BLP Simulation

Nutcha Wattanachit

12/2/2019

Ensemble/pooling Methods

TLP

Traditional linear pool finds optimal weights that maximmizes the likelihood of $f(y) = \sum_{i=1}^{k} w_i f_i(y)$.

BLP

BLP finds α, β, and weights that maximize the likelihood of

$$g_{\alpha,\beta} = (\sum_{i=1}^{k} w_i f_i(y)) b_{\alpha,\beta} (\sum_{i=1}^{k} w_i F_i(y)).$$

BLP Example: To obtain α, β, and the weights for all component models, train the BLP model on half of the data. Then, use α, β, and the weights from training to apply to the data held out for testing.

Bias-corrected TLP (bcTLP)

Bias-corrected BLP (bcBLP)

BLP with Non-central Parameter(nBLP)

nBLP finds α, β, non-central parameter λ, and weights that maximize the likelihood of

$g_{\alpha,\beta,\lambda} = (\sum_{i=1}^{k} w_i f_i(y)) b_{\alpha,\beta} (\sum_{i=1}^{k} w_i F_i(y))$.

nBLP process: To obtain α, β, λ, and the weights for all component models, train the nBLP model on half of the data. Then, use α, β, λ, and the weights from training to apply to the data held out for testing.

Component-wise BLP (cBLP)

This is the extension of the traditional BLP. We beta-transform each of the cumulative distribution functions of the component models. This is done by finding α and β that maximize the likelihood of

$$G_{i,\alpha_i,\beta_i} = B_{\alpha_i,\beta_i}[F_i(y)]$$
$$g_{i,\alpha_i,\beta_i} = f_i(y) \times b_{\alpha_i,\beta_i}[F_i(y)]$$

Then, to obtain α, β, and the weights for 21 models, we apply BLP on the beta-transformed components:

$$G_{\alpha,\beta} = B_{\alpha,\beta}\Big[\sum_{i=1}^{k} w_i B_{\alpha_i,\beta_i}(F_i(y))\Big]$$

$$g_{\alpha,\beta} = \Big(\sum_{i=1}^{k} w_i b_{i,\alpha_i,\beta_i}(F_i(y)) f_i(y)\Big) b_{\alpha,\beta}\Big(\sum_{i=1}^{k} w_i B_{i,\alpha_i,\beta_i}(F_i(y))\Big)$$

cBLP - Part 1: For each component model, train over all observations to get α_i and β_i. Then, apply α_i and β_i to beta-transform the CDF. This ends the component-wise part.

cBLP Part 2: Apply the usual BLP process on the beta-transformed component models to get the BLP ensemble.

Componentwise Bias-Corrected & Componentwise Recalibrated BLP (cbcBLP)

Simulation studies

Scenario 1: calibrated components (the exact same setup as in the Gneiting paper).
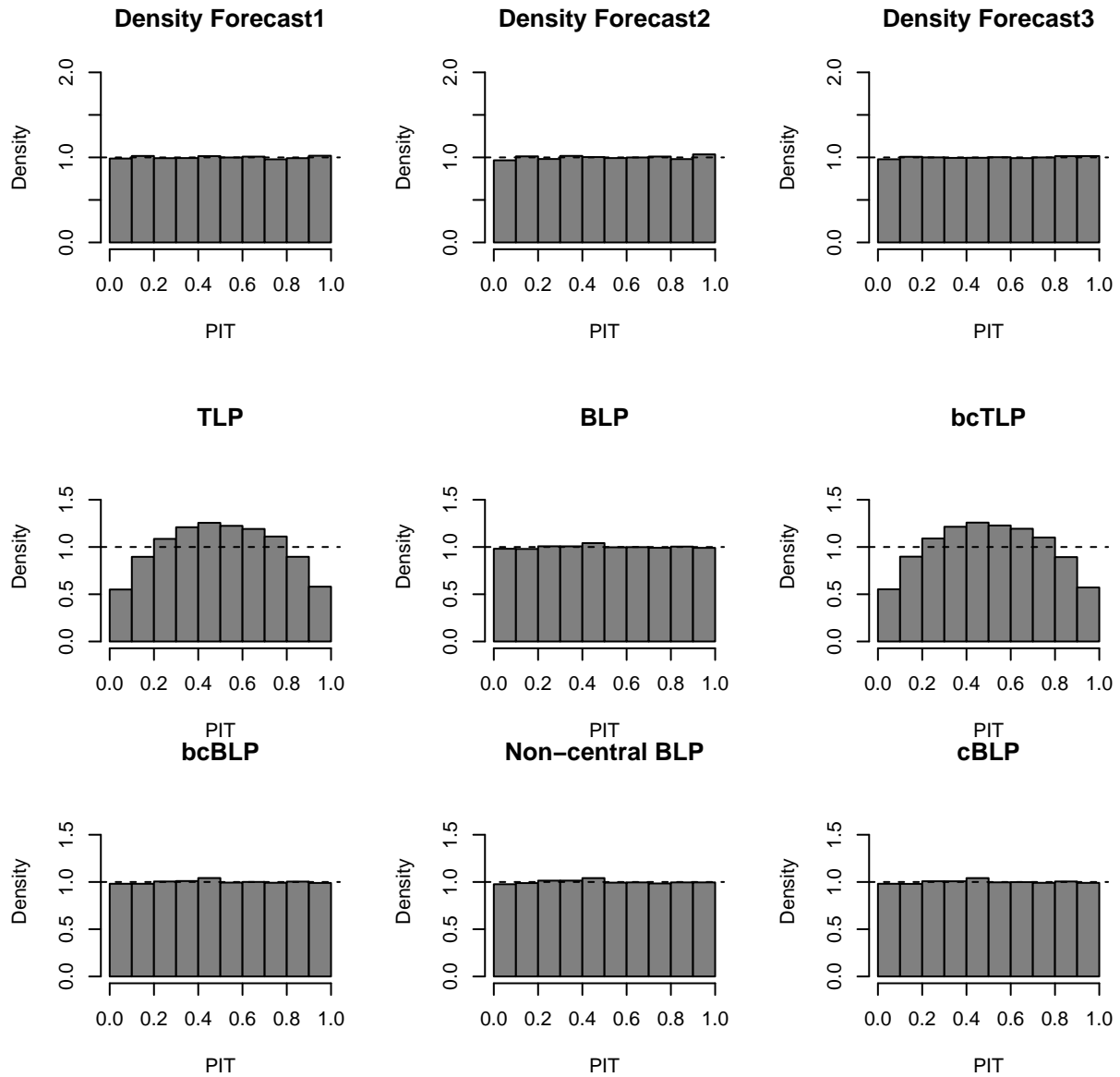
Table 1: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp		Training	Test
TLP	0.258	0.267	0.475	NA	NA	NA	f1	-2.006	-2.003
BLP	0.292	0.297	0.411	1.451	1.445	NA	f2	-2.005	-2.001
bcTLP	0.257	0.268	0.474	NA	NA	NA	f3	-1.969	-1.968
bcBLP	0.291	0.298	0.411	1.452	1.452	NA	TLP	-1.911	-1.910
nBLP	0.292	0.297	0.411	1.425	1.471	0.148	BLP	-1.871	-1.869
cBLP	0.293	0.297	0.410	1.453	1.452	NA	bcTLP	-1.911	-1.910
							bcBLP	-1.870	-1.868
							nBLP	-1.871	-1.869
							cBLP	-1.871	-1.869

Scenario 2: All component are not calibrated.

Density Forecast1

Density Forecast2

Density Forecast3

TLP

BLP

bcTLP

bcBLP

Non-central BLP
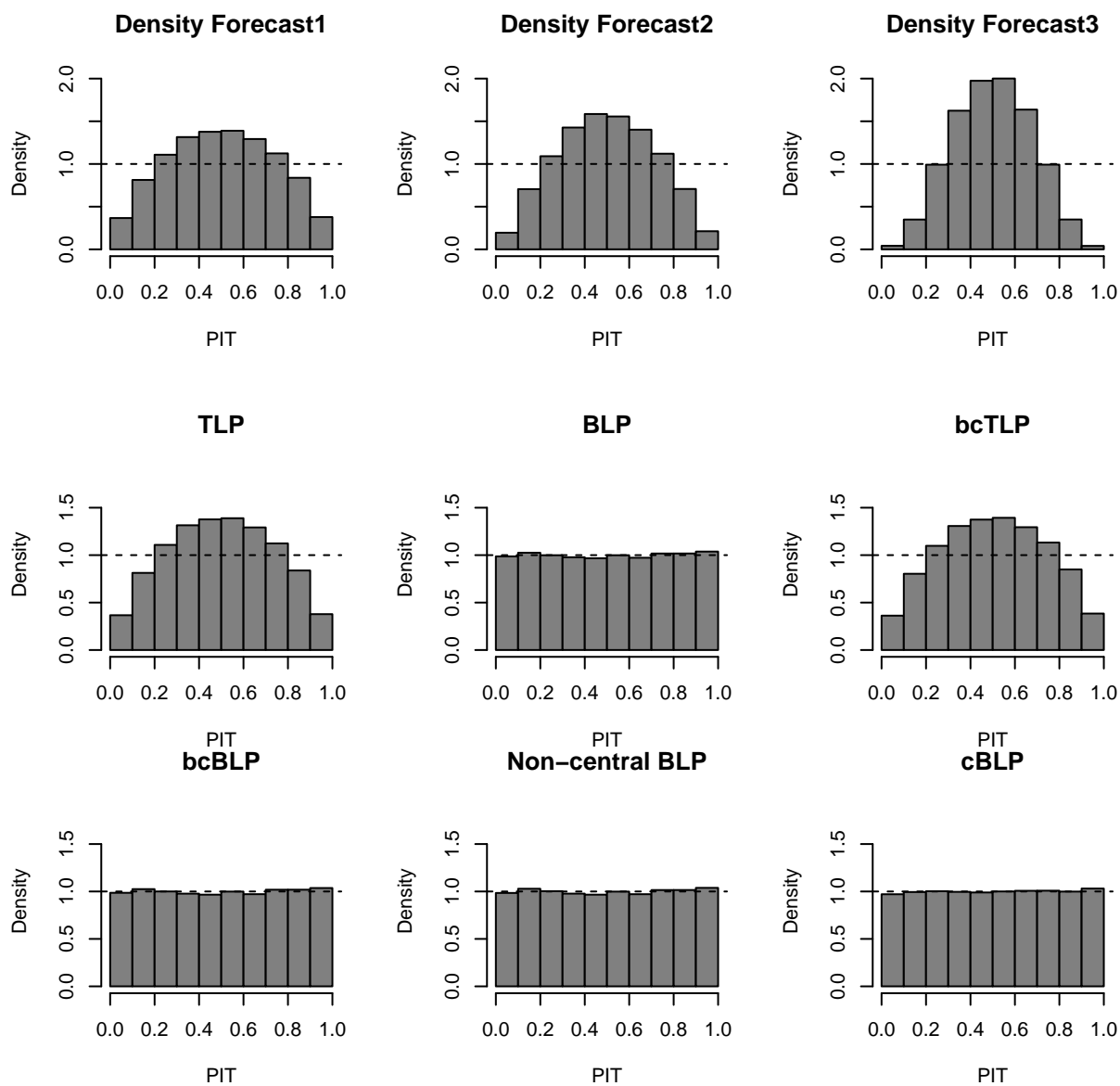
cBLP

Table 2: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp
TLP	1.000	0.000	0.000	NA	NA	NA
BLP	0.250	0.276	0.475	3.591	3.602	NA
bcTLP	1.000	0.000	0.000	NA	NA	NA
bcBLP	0.251	0.275	0.475	3.595	3.595	NA
nBLP	0.250	0.276	0.475	3.558	3.634	0.149
cBLP	0.371	0.334	0.295	1.455	1.455	NA

	Training	Test
f1	-2.090	-2.090
f2	-2.167	-2.169
f3	-2.316	-2.317
TLP	-2.090	-2.090
BLP	-1.859	-1.863
bcTLP	-2.090	-2.090
bcBLP	-1.860	-1.863
nBLP	-1.860	-1.863
cBLP	-1.882	-1.885

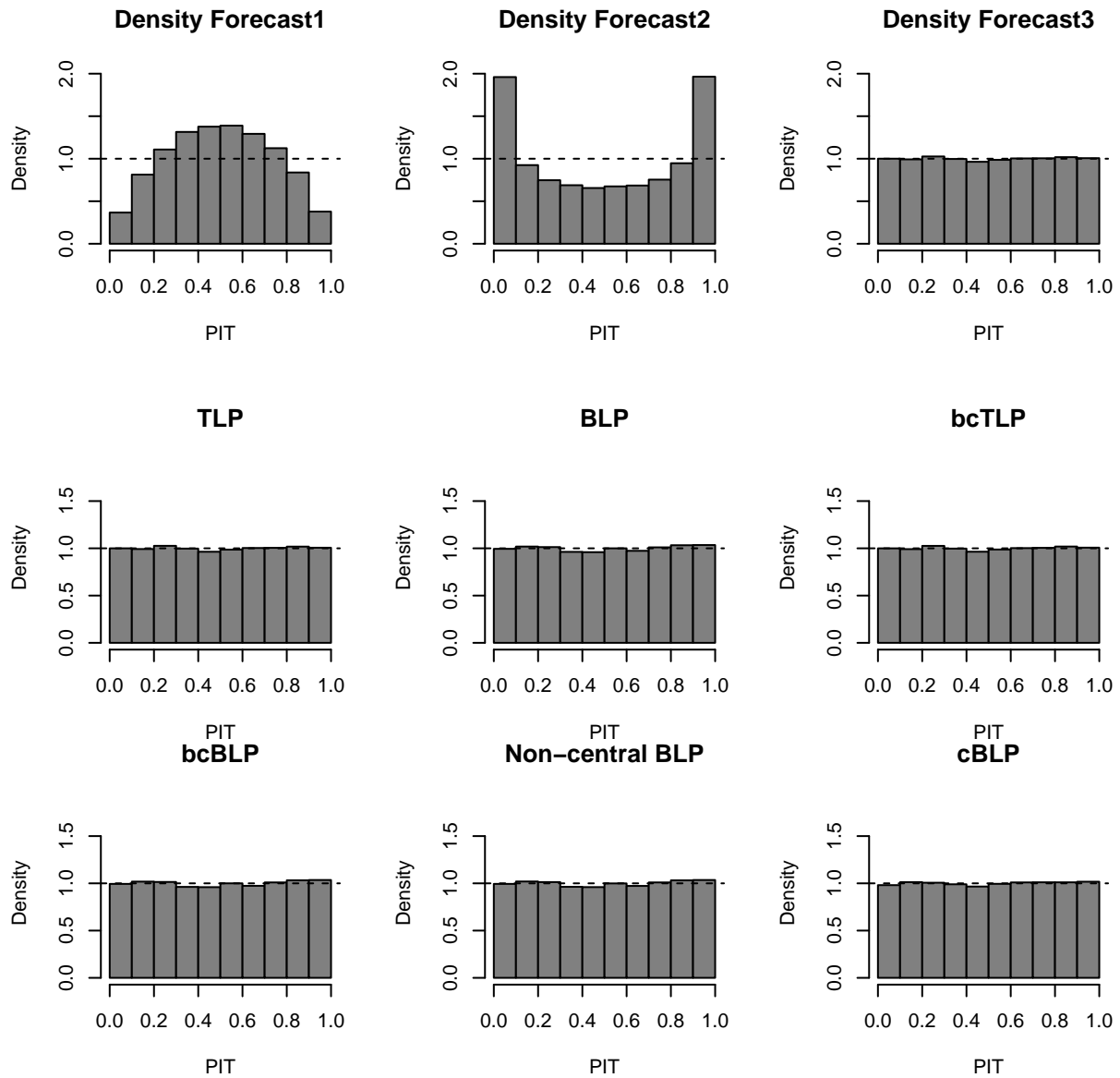## Scenario 3 : One calibrated and two miscalibrated components (one has a U-shape PIT histogram)

Density Forecast1

Density Forecast2

Density Forecast3

TLP

BLP

bcTLP

bcBLP

Non−central BLP

cBLP

Table 3: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp		Training	Test
TLP	0.000	0	1.000	NA	NA	NA	f1	-2.090	-2.090
BLP	0.521	0	0.479	1.769	1.776	NA	f2	-2.615	-2.629
bcTLP	0.000	0	1.000	NA	NA	NA	f3	-1.966	-1.969
bcBLP	0.521	0	0.479	1.773	1.773	NA	TLP	-1.966	-1.969
nBLP	0.521	0	0.479	1.765	1.779	0.021	BLP	-1.893	-1.896
cBLP	0.348	0	0.652	1.304	1.304	NA	bcTLP	-1.966	-1.969
							bcBLP	-1.893	-1.896
							nBLP	-1.893	-1.896
							cBLP	-1.983	-1.983

Scenario 4 All miscalibrated components (One has a U-shape, the other two have right and left skewed PIT histograms)
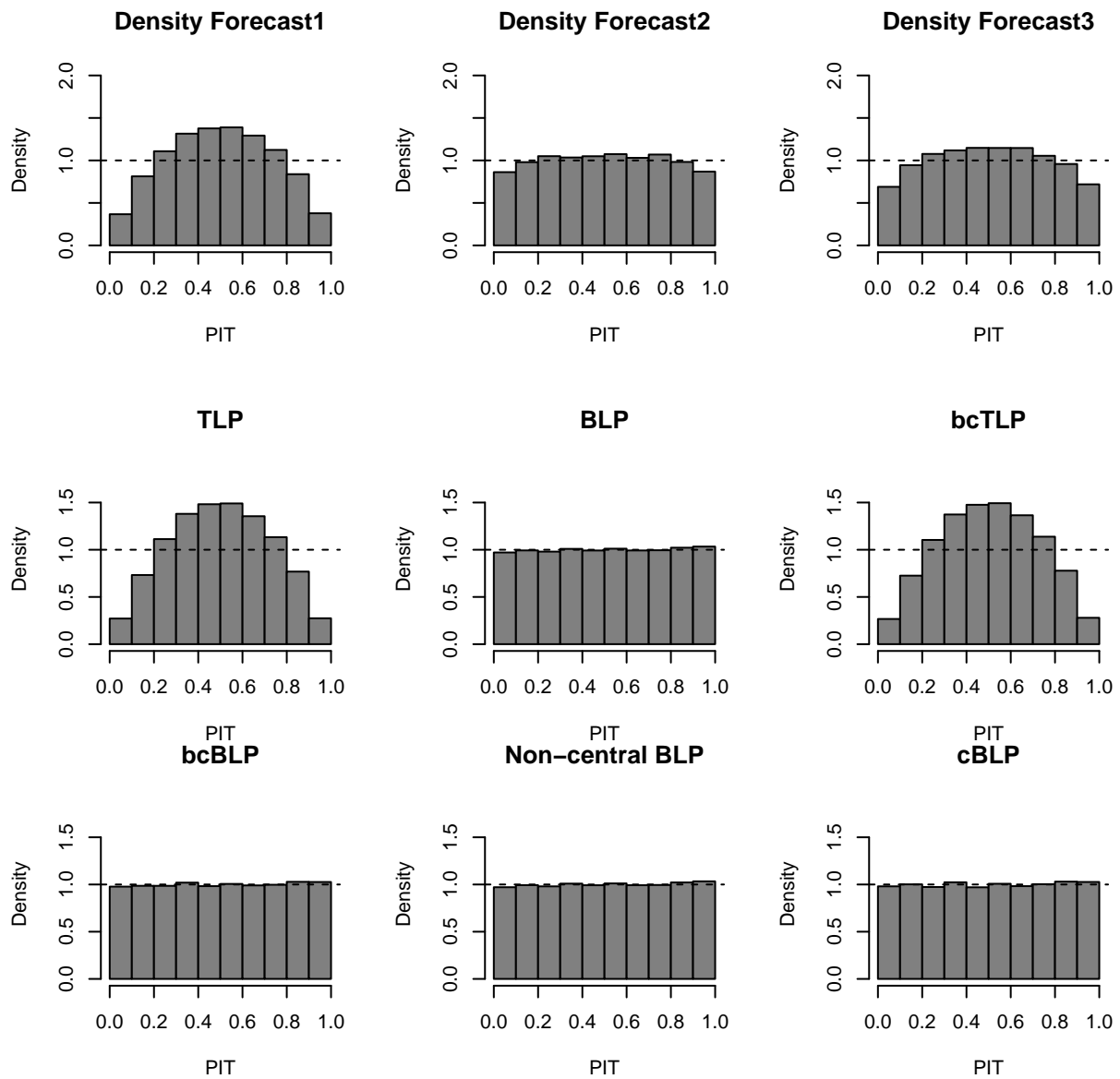
Density Forecast1

Density Forecast2

Density Forecast3

TLP

BLP

bcTLP

bcBLP

Non−central BLP

cBLP

Table 4: Model Parameters and Log Score

	w1	w2	w3	alpha	beta	ncp
TLP	1.000	0.000	0.000	NA	NA	NA
BLP	1.000	0.000	0.000	2.023	2.037	NA
bcTLP	1.000	0.000	0.000	NA	NA	NA
bcBLP	0.908	0.000	0.092	1.995	1.995	NA
nBLP	1.000	0.000	0.000	2.023	2.037	0
cBLP	0.771	0.229	0.000	1.078	1.078	NA

	Training	Test
f1	-2.130	-2.130
f2	-2.483	-2.493
f3	-3.096	-3.101
TLP	-2.130	-2.130
BLP	-2.000	-2.001
bcTLP	-2.130	-2.130
bcBLP	-1.995	-1.996
nBLP	-2.000	-2.001
cBLP	-2.288	-2.291