X_0 + a_1 X_1, (1 + a_2^2 + a_3^2) - 1) \\
f_2 &= \mathrm{N}(X_0 + a_2 X_2, (1 + a_1^2 + a_3^2) - 1) \\
f_3 &= \mathrm{N}(X_0 + a_3 X_3, (1 + a_1^2 + a_2^2) - 1.5)
\end{aligned}
$$

Table 2: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp
TLP	0.311	0.307	0.382	NA	NA	NA
BLP	0.315	0.312	0.373	1.039	1.035	NA
nBLP	0.315	0.312	0.373	1.037	1.036	0.01
cBLP	0.278	0.272	0.450	1.455	1.455	NA

	Training	Test
f1	-2.040	-2.044
f2	-2.040	-2.051
f3	-2.115	-2.116
TLP	-1.876	-1.881
BLP	-1.876	-1.880
nBLP	-1.876	-1.880
cBLP	-1.865	-1.869

	Training	Test
f1	0.101	0.101
f2	0.101	0.101
f3	0.116	0.116
TLP	0.080	0.080
BLP	0.082	0.082
nBLP	0.082	0.082
cBLP	0.083	0.083



Density Forecast1
Density Forecast2
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TLP
BLP
Non−central BLP
cBLP
Density
PIT
0.0
0.5
1.0
1.5
2.0
0.2
0.4
0.6
0.8

Scenario 3: Overdispersed components

We add constants from the variances of component density forecasts as follows:

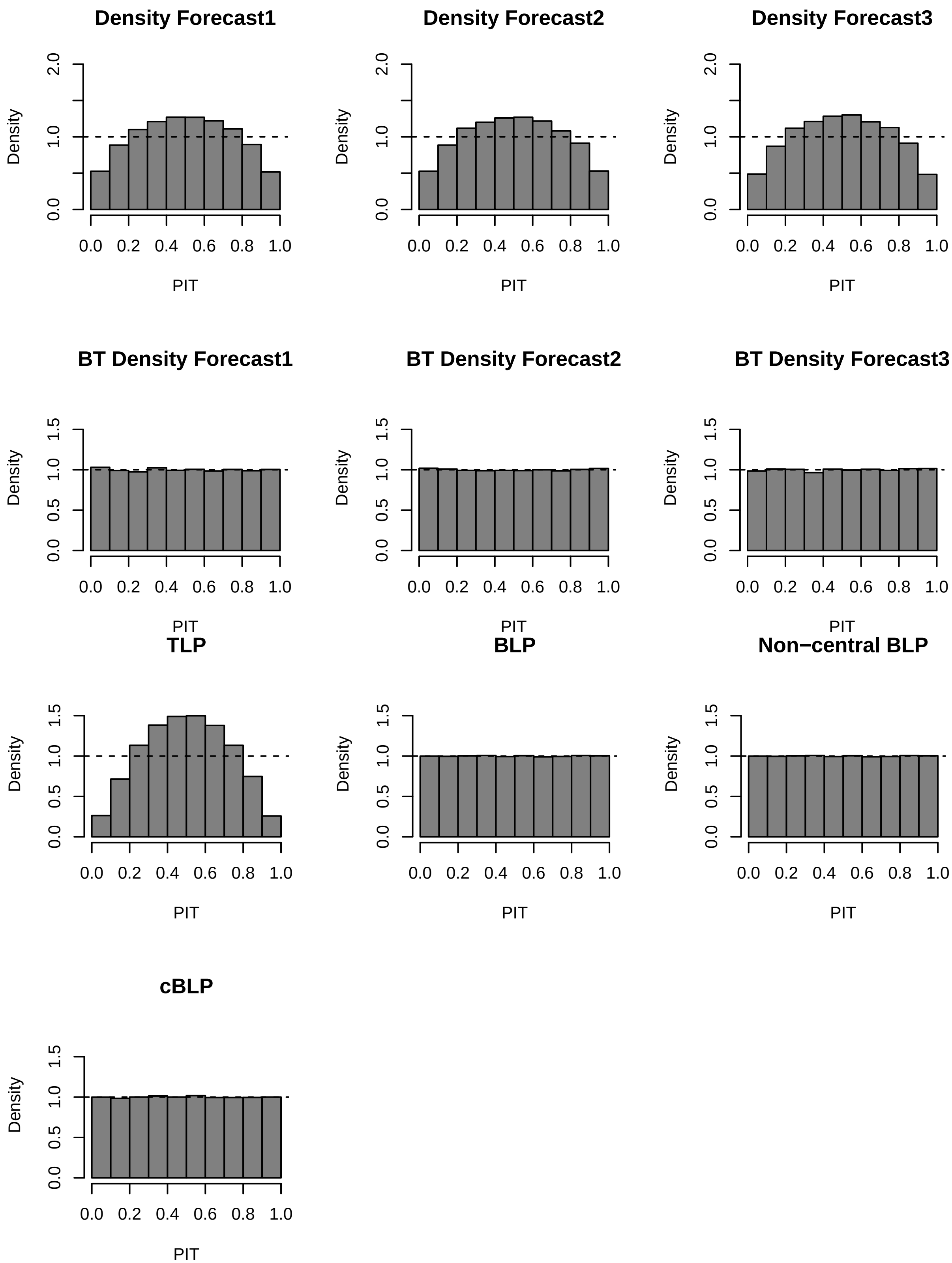
$$f_1 = \mathrm{N}(X_0 + a_1 X_1, (1 + a_2^2 + a_3^2) + 2)$$
$$f_2 = \mathrm{N}(X_0 + a_2 X_2, (1 + a_1^2 + a_3^2) + 2)$$
$$f_3 = \mathrm{N}(X_0 + a_3 X_3, (1 + a_1^2 + a_2^2) + 2)$$

Table 3: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp
TLP	0.220	0.209	0.571	NA	NA	NA
BLP	0.297	0.291	0.412	2.145	2.138	NA
nBLP	0.297	0.291	0.412	2.143	2.140	0.01
cBLP	0.301	0.295	0.404	1.455	1.455	NA

	Training	Test
f1	-2.052	-2.053
f2	-2.051	-2.056
f3	-2.022	-2.022
TLP	-2.006	-2.008
BLP	-1.861	-1.865
nBLP	-1.861	-1.865
cBLP	-1.866	-1.870

	Training	Test
f1	0.062	0.062
f2	0.062	0.063
f3	0.061	0.061
TLP	0.049	0.049
BLP	0.083	0.084
nBLP	0.083	0.084
cBLP	0.083	0.083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Scenario 4: Over- and underdispersed components

The models are defined as follows:

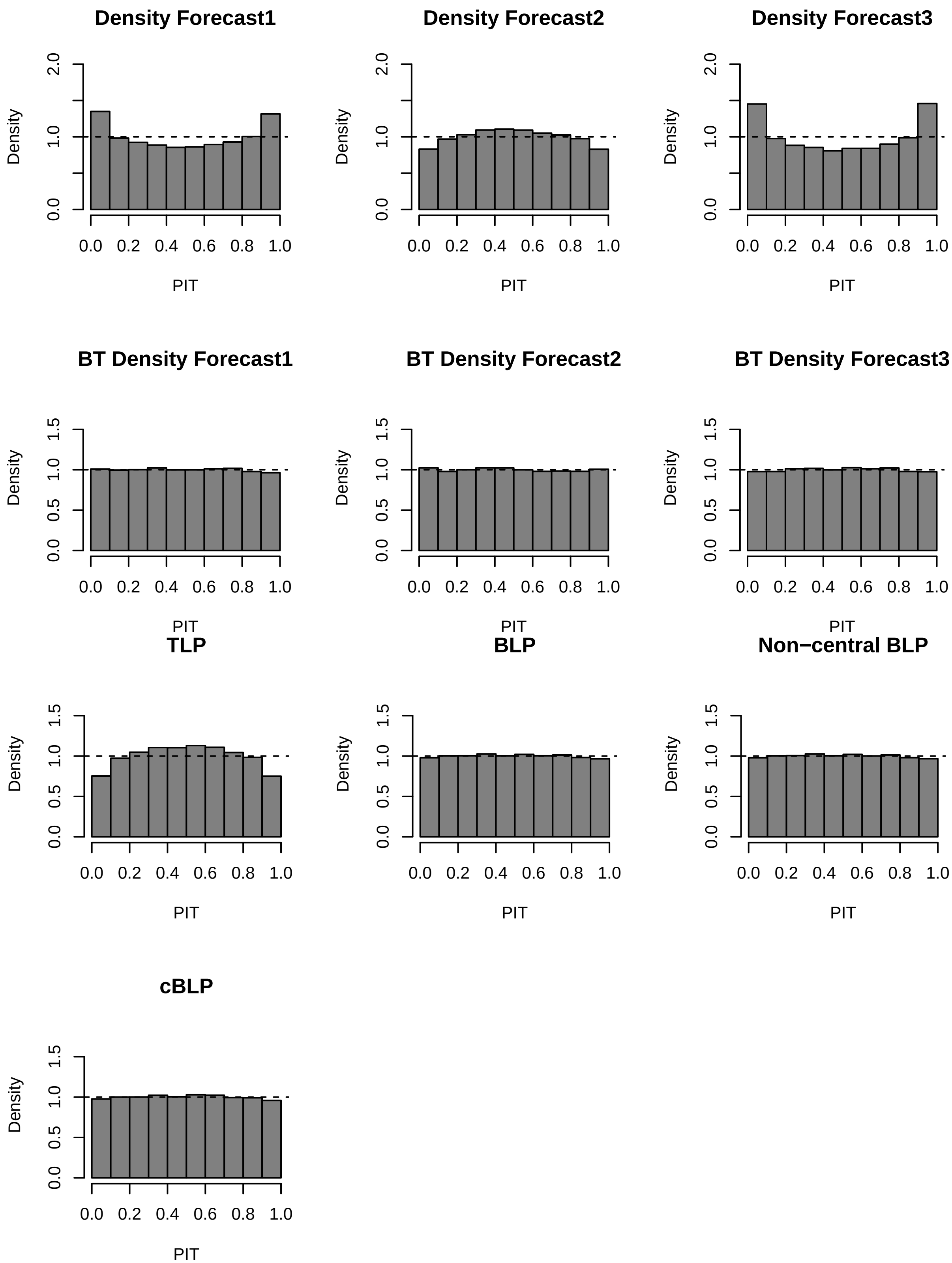
$$f_1 = \mathrm{N}(X_0 + a_1 X_1, (1 + a_2^2 + a_3^2) - 0.8)$$
$$f_2 = \mathrm{N}(X_0 + a_2 X_2, (1 + a_1^2 + a_3^2) + 0.6)$$
$$f_3 = \mathrm{N}(X_0 + a_3 X_3, (1 + a_1^2 + a_2^2) - 1)$$

Table 4: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp
TLP	0.335	0.191	0.474	NA	NA	NA
BLP	0.288	0.333	0.379	1.287	1.280	NA
nBLP	0.288	0.333	0.379	1.285	1.283	0.015
cBLP	0.345	0.198	0.456	1.436	1.436	NA

	Training	Test
f1	-2.032	-2.024
f2	-2.012	-2.011
f3	-2.017	-2.007
TLP	-1.893	-1.887
BLP	-1.877	-1.871
nBLP	-1.877	-1.871
cBLP	-1.874	-1.867

	Training	Test
f1	0.097	0.097
f2	0.076	0.076
f3	0.102	0.102
TLP	0.074	0.073
BLP	0.083	0.082
nBLP	0.083	0.082
cBLP	0.082	0.082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Comments

- BLP, nBLP, and cBLP outperforms TLP based on log score and probabilistic calibration in most scenarios.
- cBLP does better when all or most components are underdispersed. The last scenario where the sharpest component is underdispersed cBLP also does well probably because it's able to fix the dispersion of the sharp component and then assign the most weight to that component.