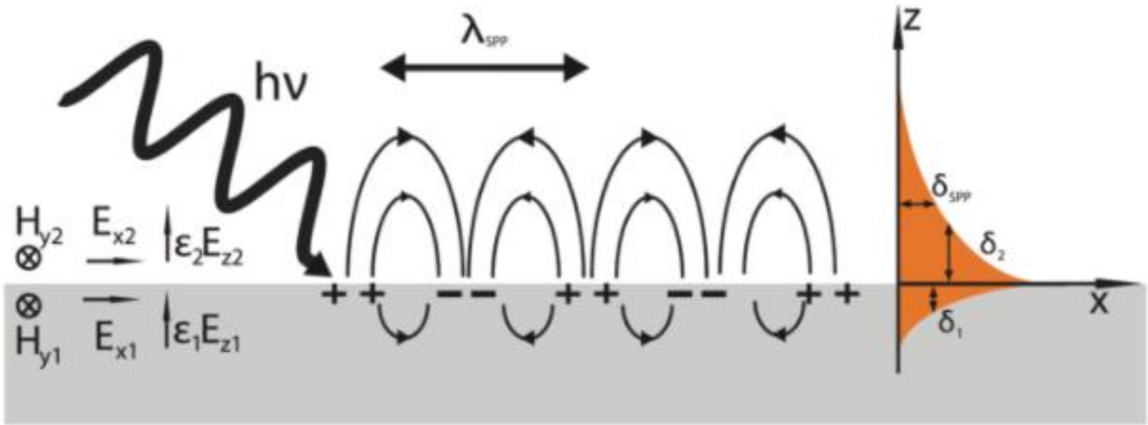
纳米光子学 Nanophotonics

第7讲：表面等离激元



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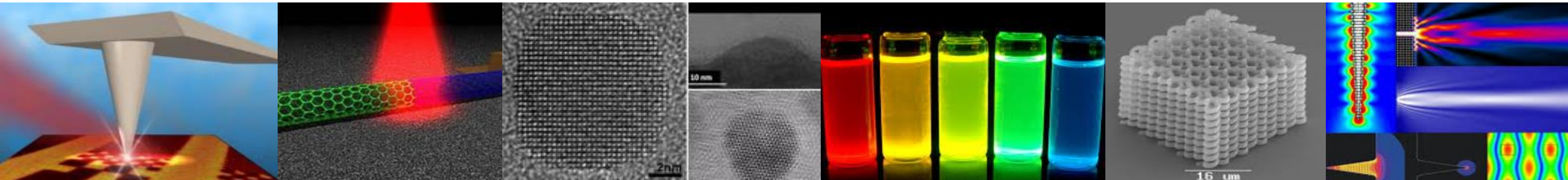


等离子体光学

- 金属光学与体积等离激元
- 表面等离子体激元
- 表面等离子体激元的激发与表征
- 局域表面等离子体
- 等离子体集成电路

上讲内容回顾

从金属Drude模型出发，分析电子受力：

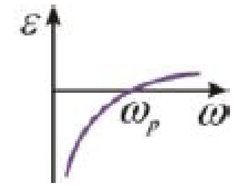
$\omega_p = \sqrt{\dfrac{Ne^2}{\varepsilon_0 m}}$ (等离子体频率)

可以导出介电常数:

$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_0\varepsilon\mathbf{E} \Rightarrow \boxed{\varepsilon(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2 + i\omega\gamma}}$

高频 ($\omega >> \gamma$)

$\varepsilon(\omega) \approx 1 - \dfrac{\omega_p^2}{\omega^2}$



① 对于高频 $\omega > \omega_p$: → $\varepsilon > 0$ → $n = \sqrt{\varepsilon} = n' + in''$ $(n' > 0, n'' = 0)$

$E = E_0 \exp(in'k_0 \cdot r)$ → 金属是透明的 (像电介质)

② 对光频 $\gamma << \omega < \omega_p$: → $\varepsilon < 0$ → $n = \sqrt{\varepsilon} = n' + in''$ $(n' \approx 0, n'' > 0)$

$E = E_0 \exp(-n''k_0 \cdot r)$, → 场指数衰减 $R \approx 1$ → 金属表面高反射

→ 当$\gamma = 0 \to$ 理想金属 , $R = 1$ ⇒ 自由电子气体

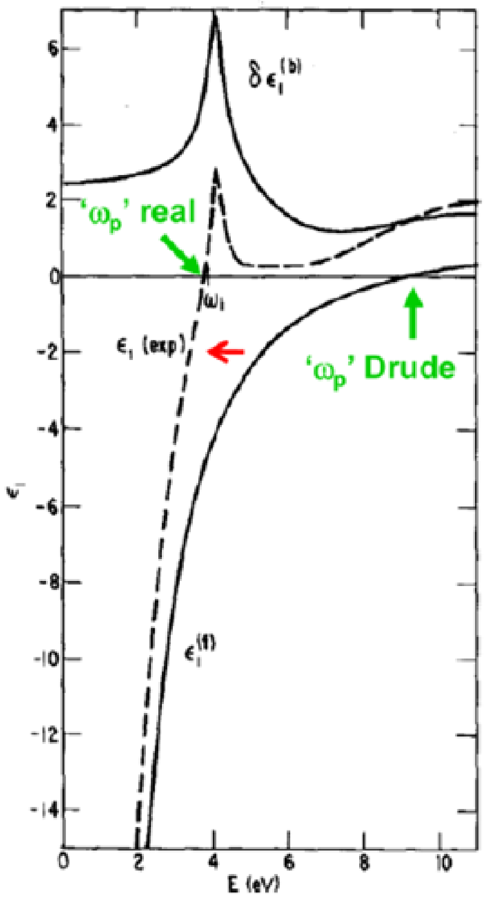
③ 对低频 $\omega << \gamma$: $\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$ → $\varepsilon'' >> \varepsilon'$

→ 折射率 $n' \approx n'' \approx \sqrt{\dfrac{\varepsilon''}{2}}$ $E = E_0 \exp(in'k_0 \cdot r)\exp(-n''k_0 \cdot r)$, $\delta = c/n''\omega$

→场迅速衰减 $R \approx 1$ →金属表面高反射率 → ω 很低时 → 理想导体

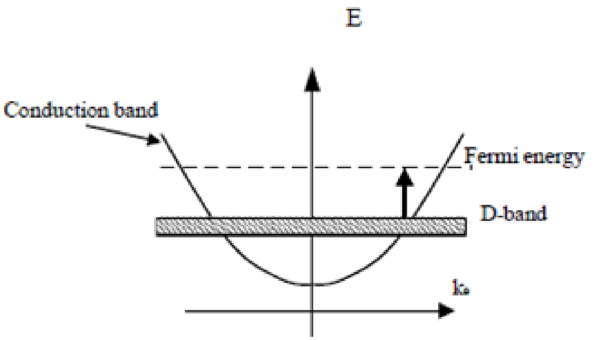
④ $\omega \approx \omega_p$: → $\varepsilon \approx 0$ → $\boxed{n = \sqrt{\varepsilon\mu} \approx 0}$ → $k = nk_0 \approx 0$ 体积等离子体共振

真实金属介电函数谱



对于真正金属，特别是贵金属 (e.g., Ag):

- 测量显示ε′有一个峰值
  且ω_p 被迁移，为什么？
- 能带间跃迁 (束缚电子被激发)



- **Drude** 模型应当用 **Lorentz**-振荡项补充:

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega^2+i\omega\gamma}+\sum_j\frac{\omega_{jp}^2}{\omega_{j0}^2-\omega^2-i\,\omega\gamma_j}$$

电磁场的麦克斯维方程组

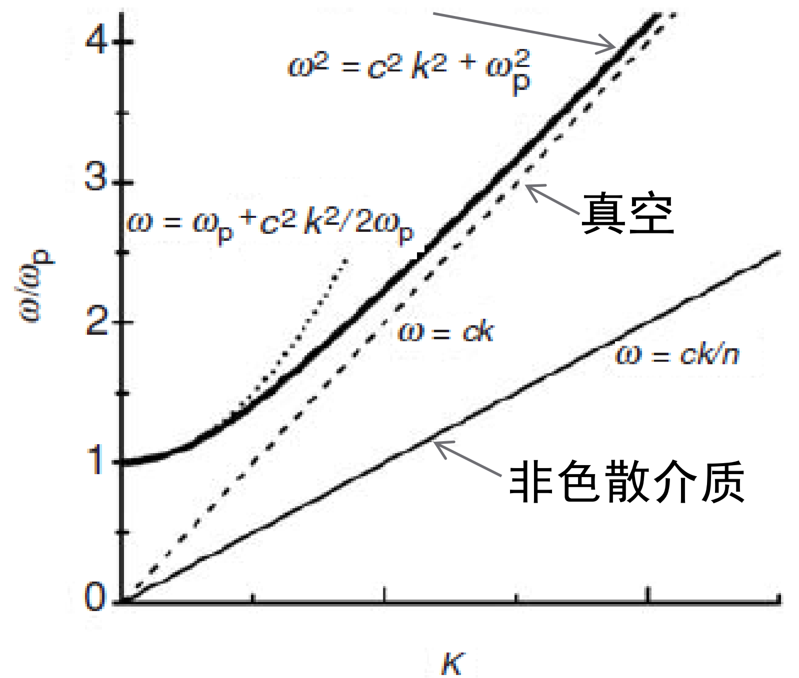
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = -\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J},$$

$$\mathbf{k}(\mathbf{k}\cdot\mathbf{E}) - k^2\mathbf{E} = -\varepsilon(\mathbf{k},\omega)\frac{\omega^2}{c^2}\mathbf{E}$$

① $\mathbf{k}\cdot\mathbf{E} = 0$ → $k^2 = \varepsilon(\mathbf{k},\omega)\frac{\omega^2}{c^2}$, $\omega = ck/\sqrt{\varepsilon}$ → $\omega^2 = c^2k^2 + \omega_p^2$

② $\mathbf{k}//\mathbf{E}$ $\quad \varepsilon(\mathbf{k},\omega) = 0$, $\omega = \omega_p$

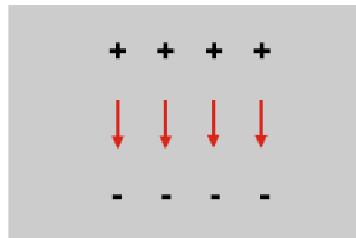
自由电子气或高频下金属



金属体积等离子体共振频率的意义

Plasmon resonance positions in vacuum

Bulk metal



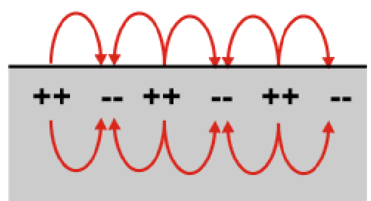
ω_p

$\varepsilon = 0$

Drude model

$$\varepsilon_m = 1 - \frac{\omega_p^2}{\omega^2}$$

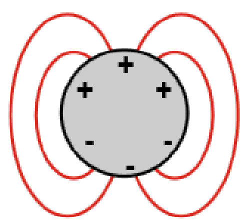
Metal surface



$\varepsilon = -1 \xrightarrow{} \omega_p/\sqrt{2}$

drude model

Metal sphere
localized SPPs



$\varepsilon = -2 \xrightarrow{} \omega_p/\sqrt{3}$

drude model

表面等离激元
Surface Plasmon Polariton (SPP)

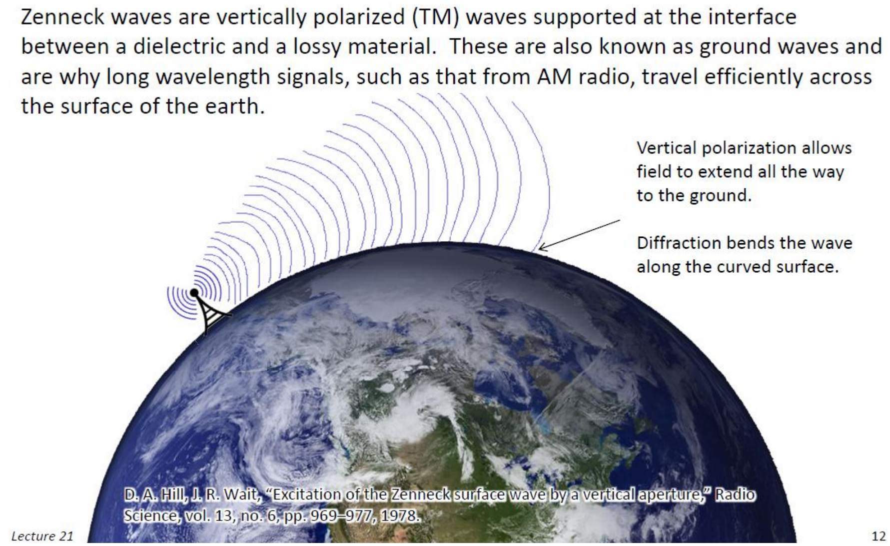
表面等离极化激元

本讲内容

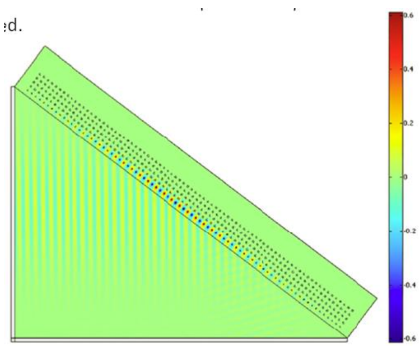
- **表面波**
- **SPP的色散关系推导**
- **SPP的产生机理**
- SPP的色散关系规律
- SPP：横波和纵波
- SPP的波长
- SPP的传导波长和损耗

各种类型表面波(surface wave)

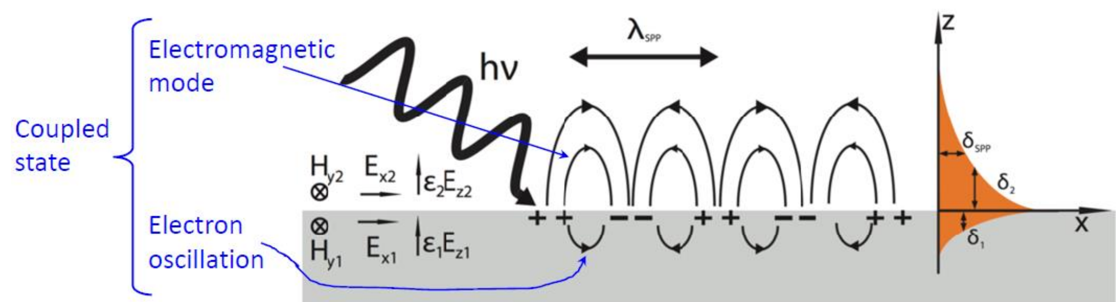
水波：水/空气界面传播的机械波



地面波：地球表面/空气界面传播的电磁表面波



共振表面波：光子晶体/介质界面传播的电磁表面波



表面等离激元：金属/介质界面传播的电磁表面波

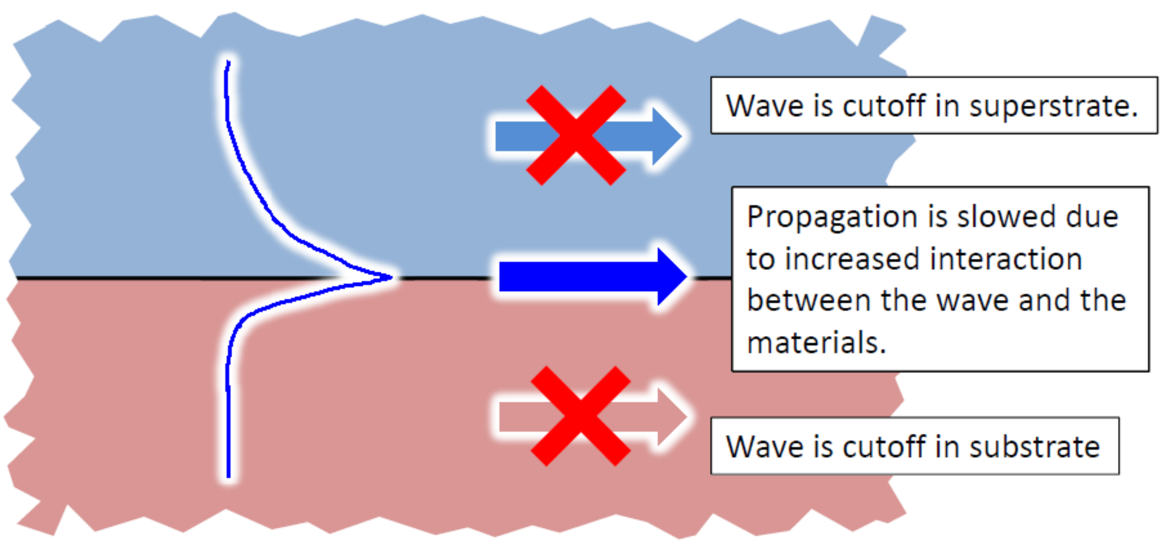
Nonlinear surface wave, Dyakonov surface waves、、、

表面等离子体激元(Surface Plasmon Polariton, SPP)

金属中第二类等离子体：

- 表面等离子体——金属和电介质界面上的等离子体振荡
- 当表面等离子体与光子耦合时——表面等离子体激元
- SPP是表面波——沿界面传播，K_{spp}

  ——法线方向呈现约束——衰减波

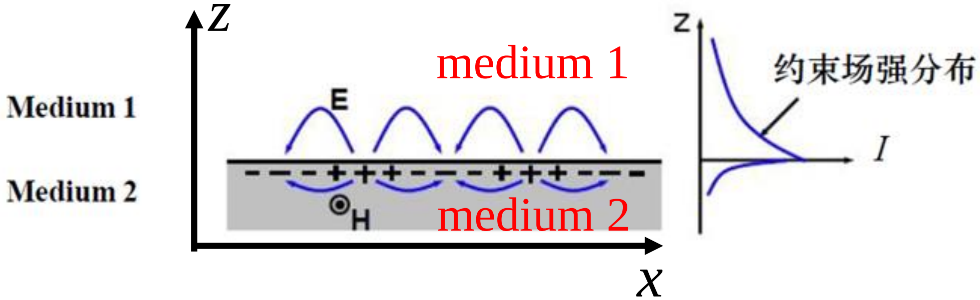


SPP的色散关系

理论假定解

用边界条件解麦克斯韦方程

要寻找的解如下:　　两种材料的界面



电磁波沿 x 方向传播，沿z方向指数分布，应该有下面的表达式:

$$\boldsymbol{E}(x,y,z) = \boldsymbol{A}\exp(\pm k_z z)\exp(\mathrm{i}\beta x), \quad \begin{cases} \text{“}-\text{” in medium 1} \\ \text{“}+\text{” in medium 2} \end{cases}$$

（x-z截面）

- $\beta = k_x$: 即波矢 **k** 的 x 分量，也称为传播常数
- ik_z: 波矢 **k** 的 z 分量，其中 k_z 为实数，即波矢 **k** 的 z 分量是一个虚数！
- 考虑波矢 **k** 位移 x-z 平面，则：

$$k^2 = k_x^2 + (\mathrm{i}k_z)^2 = \beta^2 - k_z^2 \Rightarrow \beta = \sqrt{k^2 + k_z^2} > k \Rightarrow \begin{cases} \text{SPP波长小于自由空间}\lambda \\ \text{SPP相速度小于自由空间相速度} \end{cases}$$

波方程

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{H}=\mathbf{J}_{\text{ext}}+\frac{\partial\mathbf{D}}{\partial t}$$

$\mathbf{J}_{\text{ext}}=0$

$$\mathbf{H}=\frac{1}{\mu_0}\mathbf{B}$$

$$\boldsymbol{D}=\varepsilon_0\varepsilon\boldsymbol{E}$$

$$\begin{cases}\nabla\times\boldsymbol{E}=-\dfrac{\partial\boldsymbol{B}}{\partial t} & ① \\ \nabla\times\boldsymbol{B}=\mu_0\varepsilon_0\varepsilon\dfrac{\partial\boldsymbol{E}}{\partial t} & ②\end{cases}$$

$$\begin{cases}\nabla\cdot\boldsymbol{D}=0\\ \nabla\cdot\boldsymbol{D}=\varepsilon_0\nabla\cdot(\varepsilon\boldsymbol{E})=\varepsilon_0(\boldsymbol{E}\cdot\nabla\varepsilon+\varepsilon\nabla\cdot\boldsymbol{E})\end{cases}$$

$$\Rightarrow\nabla\cdot\boldsymbol{E}=-\frac{1}{\varepsilon(\boldsymbol{r})}\boldsymbol{E}\cdot\nabla\varepsilon$$

$$\nabla\times\nabla\times\boldsymbol{E}=\nabla(\nabla\cdot\boldsymbol{E})-\nabla^2\boldsymbol{E}$$

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$\nabla\times①$

$$\nabla\times\nabla\times\boldsymbol{E}=-\frac{\partial}{\partial t}\nabla\times\boldsymbol{B}$$

② 代入上式，得到

$$\nabla\times\nabla\times\boldsymbol{E}=-\mu_0\varepsilon_0\varepsilon\frac{\partial^2\boldsymbol{E}}{\partial t^2}$$

$$\nabla\left(-\frac{1}{\varepsilon(\boldsymbol{r})}\boldsymbol{E}\cdot\nabla\varepsilon\right)-\nabla^2\boldsymbol{E}+\frac{\varepsilon}{c^2}\frac{\partial^2\boldsymbol{E}}{\partial t^2}=0$$

假定：$\varepsilon(\boldsymbol{r})=\varepsilon$与位置无关

$$\nabla^2\boldsymbol{E}-\frac{\varepsilon}{c^2}\frac{\partial^2\boldsymbol{E}}{\partial t^2}=0$$

考虑时谐电磁波

$$\boldsymbol{E}=\boldsymbol{E}(x,y,z)\exp(-\mathrm{i}\omega t)$$

$$k_0=\omega/c$$

霍姆霍兹方程：

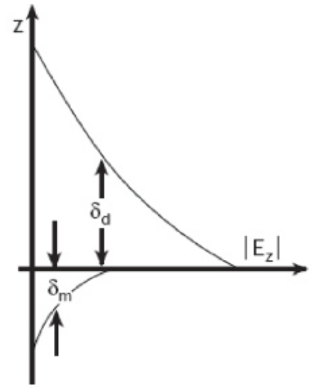
$$\nabla^2\boldsymbol{E}+k_0^2\varepsilon\boldsymbol{E}=0$$

波方程

$$\nabla^2 \boldsymbol{E} + k_0^2 \varepsilon \boldsymbol{E} = 0$$

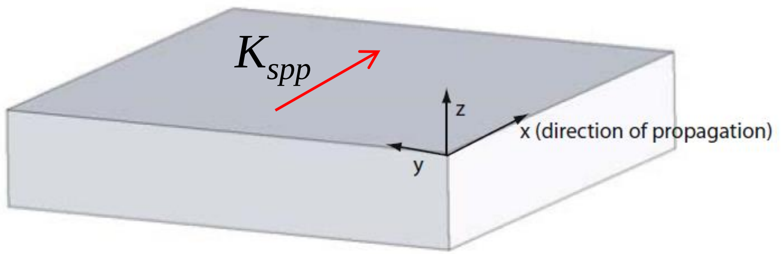
令： $\boldsymbol{E}(x,y,z) = \boldsymbol{E}(z)\exp(\mathrm{i}\beta x)$， $\beta = k_x$

波沿 x 方向传输，β 为传输常数，即 x 方向的波数 k_x



$$\boxed{\frac{\partial^2 \mathbf{E}(z)}{\partial z^2} + \left(k_0^2 \varepsilon - \beta^2\right) \mathbf{E} = 0.}$$

波动方程



z 方向衰减，要求 $k_0^2 \varepsilon - \beta^2 < 0$

令： $(\mathrm{i}k_z)^2 = k_0^2 \varepsilon - \beta^2$

$$\boxed{k_z^2 - \beta^2 + k_0^2 \varepsilon = 0}$$

k_z 与 β 的关系，但并未得到色散关系（ω - β ）

方程化为： $\dfrac{\partial^2 \boldsymbol{E}}{\partial z^2} - k_z^2 \boldsymbol{E} = 0$ $\Rightarrow$ $\boldsymbol{E} = \boldsymbol{E}_0 \exp(\pm k_z z)$

分量化与化简

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}_{\text{ext}} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\boldsymbol{E} = \boldsymbol{A} e^{-i\omega t}$$

$$\boldsymbol{H} = \boldsymbol{B} e^{-i\omega t}$$

时谐电磁波

$\mathbf{J}_{\text{ext}} = 0$

$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B}$

$\frac{\partial}{\partial t} = -i\omega$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i\omega\mu_0 H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega\mu_0 H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega\mu_0 H_z$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = -i\omega\varepsilon_0\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -i\omega\varepsilon_0\varepsilon E_z$$

$\partial x \to i\beta$

$\partial y = 0$

$\partial z \to \pm k_z$

$$\pm k_z E_y = -i\omega\mu_0 H_x$$

$$\pm k_z E_x - i\beta E_z = i\omega\mu_0 H_y$$

$$i\beta E_y = i\omega\mu_0 H_z$$

$$\pm k_z H_y = i\omega\varepsilon_0\varepsilon E_x$$

$$\pm k_z H_x - i\beta H_z = -i\omega\varepsilon_0\varepsilon E_y$$

$$i\beta H_y = -i\omega\varepsilon_0\varepsilon E_z$$

电磁波沿 x 方向传播，场量与 y 无关，$z = 0$ 两侧衰减

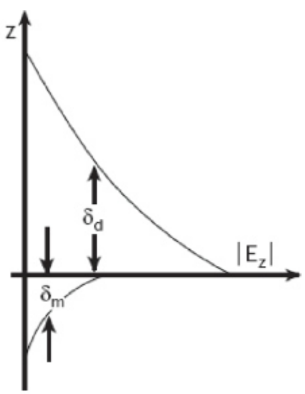
$$\boldsymbol{E}(x,z) = \boldsymbol{A} e^{\pm k_z z} e^{i\beta x} e^{-i\omega t}$$

$$\boldsymbol{H}(x,z) = \boldsymbol{H} e^{\pm k_z z} e^{i\beta x} e^{-i\omega t}$$

注意:

$k_z > 0$, 正负号已放到其前面。

$z > 0$，取$-k_z$; $z < 0$, 取 k_z



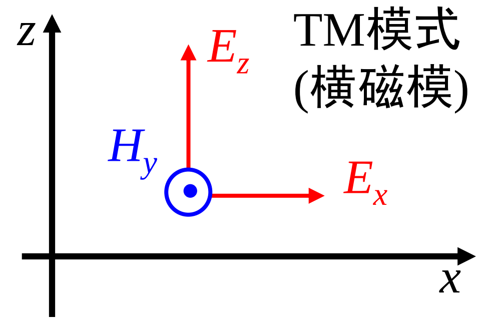
两套独立解

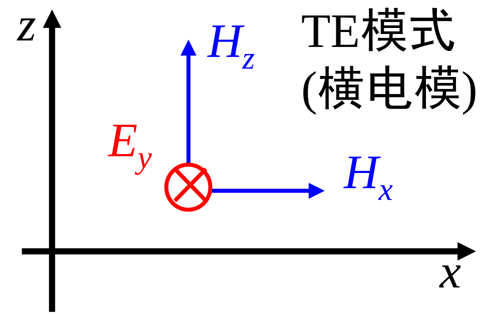
$$\pm k_z E_y = -i\omega\mu_0 H_x$$
$$\pm k_z E_x - i\beta E_z = i\omega\mu_0 H_y$$
$$i\beta E_y = i\omega\mu_0 H_z$$
$$\pm k_z H_y = i\omega\varepsilon_0\varepsilon E_x$$
$$\pm k_z H_x - i\beta H_z = -i\omega\varepsilon_0\varepsilon E_y$$
$$i\beta H_y = -i\omega\varepsilon_0\varepsilon E_z$$

两套独立解：

$$E_x = \pm k_z H_y / \mathrm{i}\omega\varepsilon_0\varepsilon$$
$$E_z = -\beta H_y / \omega\varepsilon_0\varepsilon$$
$$(k_z^2 - \beta^2 + k_0^2\varepsilon) H_y = 0$$

$$H_z = \beta E_y / \omega\mu_0$$
$$H_x = -\mathrm{i}\omega\mu_0 / (\pm k_z E_y)$$
$$(k_z^2 - \beta^2 + k_0^2\varepsilon) E_y = 0$$





参考面 x-z 平面

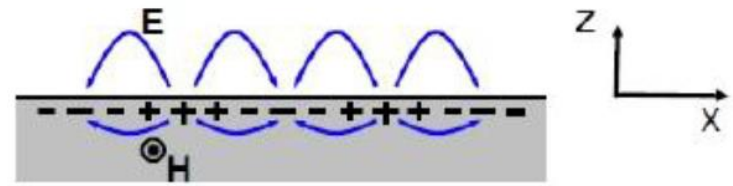
下面分别求解两种极化波对应的SPP波的波矢

已知：边界条件+波动方程

对TM波

$$\pm k_z E_x - i\beta E_z = i\omega\mu_0 H_y$$
$$i\beta H_y = -i\omega\varepsilon_0\varepsilon E_z$$
$$\boxed{\pm k_z H_y = i\omega\varepsilon_0\varepsilon E_x}$$

平行于界面分量关系



$$-k_{z1}H_{y1} = i\omega\varepsilon_0\varepsilon_1 E_{x1} \text{(in Medium 1)}$$
$$k_{z2}H_{y2} = i\omega\varepsilon_0\varepsilon_2 E_{x2} \text{ (in Medium 2)}$$

$$\frac{k_{z1}}{k_{z2}} \cdot \frac{H_{y1}}{H_{y2}} = -\frac{\varepsilon_1}{\varepsilon_2} \cdot \frac{E_{x1}}{E_{x2}}$$

边界条件: H_y 和 E_x 连续

$H_{y1} = H_{y2}$, $E_{x1} = E_{x2}$

$$\frac{k_{z1}}{k_{z2}} = -\frac{\varepsilon_1}{\varepsilon_2}$$

要保证 k_{z1}与k_{z2}均大于零

ε_1和 ε_2 必须异号——金属和电介质

对TM波

波动方程:

$$(k_z^{\,2} - \beta^2 + k_0^{\,2}\varepsilon)H_y = 0$$

$$k_z^2 = \beta^2 - k_0^2\varepsilon$$

$$\boxed{\frac{k_{z1}}{k_{z2}} = -\frac{\varepsilon_1}{\varepsilon_2}}$$

$$k_{1z}^2 = \beta^2 - k_0^2\varepsilon_1$$

$$k_{2z}^2 = \beta^2 - k_0^2\varepsilon_2$$

传播常数 β 为实数

$$\varepsilon_m \varepsilon_d < 0$$

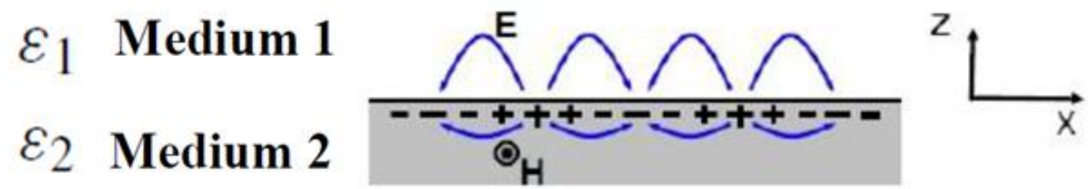
$$\Rightarrow \ \varepsilon_m + \varepsilon_d < 0$$

$$\Rightarrow \varepsilon_d < |\varepsilon_m|$$

$$\boxed{\beta = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m\varepsilon_d}{\varepsilon_m+\varepsilon_d}}}$$

SPP色散关系—SPP mode

对于TE波



$$\pm k_z E_y = -i\omega\mu_0 H_x$$
$$i\beta E_y = i\omega\mu_0 H_z$$
$$\pm k_z H_x - i\beta H_z = -i\omega\varepsilon_0\varepsilon E_y$$

$-k_{z1}E_{y1} = -i\omega\mu_0 H_{x1}$ (in Medium 1)

$k_{z2}E_{y2} = -i\omega\mu_0 H_{x2}$ (in Medium 2)

边界条件： $H_{x1} = H_{x2}$, $E_{y1} = E_{y2}$

$$E_y\left(k_{z1} + k_{z2}\right) = 0$$

由于假设TE同样是表面波形式存在，说明 k_{z1} k_{z2} 都为正值，只有， $E_{y1} = E_{y2} = 0$ 才满足上面条件

说明SPP不能是TE偏振

SPP只能是TM偏振!!

要激发SPP需要采用TM偏振的光，但还不够！为什么？

$$\beta = k_0\sqrt{\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}} = k_0\sqrt{\frac{\varepsilon_d}{1 - \frac{\varepsilon_d}{|\varepsilon_m|}}} = n_d k_0\sqrt{\frac{1}{1 - \frac{\varepsilon_d}{|\varepsilon_m|}}} > n_d k_0$$

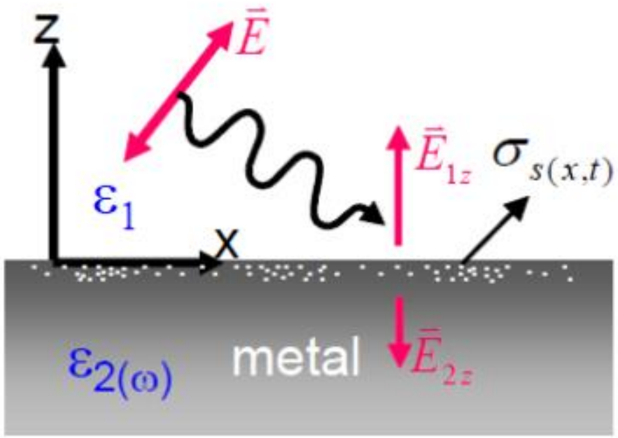
波矢不匹配！

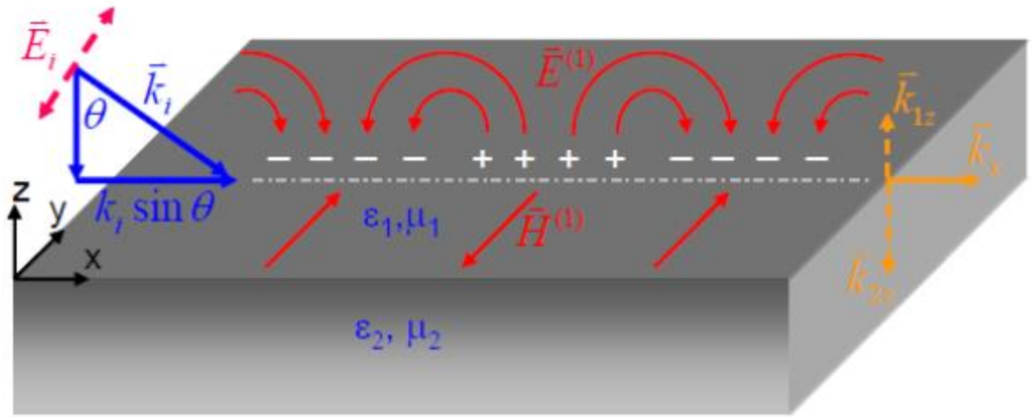
导致SPP只能是TM模式的根本原因是什么？

SPP的产生机制

SPP：横波与纵波

SPP波只能是TM模式？





- E_z在界面上的不连续性——积累表面电子
- E_x分量—— "推"电子进行振荡
- TE激发——连续E——无表面电荷——没有SPP

SPP是纵波还是横波?

横、纵电场比值和能量传输

$$\begin{cases} \mathrm{i}\beta H_y = -\mathrm{i}\omega\varepsilon_0\varepsilon E_z \\ \pm k_z H_y = \mathrm{i}\omega\varepsilon_0\varepsilon E_x \end{cases} \Rightarrow \begin{cases} E_z = -\dfrac{\beta}{\omega\varepsilon_0\varepsilon} H_y \\ E_x = \dfrac{\pm k_z}{\mathrm{i}\omega\varepsilon_0\varepsilon} H_y \end{cases}$$

因此： $\left|\dfrac{E_z}{E_x}\right| = \dfrac{\beta}{k_z} \xrightarrow{k_z^2 - \beta^2 + k_0^2\varepsilon = 0} \left|\dfrac{E_z}{E_x}\right| = \sqrt{\dfrac{\beta^2}{\beta^2 - k_0^2\varepsilon}}$

由SPP色散关系： $\beta = k_0\sqrt{\dfrac{\varepsilon_m\varepsilon_d}{\varepsilon_m + \varepsilon_d}}$

$$\Rightarrow \left|\frac{E_z}{E_x}\right| = \sqrt{\frac{\varepsilon_m\varepsilon_d}{\varepsilon_m\varepsilon_d - (\varepsilon_m + \varepsilon_d)\varepsilon}} = \begin{cases} \sqrt{-\dfrac{\varepsilon_d}{\varepsilon_m}} & \text{for metal} \\ \sqrt{-\dfrac{\varepsilon_m}{\varepsilon_d}} & \text{for dielectric} \end{cases}$$

一般情况下： $|\varepsilon_m| > |\varepsilon_d|$

- 介质一侧，横向电场大于纵向电场
- 金属一侧，横向电场小于纵向电场

x方向的平均能流密度： $S_x^d = \dfrac{\beta}{2\varepsilon_0\varepsilon_d\omega}|H_y|^2$， $S_x^m = -\dfrac{\beta}{2\varepsilon_0|\varepsilon_m|\omega}|H_y|^2$ $\quad S_x^d > S_x^m$

$\boldsymbol{S} = \dfrac{1}{2}\mathrm{Re}(\boldsymbol{E}^* \times \boldsymbol{H})$

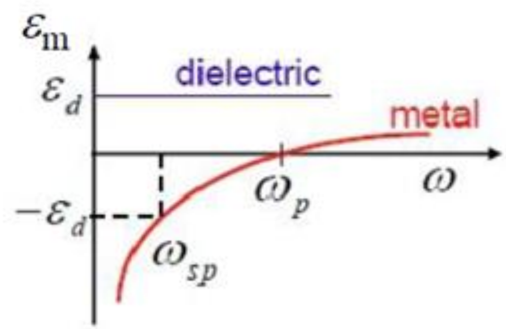
SPP色散关系的规律

SPP色散曲线

$$\beta = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m\varepsilon_d}{\varepsilon_m+\varepsilon_d}}$$

1. 非色散介质: ε_d = 常数

2. 无衰减的Drude金属: $\varepsilon_m(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

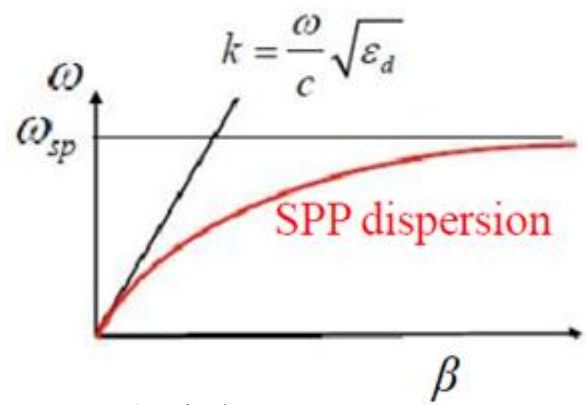


- 在低频ω: $\varepsilon_m \to -\infty$

$$\beta = \frac{\omega}{c}\lim_{\varepsilon_m\to-\infty}\sqrt{\frac{\varepsilon_m\varepsilon_d}{\varepsilon_m+\varepsilon_d}} \approx \frac{\omega}{c}\sqrt{\varepsilon_d}$$

(趋向电介质的light line)

- 当$\varepsilon_m = -\varepsilon_d$的频率$\omega$ 处: $\beta \to \infty$ (短波长限制)



此频率被称为**表面等离子体特征频率 ω_{sp}**:

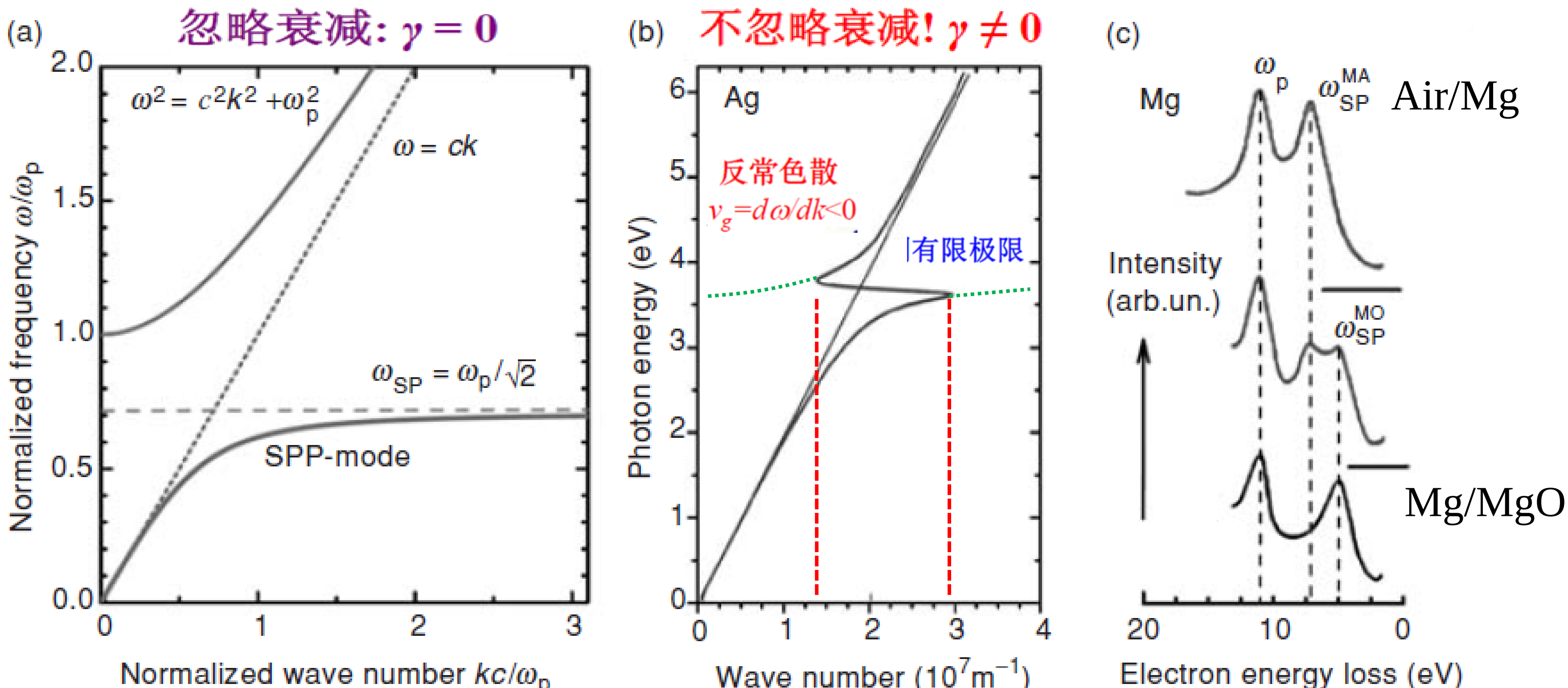
$$1 - \frac{\omega_p^2}{\omega_{sp}^2} = \varepsilon_d \quad \Rightarrow \quad \boxed{\omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}}$$

介质为空气:

$$\varepsilon_d = 1,\ \omega_{sp} = \frac{\omega_p}{\sqrt{2}}$$

SPP色散曲线

$$\boxed{\beta = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m\varepsilon_d}{\varepsilon_m+\varepsilon_d}}}$$



空气/理想金属(free electron plasma)　　空气/银　　真空/镁

- 在ω_{sp}，β达到有限极限
- 允许ω_{sp}和ω_p之间的准结合模

对空气：

$$\beta = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m(\omega)}{1+\varepsilon_m(\omega)}},\ \varepsilon_m(\omega) = 1-\frac{\omega_p^2}{\omega^2} \Rightarrow \beta = \frac{\omega}{c}\sqrt{\frac{1-\omega_p^2/\omega^2}{2-\omega_p^2/\omega^2}}$$

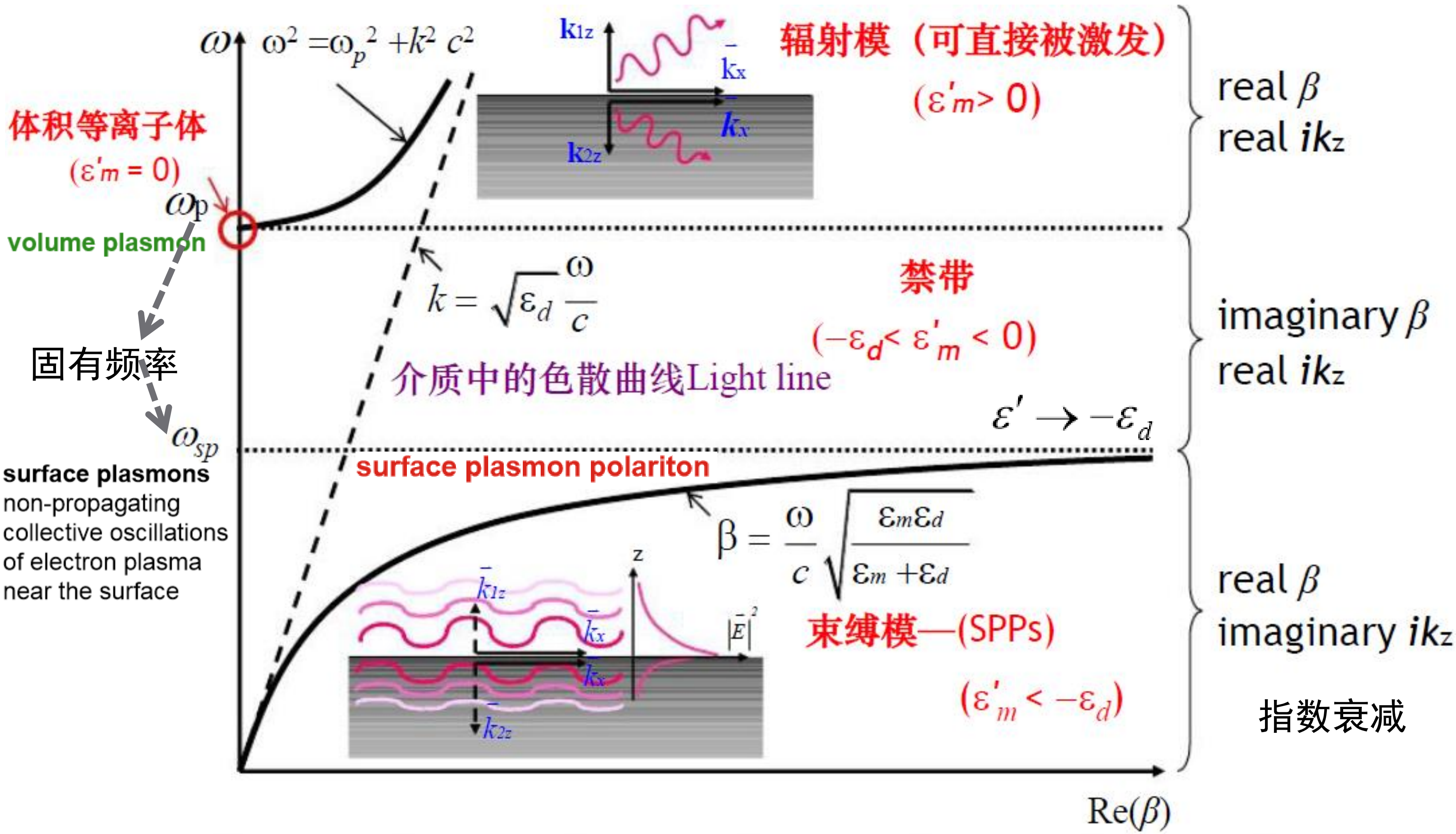
$$\omega_{sp} = \frac{\omega_p}{\sqrt{2}}$$

对介质：

$$\boxed{\omega_{SP} = \frac{\omega_p}{\sqrt{1+\varepsilon_1}}}$$

$v_g = d\omega/dk$ tends to zero for $\omega \rightarrow \omega_{SP}$

全光谱等离子体色散曲线

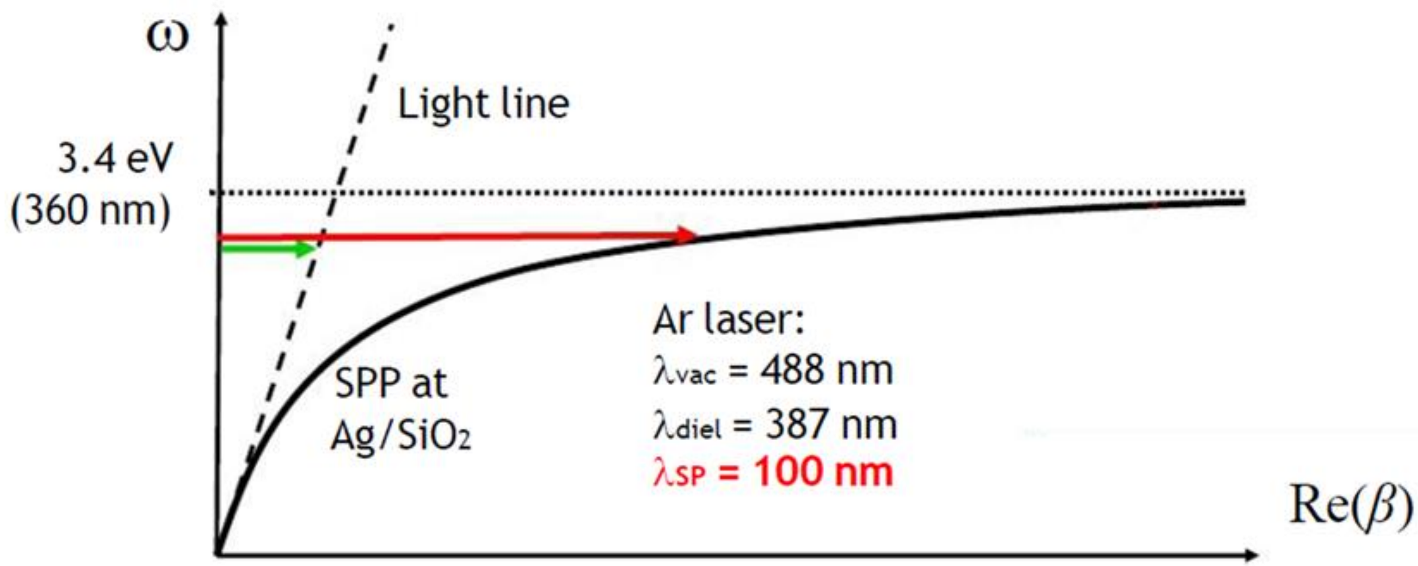


SPP的波长

SPP的波长

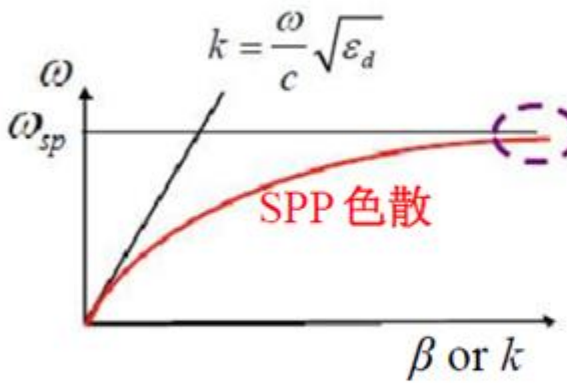
$k = \dfrac{2\pi}{\lambda}$ SPP的传播常数看做波数，则有：

$$\beta = k_{spp} = \frac{2\pi}{\lambda_{spp}} \Rightarrow \lambda_{spp} = \frac{2\pi}{k_{spp}} = \frac{2\pi}{\beta}$$



- SPP波长可以在光频达到纳米级！得到亚波长约束
- 光不能直接在平板金属表面激发SPP。⟵ 怎样激发的? - 下一讲介绍

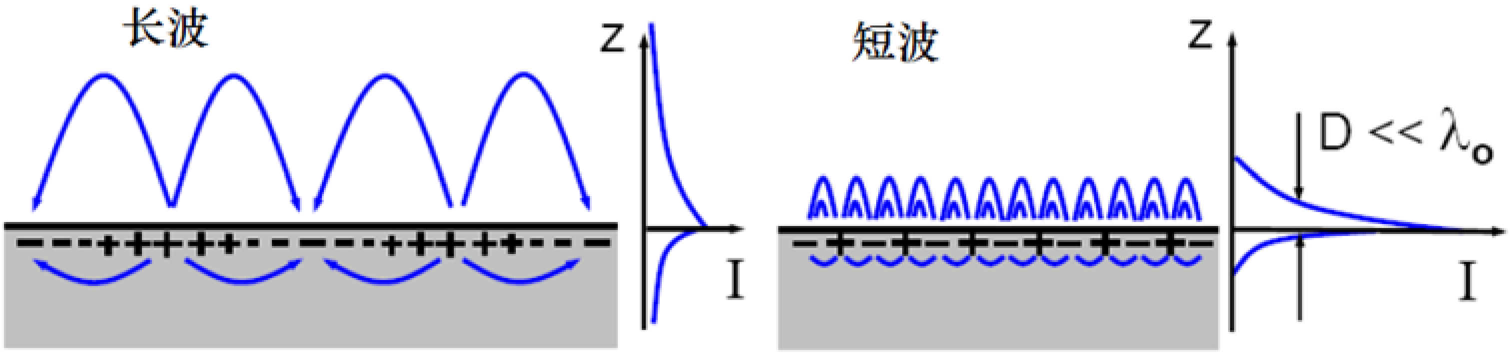
短波长极限



$\beta \to \infty \quad (\lambda \to 0)$

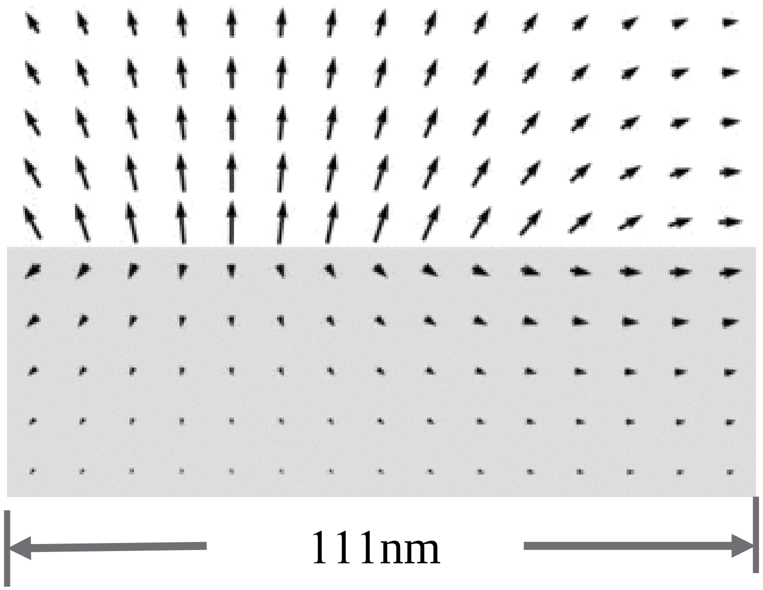
$\therefore k_z = \sqrt{\beta^2 - k_0^2 \varepsilon} \to \infty$

z 方向场迅速衰减 $e^{\pm k_z z}, \ \Delta = \frac{1}{k_z}$



- 场被约束在金属表面的一个很小的区域，造成局域场增强
- $v_g \sim 0 \to$ 不传播,准静态表面模：表面等离激元

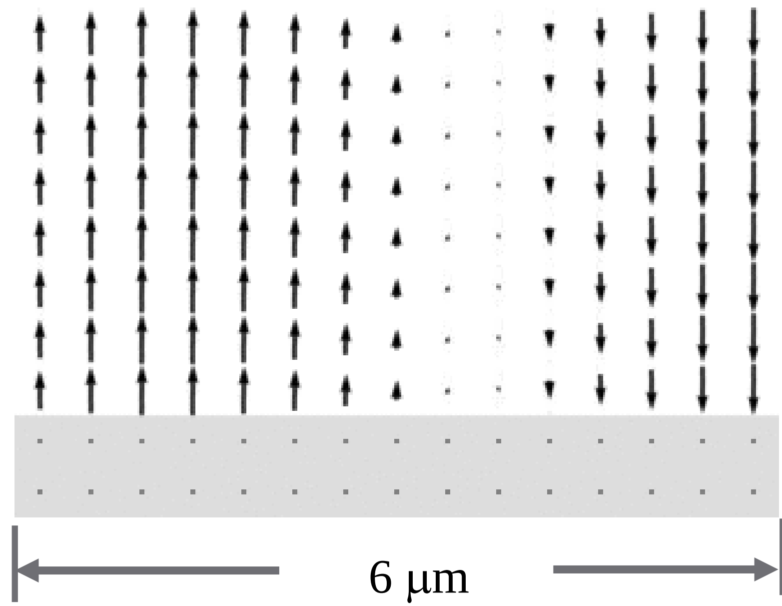
Ag-Air表面等离子体激元



E 电场

真空波长：370 nm

在此频率，Ag介电常数：−2.6+0.6i



E 电场

真空波长：10 μm

在此频率，Ag介电常数：－2700+1400i

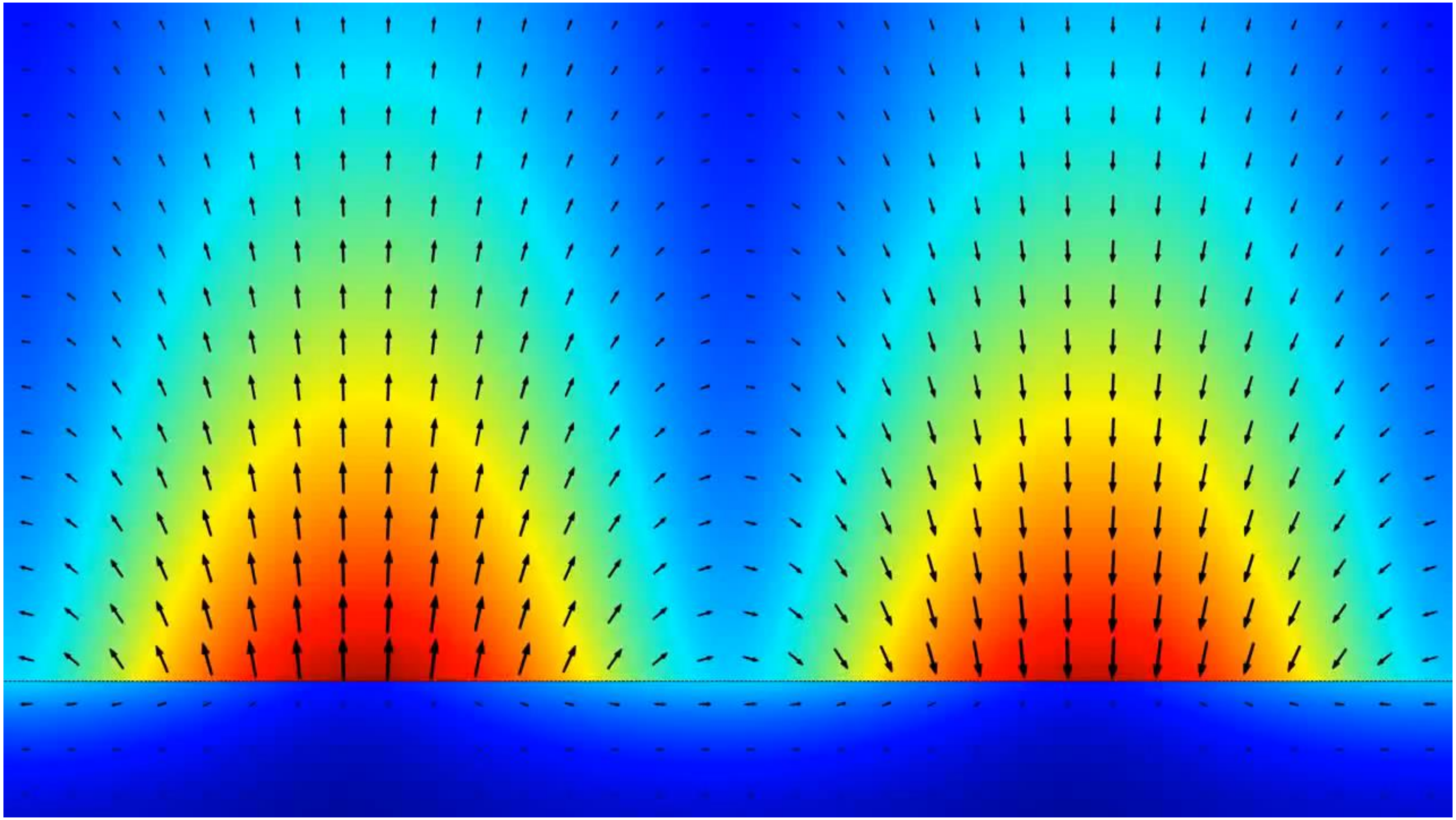
$$\lambda_{spp}\approx 10\ \mu\text{m},\ \Delta_{air}\approx 83\ \mu m,\ \Delta_m\approx 31\ nm$$

$$\beta=\frac{2\pi}{\lambda_{spp}}=\frac{2\pi}{\lambda_0}\sqrt{\frac{\varepsilon_m}{1+\varepsilon_m}}\Rightarrow\lambda_{spp}=\sqrt{\frac{1+\varepsilon_m}{\varepsilon_m}}\lambda_0$$

$$k_z=\sqrt{\beta^2-\varepsilon_i k_0^2},\ \beta=k_0\sqrt{\frac{\varepsilon_m}{1+\varepsilon_m}}\Rightarrow k_z=k_0\sqrt{\frac{\varepsilon_m}{1+\varepsilon_m}-\varepsilon_i}$$

$$\Delta=\frac{1}{k_z}=\frac{\lambda_0}{2\pi}\sqrt{\frac{1+\varepsilon_m}{\varepsilon_m-\varepsilon_i-\varepsilon_i\varepsilon_m}},\ \Delta_{air}=\frac{\lambda_0}{2\pi}\sqrt{-(1+\varepsilon_m)},\quad \Delta_m=\frac{\lambda_0}{2\pi}\sqrt{-\frac{\varepsilon_m+1}{\varepsilon_m^2}}$$

$$\lambda_{spp}\approx 290\ \text{nm},\ \Delta_{air}\approx 75\ nm,\ \Delta_m\approx 29\ nm$$



SPP的传播距离与损耗

SPP的传播距离

metal/air interface $k_x = k_x' + ik_x'' = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m}{\varepsilon_m + 1}}$

$$\mathbf{E}(x) = \mathbf{E}_0 e^{ik_x x} = \mathbf{E}_0 e^{ik_x' x} e^{-k_x'' x}$$

propagating term   exponential decay in x-direction

$$I \propto E^2 = E_0^2 e^{-2k_x'' x}$$

$$\boxed{L_x = \frac{1}{2k_x''}}$$

propagation length

intensity !

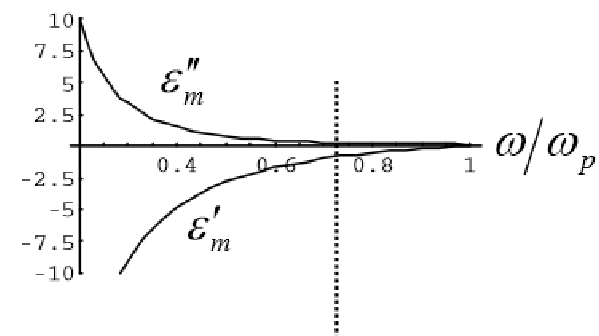
Example silver: $\lambda = 514.5\ \mathrm{nm} : L_x = 22\ \mu\mathrm{m}$

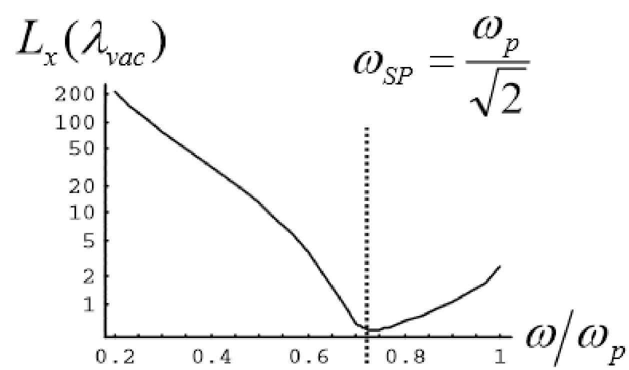
$\lambda = 1060\ \mathrm{nm} : L_x = 500\ \mu\mathrm{m}$

$\varepsilon_d = 1$

$\varepsilon_m = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$ **Drude model**

$\gamma = 0.2$





SPP的传播距离和损耗

三个特征尺度（重要！）：

δ_m: 金属中的衰减长度 → 场约束

δ_d:电介质中的衰减长度 → 场约束

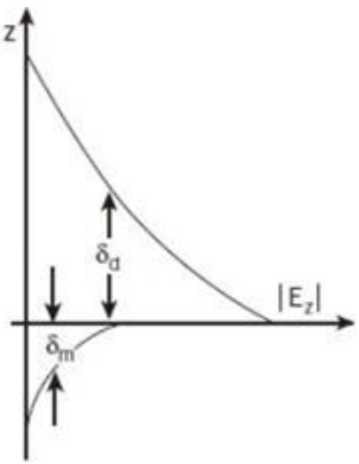
δ_{sp}: SPP的传播长度 → 损耗

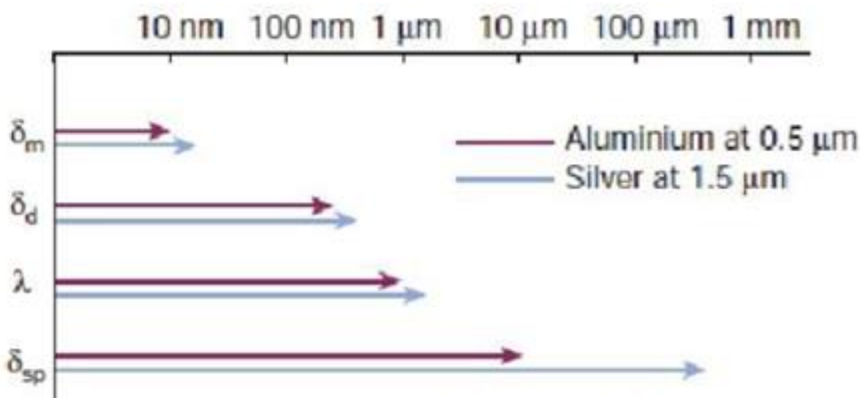
$$\delta_m = \frac{1}{2k_{zm}}$$

$$\delta_d = \frac{1}{2k_{zd}}$$

$$\delta_{sp} = \frac{1}{2\beta''}$$

衰减长度（传播长度）定义为**振荡强度**减小为1/e的长度

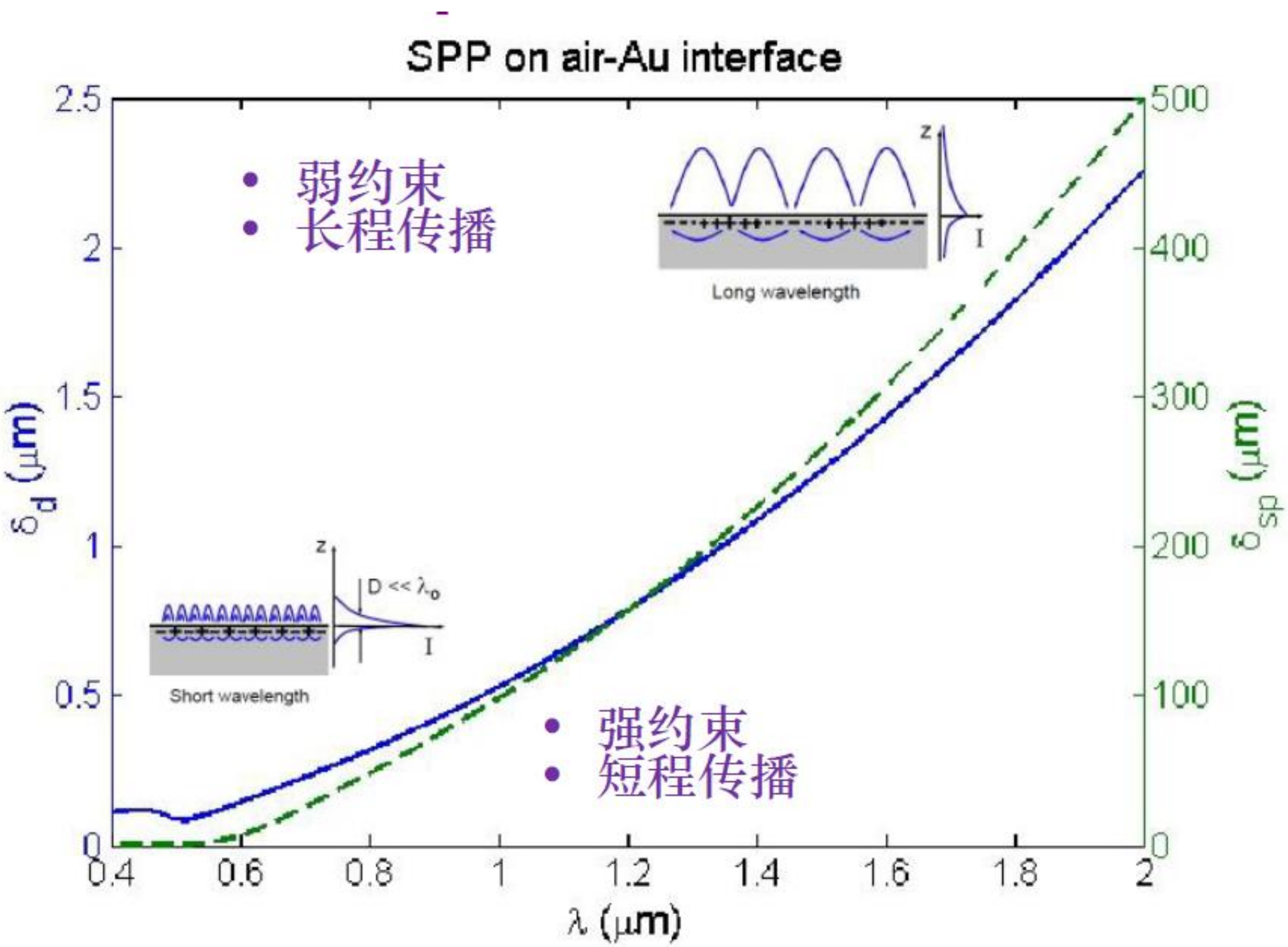




W. L. Barnes, et al., Nature **424**, 824 (2003).

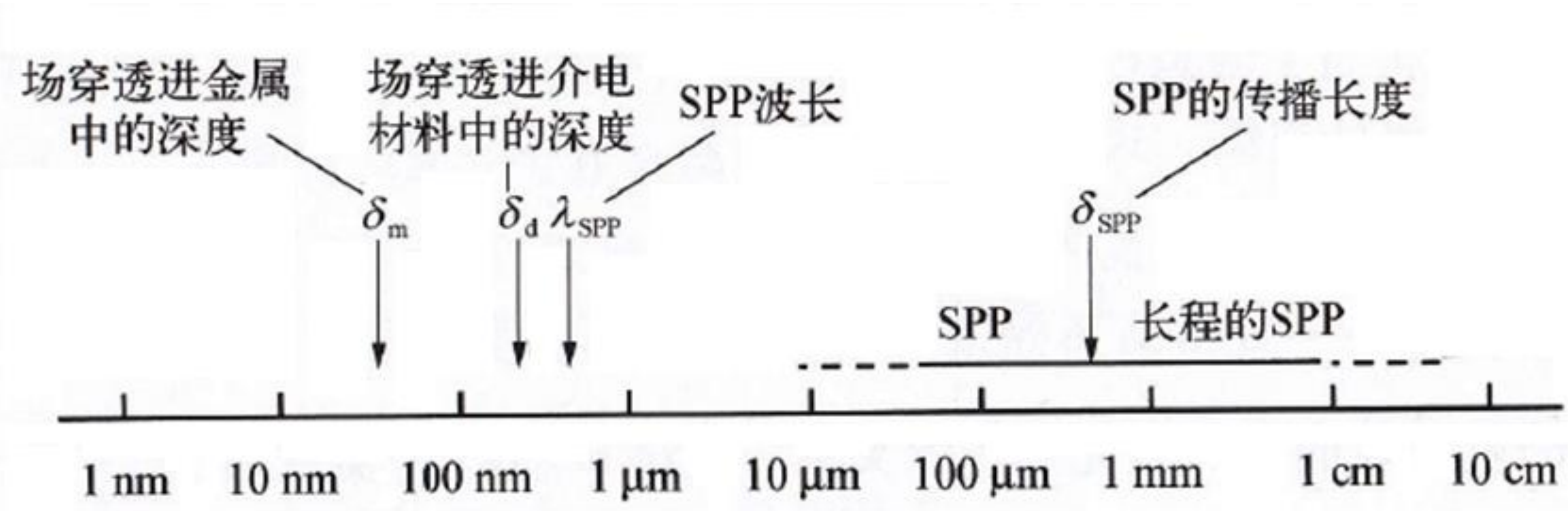
- 我们希望传播距离更长和场约束更强。
- 因此，$\boldsymbol{\delta_{sp} / \delta_d}$是等离子体设备的关键测量值，希望其尽可能大！

金表面的 δ_d 和 δ_{sp}



表面等离子体中典型的约束和损耗之间的折中

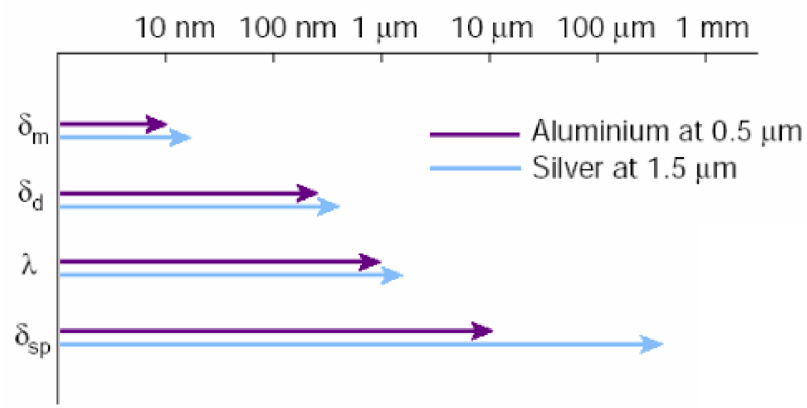
SPP的特征尺度



SPP 的特征尺度

金属的Z方向吸收导致表面等离子体波耗散——有限传播距离

δ_{sp}、δ_d、δ_m 的指导意义



Nature **424,** 825 (2003)

- 光子回路最大的尺度 δ_{sp}
- 波长一半、元件特征高度 δ_d
- 金属结构的最小的特征高度 δ_m

- 可见光波段传播距离为μm级
- 保证约束下增加传播距离
- 传播距离能够达到cm量级

There are three characteristic length scales that are important for SP-based photonics in addition to that of the associated light. The propagation length of the SP mode, δ_{SP}, is usually dictated by loss in the metal. For a relatively absorbing metal such as aluminium the propagation length 2 μm at a wavelength of 500 nm. For a low loss metal, for example, silver, at the same wavelength it is increased to 20 μm. By moving to a slightly longer wavelength, such as 1.55 μm, the propagation length is further increases towards 1 mm. The propagation length sets the upper size limit for any photonic circuit based on SPs. The decay length in the dielectric material, δ_d, is typically of the order of half the wavelength of light involved and dictates the maximum height of any individual features, and thus components, that might be used to control SPs. The ratio of δ_{SP}:δ_d thus gives one measure of the number of SP-based components that may be integrated together. The decay length in the metal, δ_m, determines the minimum feature size that can be used; as shown in the diagram, this is between one and two orders of magnitude smaller than the wavelength involved, thus highlighting the need for good control of fabrication at the nanometre scale. The combinations chosen give an indication of range from poor (Al at 0.5 μm) to good (Ag at 1.5 μm) SP performance.

小结

表面等离子体激元(SPPs)：

- 被约束的表面波；
- 横向和纵向振荡；
- TM激发；
- 色散关系；
- 全光谱等离子色散；
- 亚波长约束；
- SPP波长；三个特性长度；
- 约束和损耗间平衡