GP108 :: MINI PROJECT
CALIBRATION OF A THERMISTOR

Practice Question:

Table 1 gives the results of an experiment (similar to the experiment that you are supposed to conduct as phase I of the Mini Project) conducted to calibrate a thermistor and the corresponding variation is plotted in Figure 1.

| Temperature (K) | 323 | 333 | 343 | 353 | 363 | 373 |
|---|---|---|---|---|---|---|
| R (Ω) | 62 | 42 | 20 | 18 | 12 | 5 |

Table 1

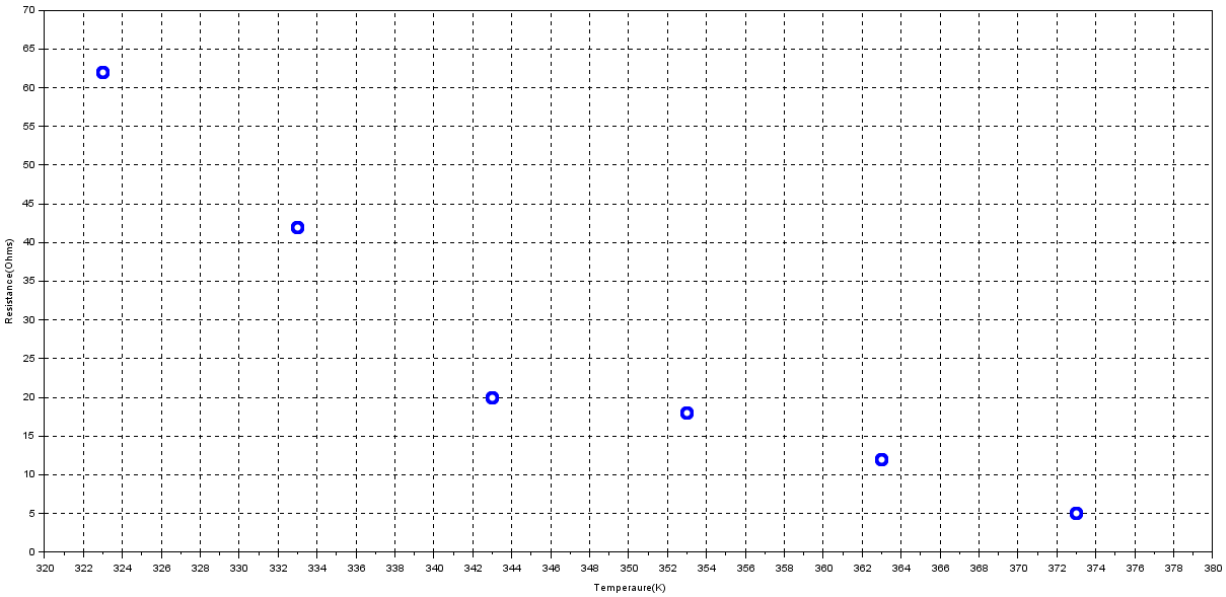


Figure 1: variation of measured Resistance (R) of the thermometer with temperature (T)

The resistance (R) Vs temperature (T) characteristic of a thermistor can be approximated by the equation $R = R_0 e^{\lambda}$, where $\lambda = \beta(\frac{1}{T} - \frac{1}{T_0})$ and $R_0$ is the resistance at temperature $T_0 = 300\,K$. $\beta$ is the material constant of the thermistor. Estimate $R_0$ and $\beta$ using Least Mean Square (LMS) error.

Solution:

>> Mathematical formulation:

$$R = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)} \tag{1}$$

Therefore,

$$\ln(R) = \ln(R_0) + \beta\left(\frac{1}{T} - \frac{1}{T_0}\right) \tag{2}$$

$$\ln(R) = \ln(R_0) + \beta\left(\frac{1}{T} - \frac{1}{T_0}\right) \tag{3}$$

$$\ln(R) = \frac{\beta}{T} + \ln(R_0) - \frac{\beta}{T_0} \tag{4}$$

Now, the measurements given in Table 1 can be expressed in the matrix form as (5)

$$\begin{bmatrix} \ln(R_1) \\ \vdots \\ \ln(R_6) \end{bmatrix} = \begin{bmatrix} 1/T_1 & 1 \\ \vdots & \vdots \\ 1/T_6 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ \ln(R_0) - \frac{\beta}{T_0} \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_6 \end{bmatrix} \tag{5}$$

Where e is error. This can be written as,

$$Y = XC + E \tag{6}$$

It can be proved that, for Least Mean Square (LMS) error,

$$C = (X^T X)^{-1} X^T Y \tag{7}$$

By substituting from Table 1,

$$Y = \begin{bmatrix} 4.13 \\ 3.74 \\ 3.00 \\ 2.89 \\ 2.49 \\ 1.61 \end{bmatrix}, X = \begin{bmatrix} 3.10 \times 10^{-3} & 1 \\ 3.00 \times 10^{-3} & 1 \\ 2.92 \times 10^{-3} & 1 \\ 2.83 \times 10^{-3} & 1 \\ 2.76 \times 10^{-3} & 1 \\ 2.68 \times 10^{-3} & 1 \end{bmatrix} \rightarrow C = \begin{bmatrix} \beta \\ \ln(R_0) - \frac{\beta}{T_0} \end{bmatrix} = \begin{bmatrix} 5645.68 \\ -13.28 \end{bmatrix}$$

Therefore,

$$\beta = 5645.68\ K,\ \ R_0 = 252.32\ \Omega \tag{8}$$

Therefore, the equation for the above thermistor can be given by equation (9).

$$R = 252.32e^{5645.68\left(\frac{1}{T} - \frac{1}{300}\right)} \tag{9}$$

The corresponding curve to equation (9) is shown in Figure (2).

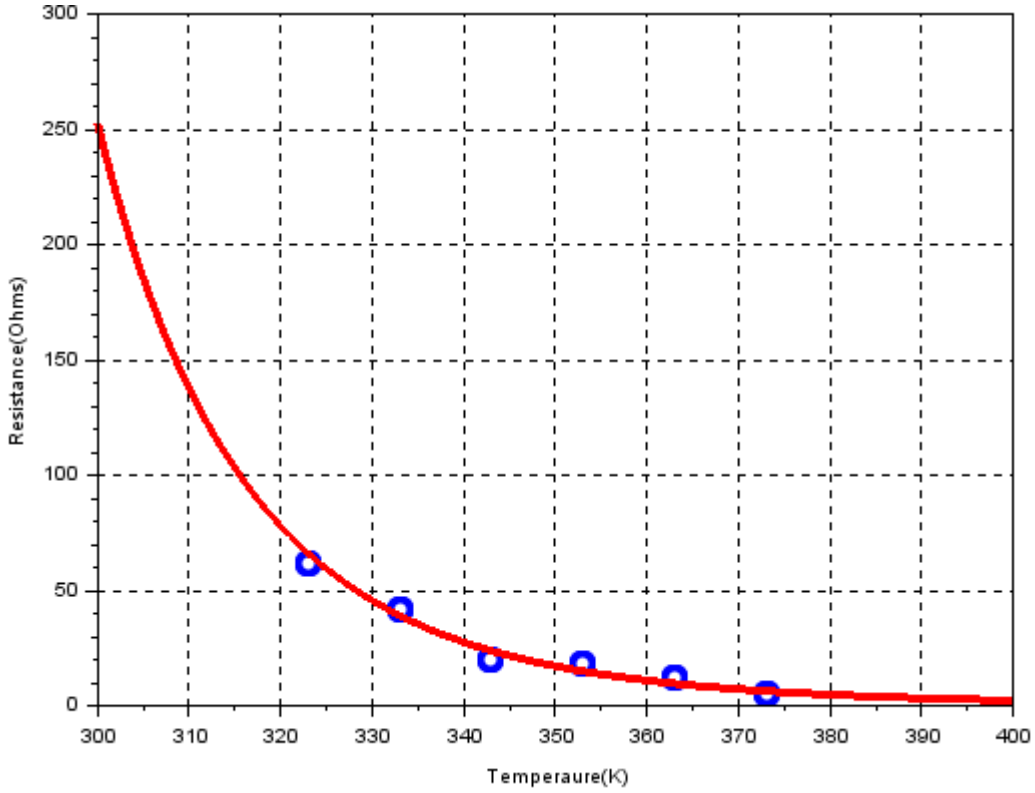


Figure 2: The curve fitted to the measurement using LMS