Department of Electronic and Telecommunication Engineering
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EN4553 - Machine Vision

Assignment 3

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This report is submitted in partial fulfilment of the requirements
for the module EN4553 - Machine Vision

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180066F_assignment3

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1 Answers

Import libraries

[1]:
```
import pandas
import numpy as np
from sklearn.linear_model import LinearRegression
import matplotlib.pyplot as plt
from typing import Tuple
```

1.1 Answer to (1) (a)

[2]:
```
def load_dataset(
src_dir: str
) -> Tuple[np.ndarray, np.ndarray, np.ndarray, np.ndarray, np.ndarray]:
    """Load the dataset as a set of numpy arrays.
    Args:
    src_dir: Directory where dataset files are stored.
    Returns:
    (x_train, y_train, x_val, y_val, x_test) tuple where each array is one␣
 ↪dimensional.
    """
    if(src_dir!=''):
        src_dir = src_dir + '/'

    x_train = np.loadtxt(src_dir+'x_train.txt')
    y_train = np.loadtxt(src_dir+'y_train.txt')
    x_val = np.loadtxt(src_dir+'x_val.txt')
    y_val = np.loadtxt(src_dir+'y_val.txt')
    x_test = np.loadtxt(src_dir+'x_test.txt')

    return x_train, y_train, x_val, y_val, x_test
```

1.2 Answer to (1) (b) (i)

[3]:
```
def get_features(x: np.ndarray, n: int) -> np.ndarray:
    """Creates n-th degree polynomial features for the given vector x.
    Example usage:
    get_features(np.array([1.0, 2.0, 3.0]), 3) outputs
    np.array([[ 1., 1., 1.],
    [ 2., 4., 8.],
    [ 3., 9., 27.]])
    Args:
    x: A numpy array of shape (num_examples, ) or (num_examples, 1).
    n: The degree of the polynomial features.
    Returns:
    A matrix of shape (num_examples, n) where the j-th column is equal to
    the vector x raised, elementwise, to the power j.
    """
    poly_feature_matrix = np.zeros([n,x.size])

    for i in range(0,n):
        poly_feature_matrix[i,:] = np.power(x,i+1)

    poly_feature_matrix = np.transpose(poly_feature_matrix)

    return poly_feature_matrix
```

1.3 Answer to (1) (b) (ii)

Method 1: MSE calculation from the equation

[4]:
```
def fit_and_evaluate(
    x_train: np.ndarray, y_train: np.ndarray,
    x_val: np.ndarray, y_val: np.ndarray,
    n: int
    ) -> Tuple[float, float]:
    """Fits an n-th degree polynomial and outputs train and validation MSE.
    Fits a linear regression model y = sum_{i=1}^n w_i x^i to the given train
    set and outputs the mean-squared-error (MSE) on train and validation sets.
    Args:
    x_train: Input features for the train set. Has shape (num_train, )
    y_train: Targets (labels) for the train set. Has shape (num_train, )
    x_val: Input features for the validation set. Has shape (num_val, )
    y_val: Targets (labels) for the validation set. Has shape (num_val, )
    n: The degree of the polynomial fit. See the above equation.
    Returns:
    (train_mse, val_mse), tuple of MSE on train and validation sets.
    """
    # Fit the model on the train set.
    x_train_features = get_features(x_train,n)
```

```python
    model = LinearRegression(fit_intercept=False).fit(x_train_features,y_train)

    # Generate model predictions for the train set and calculate the MSE.
    y_predict_train = model.predict(x_train_features)
    train_mse = np.power((y_train - y_predict_train),2).sum()/x_train.size

    # Similarly, calculate the MSE on the val set.
    x_val_features = get_features(x_val,n)
    y_predict_val = model.predict(x_val_features)
    val_mse = np.power((y_val - y_predict_val),2).sum()/x_val.size

    return train_mse, val_mse
```

Method 2: MSE calculation from sklearn.metrics.mean_squared_error

[5]:
```python
from sklearn.metrics import mean_squared_error as mse

def fit_and_evaluate_(
    x_train: np.ndarray, y_train: np.ndarray,
    x_val: np.ndarray, y_val: np.ndarray,
    n: int
    ) -> Tuple[float, float]:
    """Fits an n-th degree polynomial and outputs train and validation MSE.
    Fits a linear regression model y = sum_{i=1}^n w_i x^i to the given train
    set and outputs the mean-squared-error (MSE) on train and validation sets.
    Args:
    x_train: Input features for the train set. Has shape (num_train, )
    y_train: Targets (labels) for the train set. Has shape (num_train, )
    x_val: Input features for the validation set. Has shape (num_val, )
    y_val: Targets (labels) for the validation set. Has shape (num_val, )
    n: The degree of the polynomial fit. See the above equation.
    Returns:
    (train_mse, val_mse), tuple of MSE on train and validation sets.
    """
    # Fit the model on the train set.
    x_train_features = get_features(x_train,n)
    model = LinearRegression(fit_intercept=False).fit(x_train_features,y_train)

    # Generate model predictions for the train set and calculate the MSE.
    y_predict_train = model.predict(x_train_features)
    train_mse = mse(y_train, y_predict_train)

    # Similarly, calculate the MSE on the val set.
    x_val_features = get_features(x_val,n)
    y_predict_val = model.predict(x_val_features)
    val_mse = mse(y_val, y_predict_val)
```

```
    return train_mse, val_mse
```

1.4 Subsequent Python-based implementation

[7]:
```
[x_train, y_train, x_val, y_val, x_test] = load_dataset('')
```

[8]:
```
upper_bound = 10
mse_array = np.zeros([upper_bound,2])

for i in range(0,upper_bound):
    mse_array[i,:] = fit_and_evaluate(x_train, y_train, x_val, y_val,i+1)
```

1.5 Answer to (1) (c)

[9]:
```
n_values = range(1,upper_bound+1)

mse_df = pandas.DataFrame(mse_array) #for saving the MSEs in a csv
mse_df['n'] = n_values
mse_df.columns = ['Training set MSEs','Validation set MSEs', 'n']
mse_df = mse_df[['n', 'Training set MSEs','Validation set MSEs']]

print(mse_df.to_string(index=False))
mse_df.to_csv('MSEs against n - Training and Validation.csv', index=False)
```

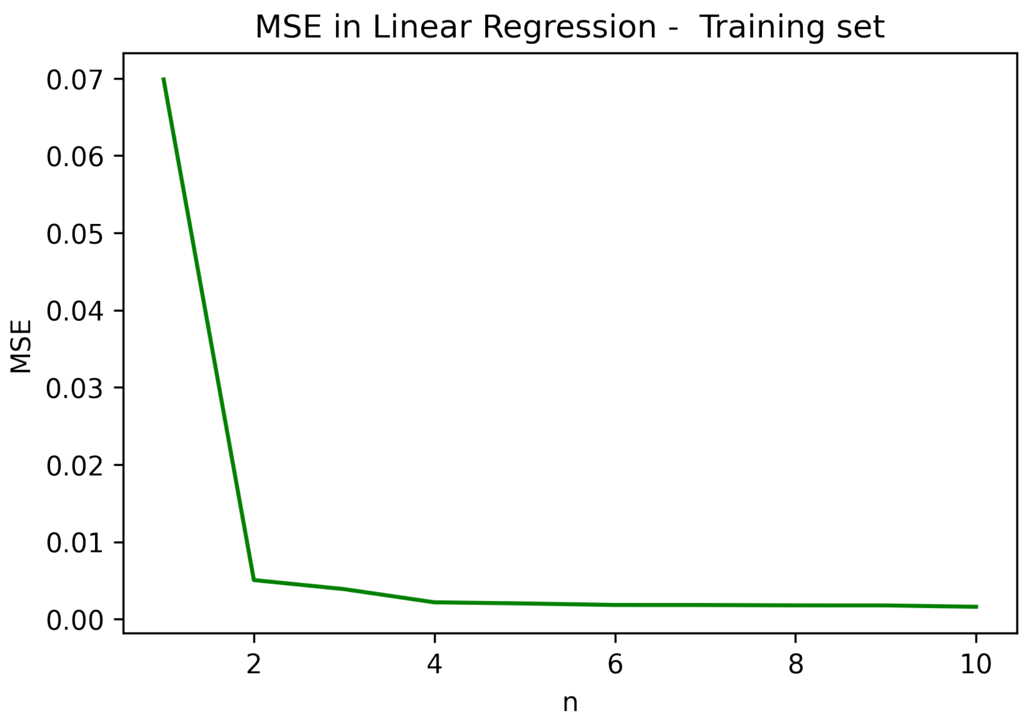
```
 n  Training set MSEs  Validation set MSEs
 1           0.069869             0.072270
 2           0.005047             0.003894
 3           0.003874             0.002462
 4           0.002166             0.004120
 5           0.002020             0.004304
 6           0.001821             0.004682
 7           0.001814             0.004652
 8           0.001773             0.004996
 9           0.001766             0.004608
10           0.001589             0.004163
```

n	$Training\,set\,MSEs$	$Validation\,set\,MSEs$
1	0.069868788	0.072270038
2	0.005047321	0.003893858
3	0.003873768	0.002461939
4	0.002165988	0.00412025
5	0.002020267	0.004303564
6	0.001821074	0.004682199
7	0.00181447	0.004651991
8	0.001773034	0.004996135
9	0.001766428	0.004607625
10	0.001588783	0.004162643

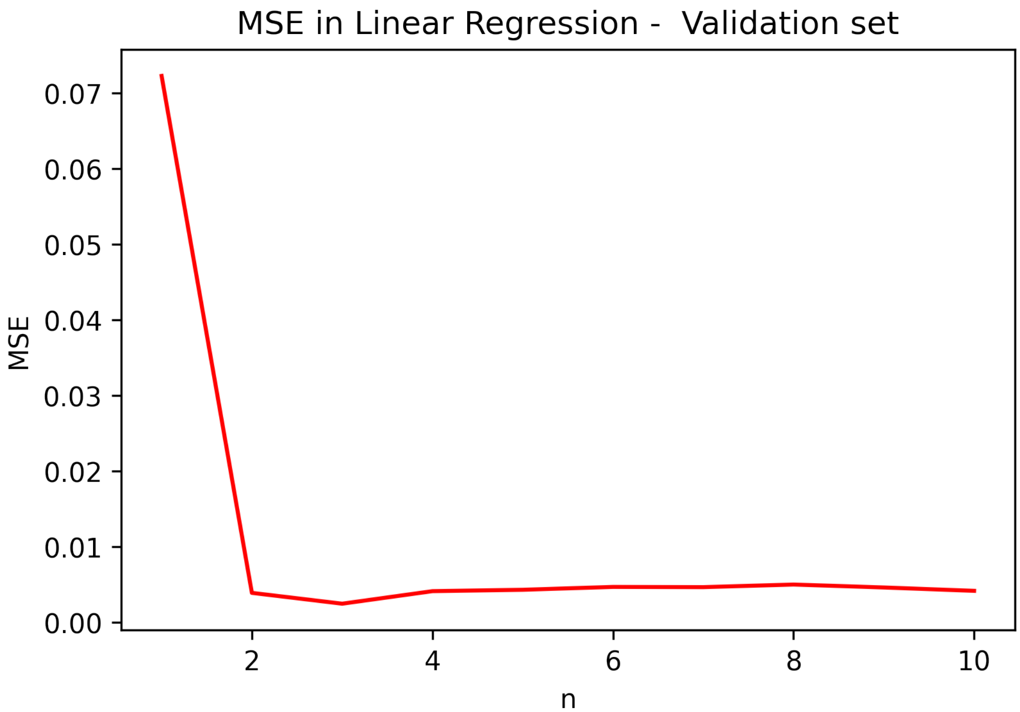
Plotting MSE vs n graphs

[10]:
```
plt.rcParams["figure.dpi"] = 300 #change the deafult figure resolution
```

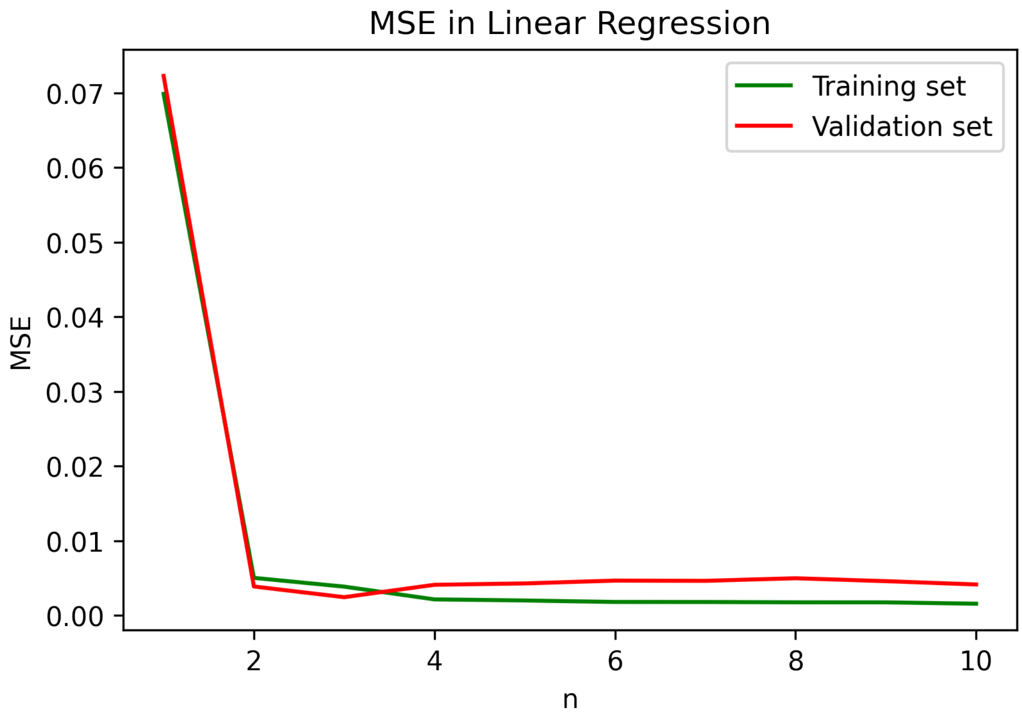
[11]:
```
plt.figure()
plt.plot(n_values, mse_array[:,0], 'g')
plt.title('MSE in Linear Regression -  Training set')
plt.xlabel('n')
plt.ylabel('MSE')
plt.savefig('MSE_in_train_set')
```



[12]:
```
plt.figure()
plt.plot(n_values, mse_array[:,1], 'r')
plt.title('MSE in Linear Regression -  Validation set')
plt.xlabel('n')
plt.ylabel('MSE')
plt.savefig('MSE_in_val_set')
```



[13]:
```
plt.figure()
plt.plot(n_values, mse_array[:,0], 'g')
plt.title('MSE in Linear Regression')
plt.xlabel('n')
plt.ylabel('MSE')
plt.plot(n_values, mse_array[:,1],'r')
plt.legend(['Training set','Validation set'])
plt.savefig('MSE_in_both_train_val')
```



Through observing the calculated MSEs for both training and validation sets (as depicted in the above table: Training set MSEs and Validation set MSEs against n) and the above plots (MSE in Linear Regression - Validation set and MSE in Linear Regression - Training set),

I would select,

n=3

because validation MSE is minimum at n=3 (i.e. MSE (val, n=3) = 0.002461939). When n is above 3, the model tends to overfit to the training set and thereby, the validation MSE is getting higher

1.6 Answer to (1) (d)

[14]:
```
n = 3 #choosen n value by considering the minimum validation MSE
x_train_features = get_features(x_train,n)
model = LinearRegression(fit_intercept=False).fit(x_train_features,y_train)

x_test_features = get_features(x_test,n)
y_test_predict = model.predict(x_test_features)
np.savetxt('180066F_y_predict_test.txt',y_test_predict)
```

Here, the numbers are mentioned upto 15 decimal precision. In the submitted txt file, which comprises the predictions for x_test.txt, has the full decimal precision numbers.

Test Input	Prediction
0.408099946310876	-0.919671771036896
0.670388746049254	-1.83759333117145
0.0661556135454454	-0.128486867808112
0.893691290219071	-2.97271229839415
0.885182152056329	-2.92212846332498
0.559962322348422	-1.40523039759975
0.569050898874253	-1.43806176671578
0.799460547260773	-2.44657121294273
0.204027296185715	-0.41286435966323
0.208430386536827	-0.422520971814956
0.930781087124192	-3.20067119257404
0.760865034028111	-2.25166723619235
0.336048445040816	-0.724998235534161
0.990245680209208	-3.59246534298857
0.333691819365997	-0.718956712554085
0.39277843289821	-0.876579730891778
0.638393537851154	-1.70454283399612
0.408131236780316	-0.919760767420466
0.0953792310544752	-0.186414768223298
0.91185999934503	-3.08284623037008
0.445001121511756	-1.02754528458343
0.679081613895861	-1.87490441167279
0.52412246567585	-1.28020853000785
0.191442752493149	-0.385502792755026
0.388115260121508	-0.863653857358733
0.347535005802599	-0.754725852452716
0.883251289154815	-2.9107377309109
0.381394251161921	-0.84517614122298
0.419448838767459	-0.952220734819361
0.147699416448401	-0.292905176571102
0.368495115882985	-0.810207294780408
0.373091998309543	-0.822595585552324
0.541059292690096	-1.33842029745055
0.584682190223179	-1.49563132673189
0.214194252414648	-0.43522933467822
0.547257942221742	-1.3601117541729
0.462133696474829	-1.07969378478299
0.0653865032185863	-0.126974365306795
0.207156209546218	-0.419721978323085
0.724589557123306	-2.07866987198403
0.692875417627095	-1.93515520406631
0.515998575871676	-1.25282581724715
0.751196923592415	-2.20461440315651
0.317165526794562	-0.677124578180544
0.348379238272975	-0.756929345362774
0.514590590436605	-1.24811498090803
0.253593293838615	-0.524281814222741

0.0628094636778552	-0.121910523280106
0.500936816945525	-1.20296067085017
0.10882237983052	-0.213400893035868
0.0620688791457386	-0.120456428511959
0.266392937246184	-0.554093357909752
0.409167876071488	-0.922711490927989
0.467247668079637	-1.09552410271034
0.325219924194979	-0.697396380494755
0.783841338870037	-2.36631567962249
0.472952650809612	-1.11332999486774
0.895792678814873	-2.98530161283728
0.28199538302482	-0.591064299523372
0.806045057403976	-2.48097664955697
0.651114856927037	-1.75664890102496
0.635423454389892	-1.69252601544629
0.45833418454779	-1.06801176828598
0.796257413302684	-2.42995755262453
0.814456362832738	-2.52542747313862
0.0311586307860537	-0.0601681849353039
0.42335925799338	-0.96356299514531
0.12238995737602	-0.240885560529547
0.90572819998437	-3.04535055612035
0.253945599730137	-0.525096311619286
0.038308356058754	-0.0740499027047419
0.0857533271356165	-0.167229447415438
0.915022386082039	-3.10231501289385
0.471920550255411	-1.1100972031223
0.39968404772135	-0.895882739169757
0.846587551839903	-2.70048983998901
0.471760564203187	-1.10959654333021
0.616885694138676	-1.6187724320152
0.0781043631530421	-0.152060078085247
0.665160281105256	-1.81539421332646
0.657800235366057	-1.78445003331471
0.18001702402403	-0.360954120909253
0.0926746014298354	-0.181013033929305
0.239354141687592	-0.49164034167759
0.737078897670672	-2.13714583937989
0.0368146585291757	-0.0711469652081825
0.829011860644231	-2.60368887008923
0.297333644210386	-0.628118020415994
0.278810052076353	-0.583458542557061
0.901032838663546	-3.01686398410026
0.871822698703951	-2.84397510140459
0.705002693900722	-1.9892035601283
0.582709178495198	-1.48828686984564
0.520297333057035	-1.26727238163658
0.354753766929057	-0.773650739245488

0.461088142715176	-1.07647239324319
0.0235539030179635	-0.0454380818035489
0.702190396457144	-1.97657913371069
0.449636976557011	-1.0415230137195
0.373624880453876	-0.82403690596411