Essential Mathematics for AI

Solving the systems of linear equations

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*Our team choose the first equations**

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1. First system:

$$\begin{cases} x_1 - 3x_2 = 5 \\ -x_1 + x_2 + 5x_3 = 2 \\ x_2 + x_3 = 0 \end{cases}$$

Present the system as arguemented matrix $[A|b]$:

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Then, do some elementary transformations to make row-echelon form:

$$\begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} (I) \rightsquigarrow \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ -1 & 1 & 5 & 2 \end{bmatrix} (II) \rightsquigarrow \begin{bmatrix} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{bmatrix}$$

(I) Swap the rows to easier reach the row-echelon form
(II) Add R_1 to R_3, add $2R_2$ to R_3
Transform the matrix back into the system of equations and find the unknowns:

$$\begin{cases} x_1 - 3x_2 = 5 \\ x_2 + x_3 = 0 \\ 7x_3 = 7 \end{cases} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = -1 \\ x_3 = 1 \end{cases}$$

In the figure 1, *the point* (x_1, x_2, x_3) of the intersection of three planes, marked in red, is the *solution of the system.*

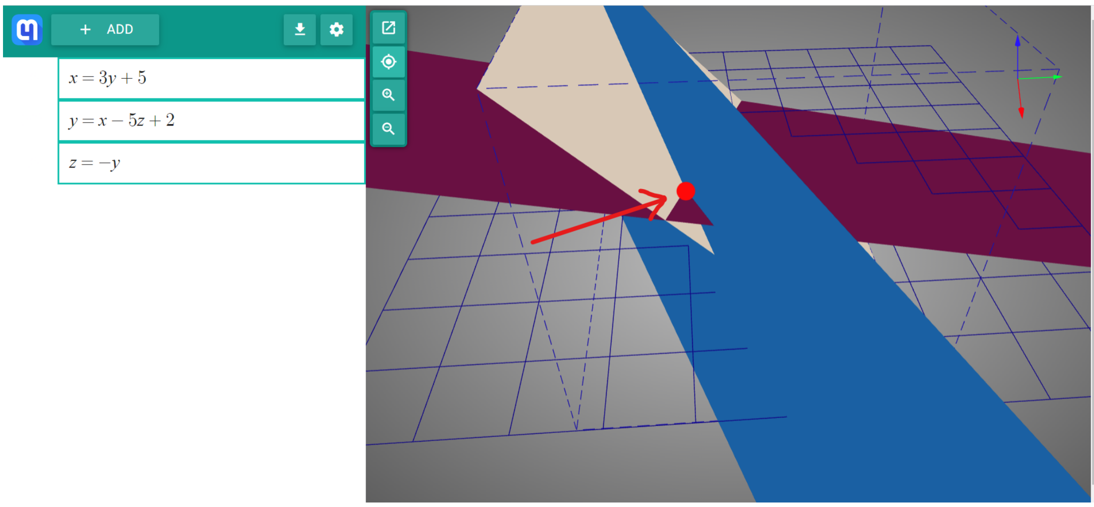

Figure 1: The first system

2. Second system:

$$\begin{cases} 2x_1 + x_2 + 4x_3 = -1 \\ x_1 + x_2 + x_3 = -1 \\ x_1 - x_2 + 5x_3 = 1 \end{cases}$$

Present the system as arguemented matrix $[A|b]$:

$$\begin{bmatrix} 2 & 1 & 4 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 5 & 1 \end{bmatrix}$$

Then, do some elementary transformations to make row-echelon form:

$$\begin{bmatrix} 2 & 1 & 4 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 5 & 1 \end{bmatrix} (I) \rightsquigarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & 5 & 1 \\ 2 & 1 & 4 & -1 \end{bmatrix} (II) \rightsquigarrow$$
$$(II) \rightsquigarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & -2 & 4 & 2 \\ 0 & -1 & 2 & 1 \end{bmatrix} (III) \rightsquigarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & -2 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(I) Swap the rows to easier reach the row-echelon form
(II) Subtract R_1 from R_2, subtract $2R_1$ from R_3
(III) Add R_2 divided by -2 to R_3

As we can see the ruslt matrix has the zero line that means x_3 is a free variable and the system has an infinity number of solutions.

In the figure 2, the *intersection* of three planes is the solution of the system, in this case it is a *straight line*, so the number of solutions *is infinite*. In the figure 3, we see that there are only 2 planes left, but the solution of the system is the same straight line.

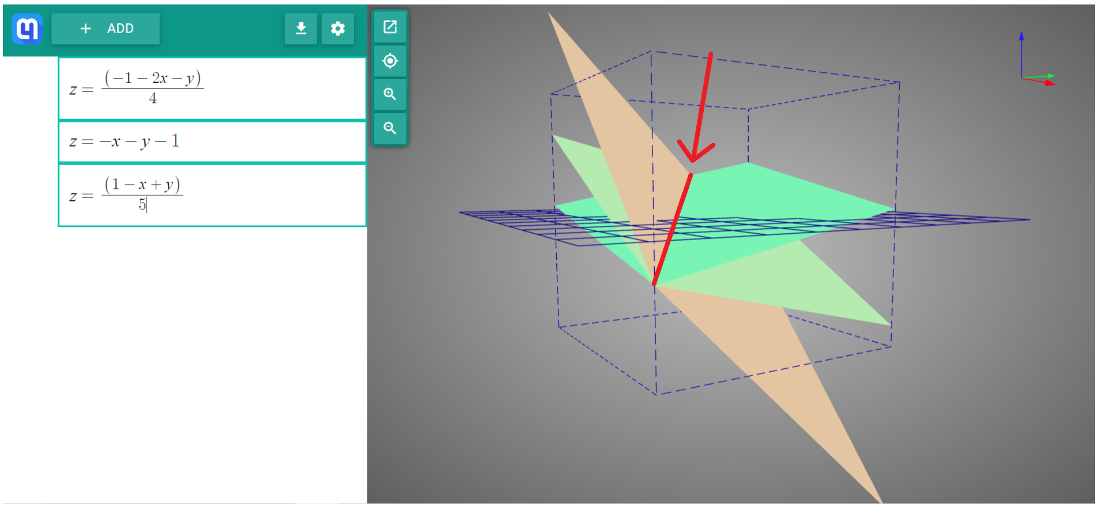

Figure 2: The original system

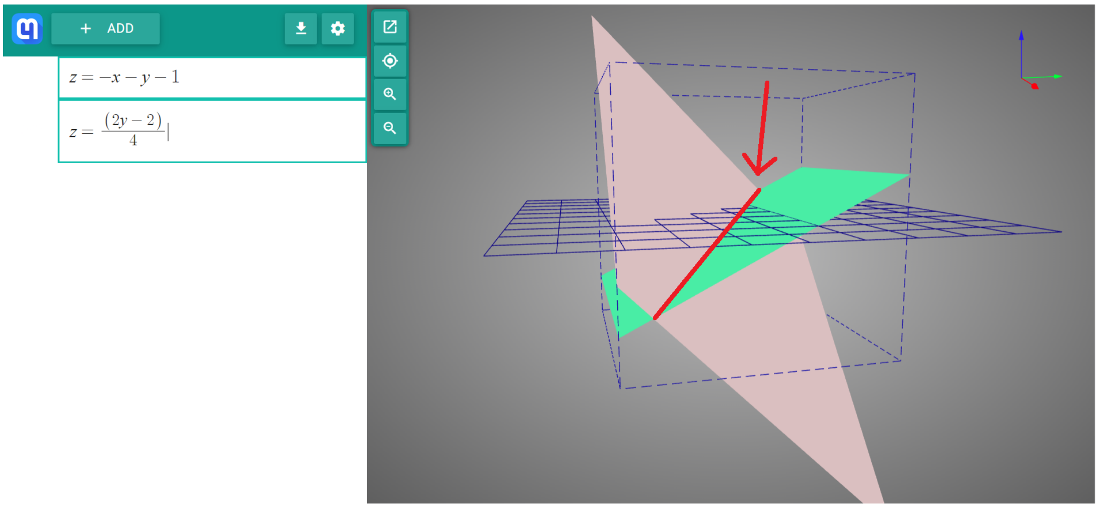

Figure 3: The converted system

3. Third system:

$$\begin{cases} x_1 + x_3 = 1 \\ -x_1 + x_2 = 0 \\ x_2 + x_3 = 0 \end{cases}$$

Present the system as arguemented matrix $[A|b]$:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Then, do some elementary transformations to make row-echelon form:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} (I) \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} (II) \rightsquigarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

(I) Add R_1 to R_2
(II) Subtract R_2 from R_3

In the third row we get impossible equality — $0 = -1$, so this system *doesn't have solution.*
As you see, there is *no intersection* of three plane in one point/line in the figure 4.

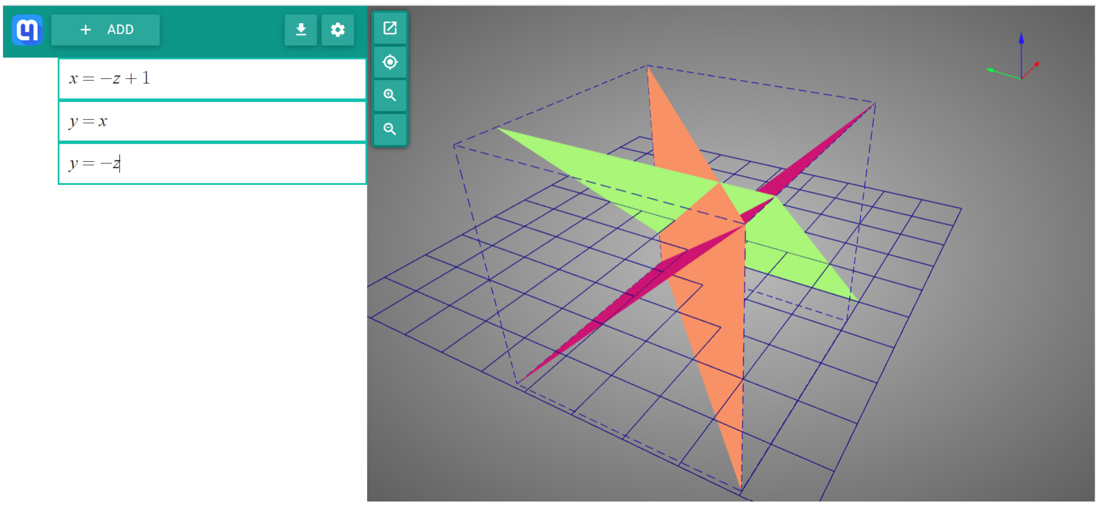

Figure 4: The third system