Wiener and Adaptive Filtering

BM4151 – Biosignal Processing

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Contents

1 Wiener Filtering **1**

1.1 Discrete-time domain implementation of the Wiener filter 1

1.2 Frequency domain implementation of the Wiener filter 10

1.3 Effect on non-stationary noise on the Wiener filtering 12

2 Adaptive filtering **14**

2.1 Least mean square algorithm 15

2.2 Recursive least square algorithm 16

1 Wiener Filtering

Wiener filters are a set of optimum filters which gives the best possible filter for a given time-series by considering statistical characteristics of its signal and noise. In contrast to the conventional FIR or IIR filters, in wiener filtering is not an ad hoc filter type in which the filter specifications are derived from trial-and-error method.

Here, optimization is implemented via the path of minimization of the mean square error (MSE) between the desired signal and the filtered signal, while the weights of the Wiener filter are calculated using the Wiener-Hopf equation.

$$w_0 = \Phi_X^{-1}\Theta_{Xy} \tag{1}$$

where w_0 is the optimal weight matrix of the Wiener filter, Φ_X is the autocorrelation of the input sampled signal and Θ_{Xy} is the cross-correlation between the input sampled signal and the desired signal. Since noise and signal processes are assumed to be independent in the Wiener method, the Wiener-Hopf equation could be extended as:

$$w_0 = (\Phi_Y + \Phi_N)^{-1}\Theta_{Xy} \tag{2}$$

where Φ_Y is the autocorrelation of the desired signal and Φ_N is the autocorrelation of the noise. Therefore, equation 2 could be utilized to estimate the filter coefficients of the Wiener optimal filter.

1.1 Discrete-time domain implementation of the Wiener filter

The noisy ECG signal is obtained by adding $10dB$ Gaussian white noise and a low-frequency sinusoidal noise to the given ideal ECG signal.

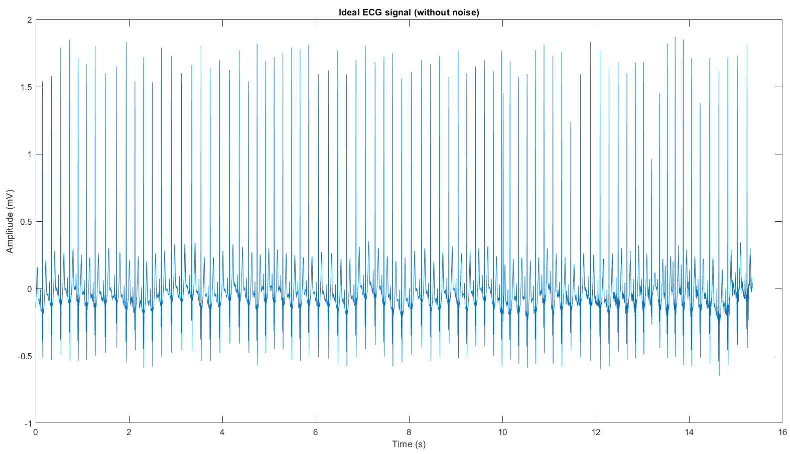


Figure 1: Ideal ECG signal

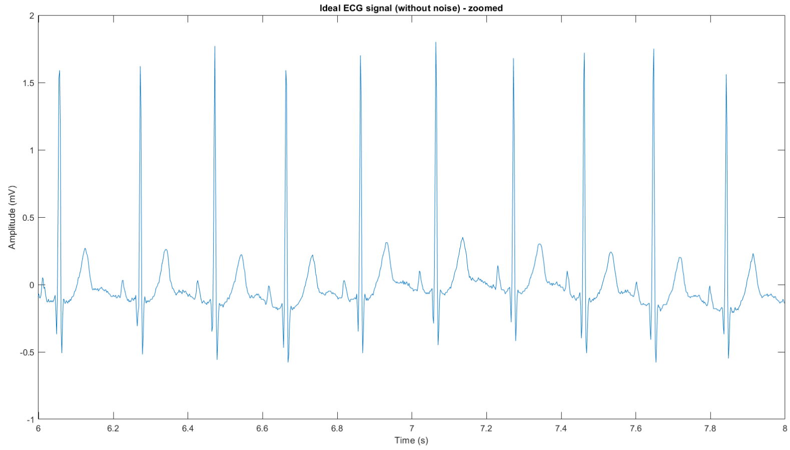


Figure 2: Ideal ECG signal (zoomed)

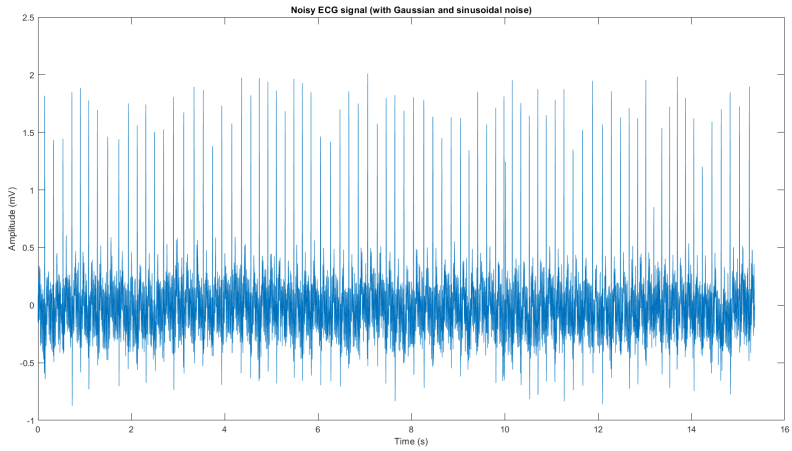


Figure 3: Noisy ECG signal with added Gaussian and sinusoidal noise

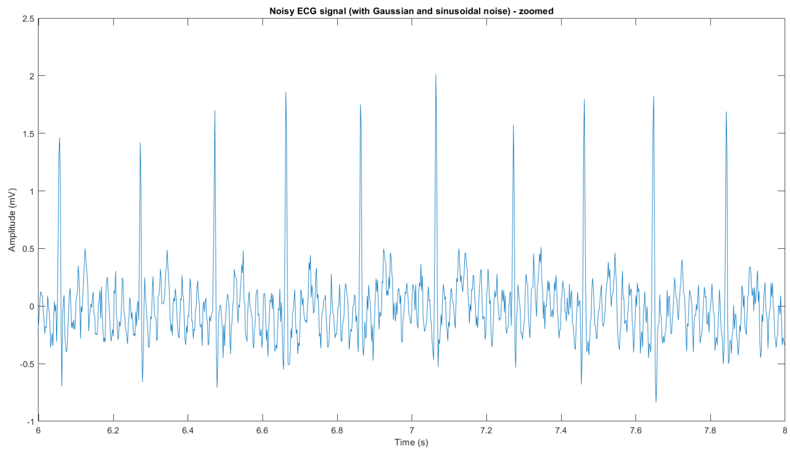


Figure 4: Noisy ECG signal with added Gaussian and sinusoidal noise (zoomed)

Part 1 – Extract single beat from the signal The desired signal for the Wiener filter implementation is randomly selected from the ideal ECG signal as a single ECG pulse. In addition, a noisy signal is obtained from the end of the T wave to the start of the P wave of the following ECG pulse (i.e. thus, an isoelectric segment from the noisy signal).

Since the isoelectric segment is comparatively shorter in length, and it is needed to have same length between the desired ECG pulse and the isoelectric segment (for prospective calculations), the isoelectric segment is replicated 4 times to match the length of the ECG pulse.

$$l_{ECG\_pulse} = 84 \tag{3}$$

$$l_{isoelectric} = 21 \tag{4}$$

where l_{ECG_pulse} is the sample length of the ECG pulse and $l_{isoelectric}$ is the sample length of the isoelectric segment.

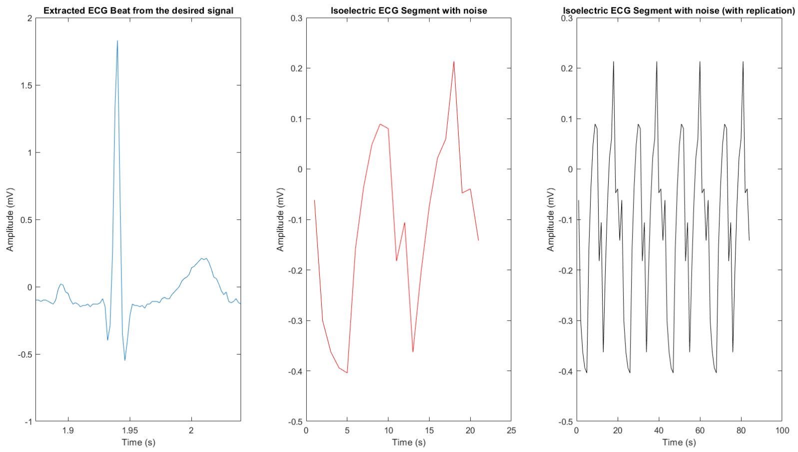


Figure 5: Extracted singe ECG beat and replicated isoelectric segment

Afterwards, for an arbitrarily selected filter order of 18, the following weight matrix is obtained using the Weiner-Hopf equation.

$$\begin{aligned} w_T = [&0.6696, 0.3104, -0.1226, -0.2292, -0.0629, 0.0755, 0.0653, 0.0433, 0.0328, \\ &0.0268, -0.0020, -0.0108, 0.0038, 0.0229, 0.0227, 0.0094, 0.0011, -0.0323] \end{aligned} \tag{5}$$

Then, the optimum filter order is discovered by using the MSE as the parameter between the desired signal and the filtered signal for order values ranging from 2 to 50.

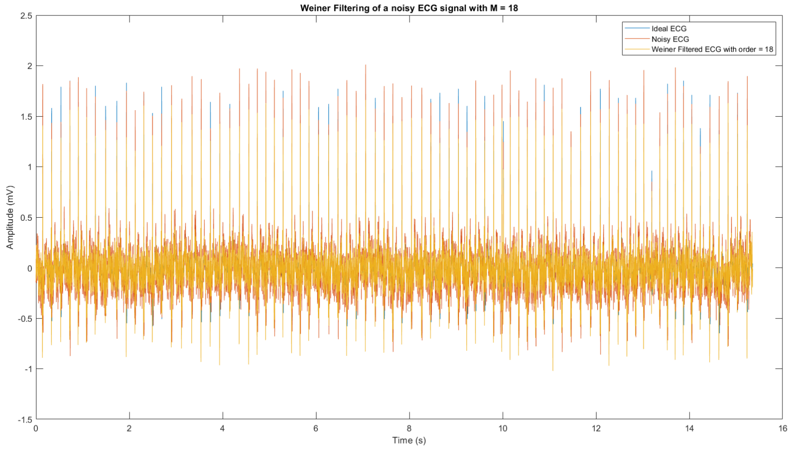


Figure 6: Wiener filtered signal with order 18

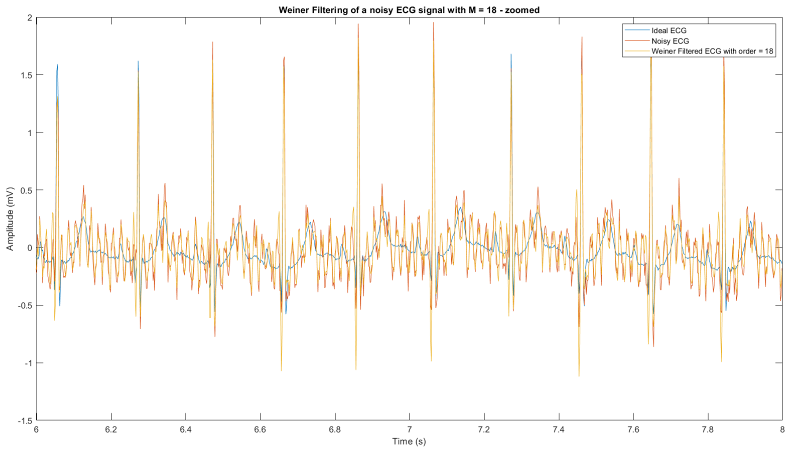


Figure 7: Wiener filtered signal with order 18 (zoomed)

Then the optimum filter order is obtained through the lowest MSE value as shown in the above graph. Thus, the optimal filter order obtained is 7. The magnitude and phase responses of that optimal filter are presented below.

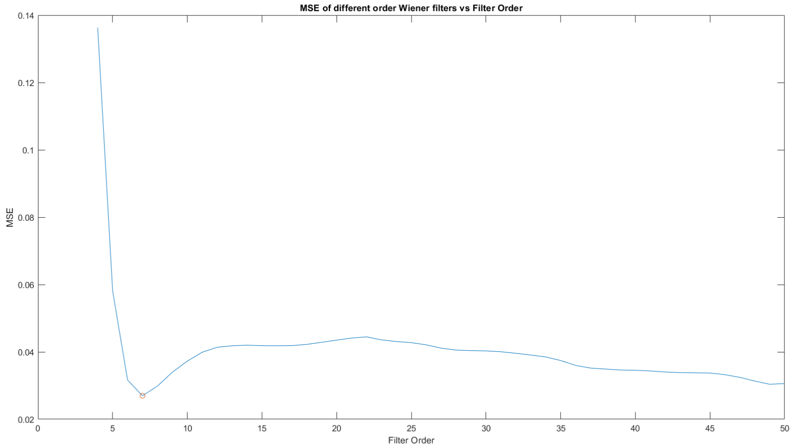


Figure 8: MSE vs filter order in Wiener filtering method on noisy ECG

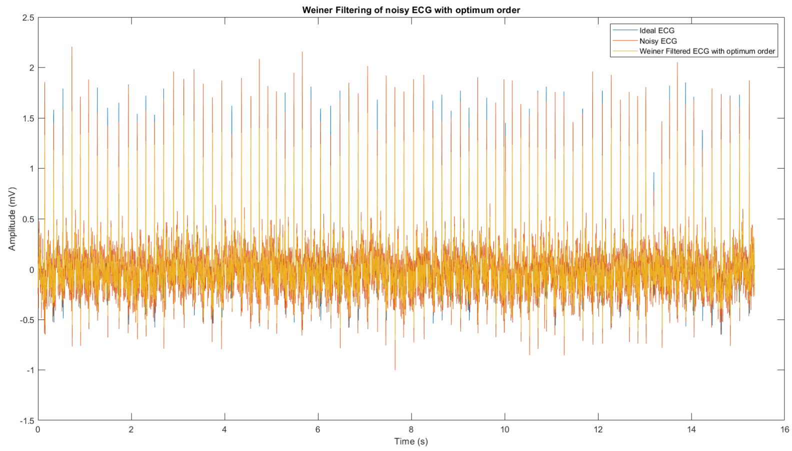


Figure 9: Wiener filtering of noisy ECG with optimum order of 7

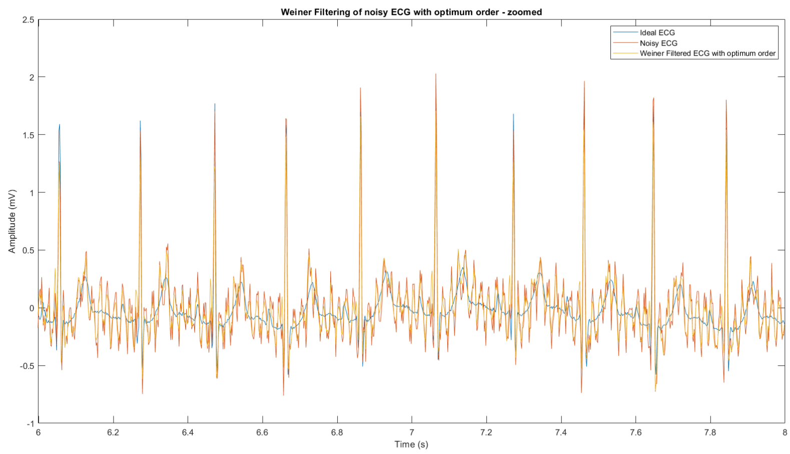


Figure 10: Wiener filtering of noisy ECG with optimum order of 7 (zoomed)

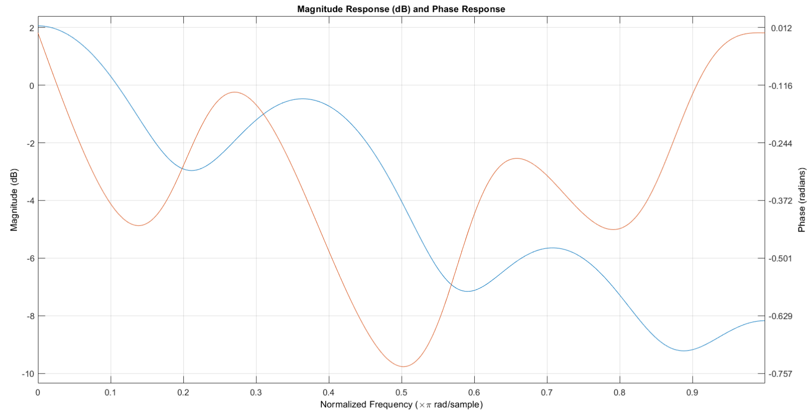


Figure 11: Magnitude (blue) and phase (orange) responses of the optimum order filter

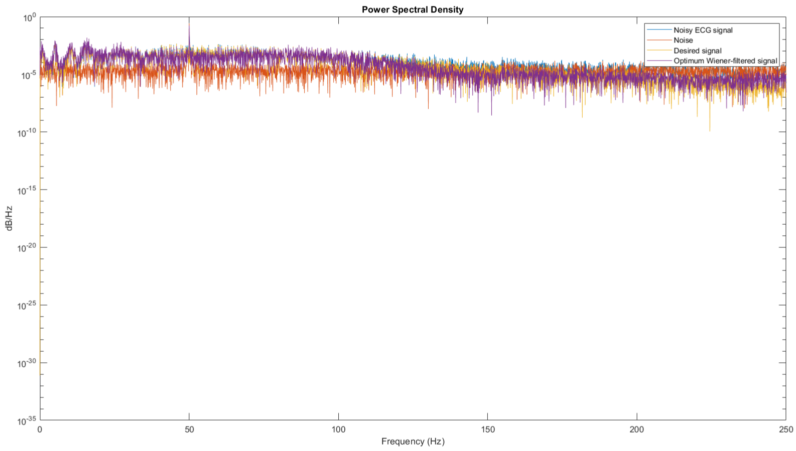


Figure 12: Power spectral density

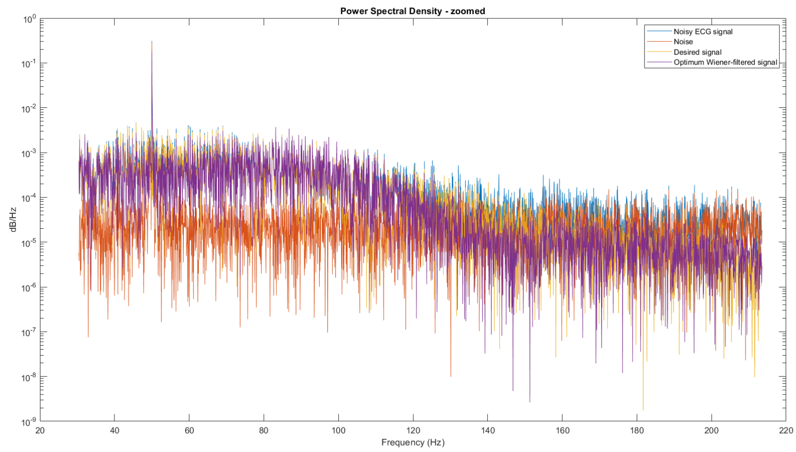


Figure 13: Power spectral density (zoomed)

According to the above plots, it is observable that even though the optimal Weiner filter has the potential to remove high-frequency noise, it performs poorly in removing low-frequency noise and the added sinusoidal noise of $50Hz$ (which is indicated by the frequency spike at $50Hz$ in the filtered spectrum).

Through a linear model of the ECG pulse However, since it is almost impossible to find ideal ECG signals in practical applications, a linearly modelled ECG pulse is utilized as an alternative to obtaining the optimum Wiener filter order. The manipulated linear-modelled ECG pulse is visualized below.

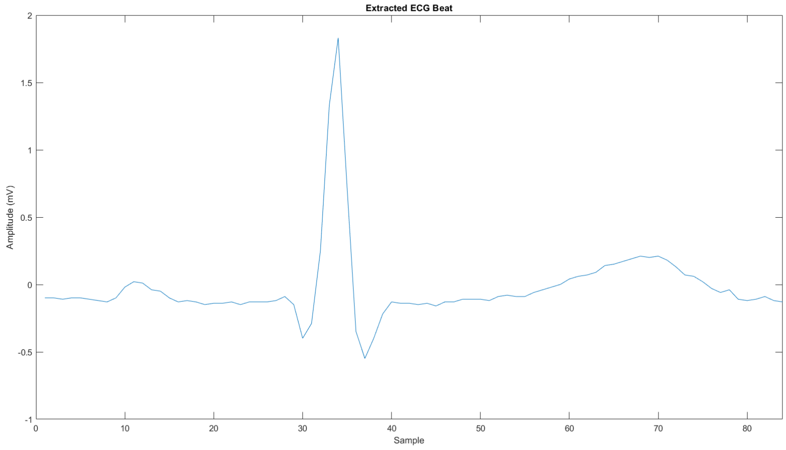


Figure 14: Extracted ECG beat from the ideal ECG signal

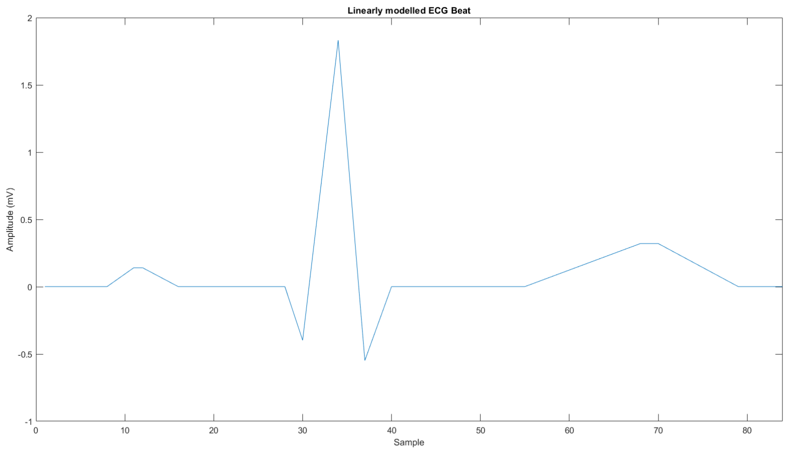
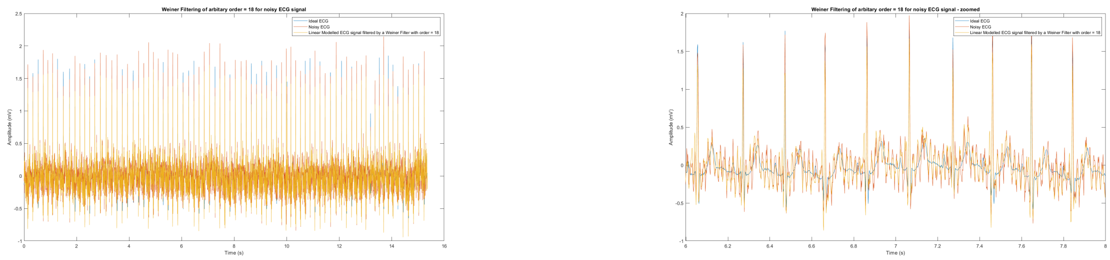


Figure 15: Linear-modelled ECG beat



(a) (b)

Figure 16: Wiener filtering of arbitrary order 18 for noisy ECG signal

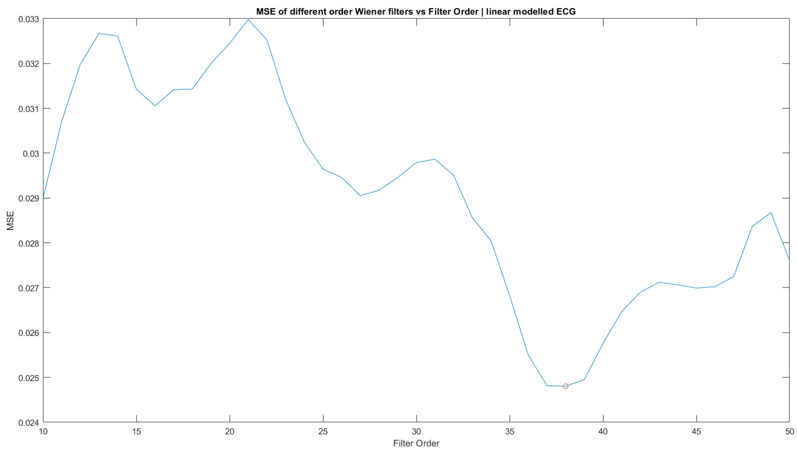


Figure 17: MSE of different order filters vs order in linear-modelled ECG

The plot: The MSE vs. the order for the Wiener filter shows that the optimal filter could be obtained when the filter order is 38.

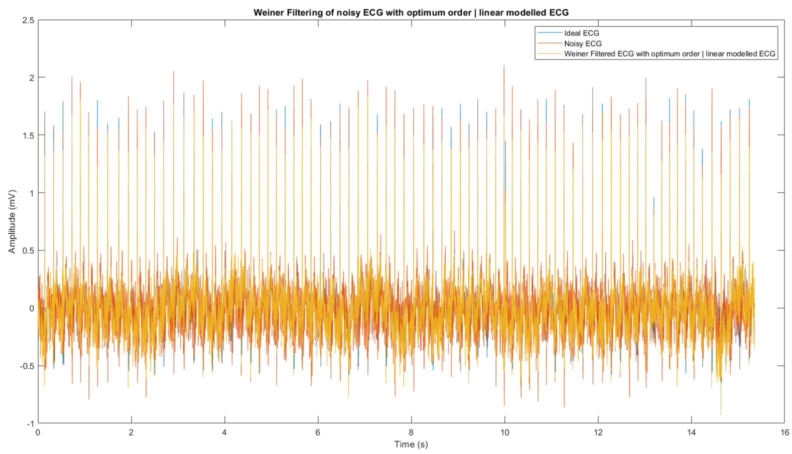


Figure 18: Wiener filtering of noisy ECG with optimum order of 38 in linear-modelled ECG

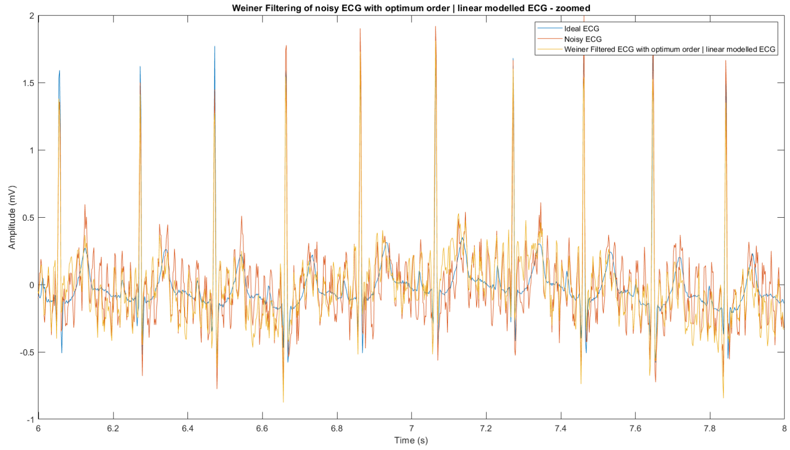


Figure 19: Wiener filtering of noisy ECG with optimum order of 38 in linear-modelled ECG (zoomed)

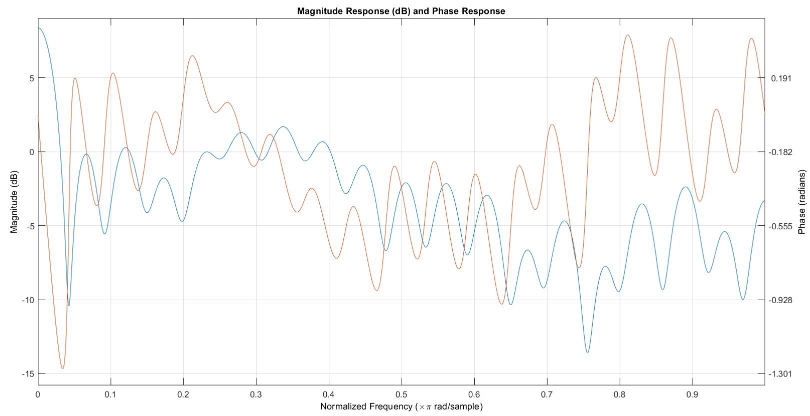


Figure 20: Magnitude (blue) and phase (orange) responses of the optimum order filter

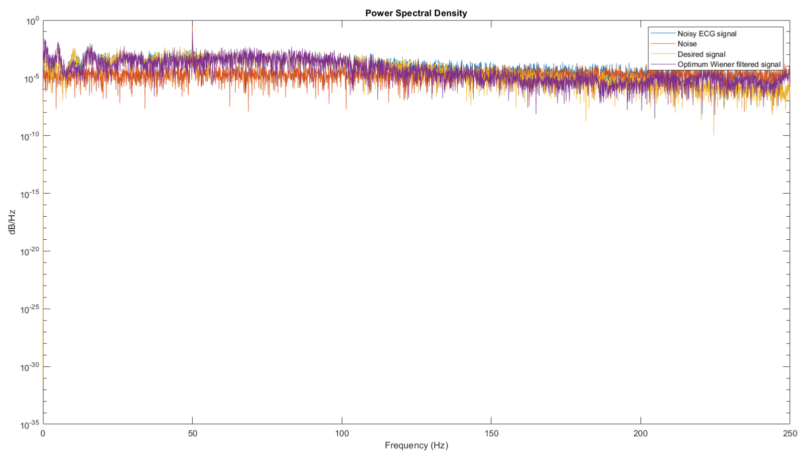


Figure 21: Power spectral density

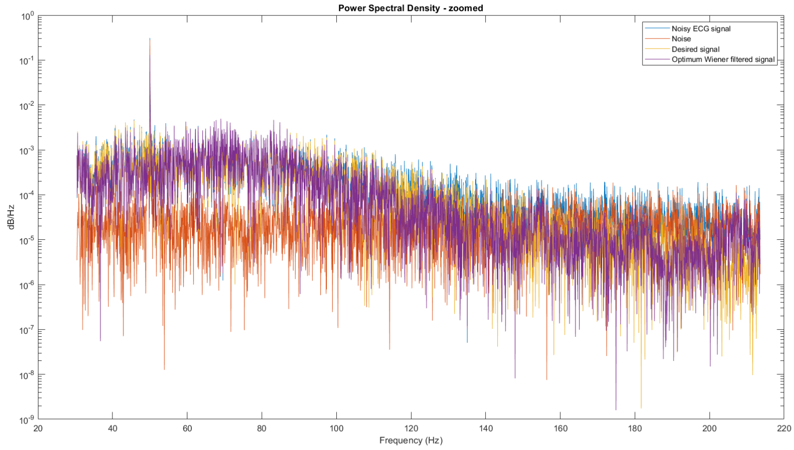


Figure 22: Power spectral density (zoomed)

Through the comparison of the above plots of Figures 13 and 22, it could be deduced that although the $50Hz$ peak is still present in the linear modelled approach as well, the high-frequency noise removal is better in the ideal ECG method (due to higher order in the optimal filter of later method tends to have a periodic effect in filtering).

1.2 Frequency domain implementation of the Wiener filter

Frequency domain implementation of the Wiener filter is based on the following equation that is utilized to find the respective filter coefficients.

$$W(f) = \frac{S_{YY}(f)}{S_{YY}(f) + S_{NN}(f)} \tag{6}$$

where $S_{YY}(f)$ is the power spectral density (PSD) of the actual signal $y(n)$ (through feeding the ideal signal) and it is equivalent to the sqared of the absolute value of the Fourier transform of signal $y(n)$. Further, $S_{NN}(f)$ is the power spectral density of noise.

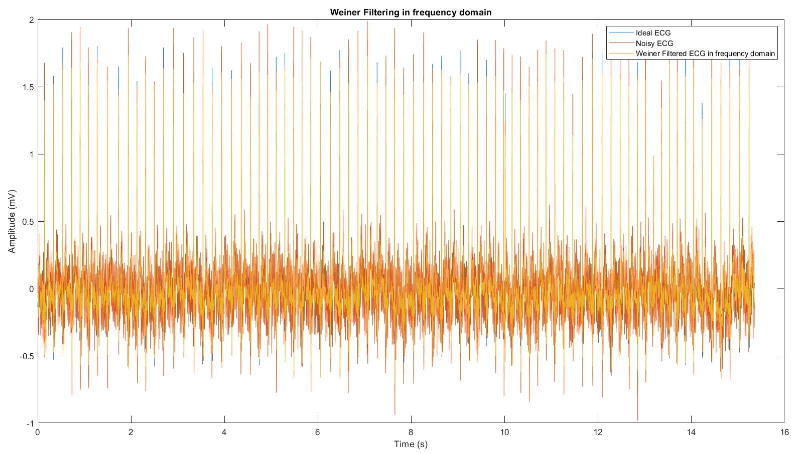


Figure 23: Wiener filtering in frequency domain

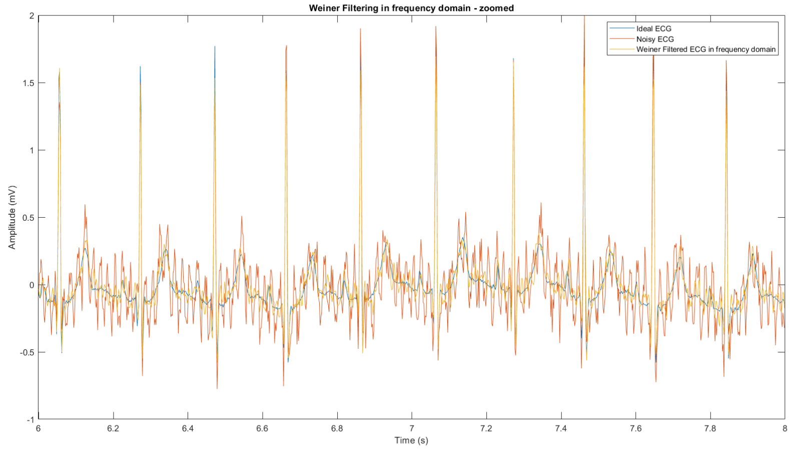


Figure 24: Wiener filtering in frequency domain (zoomed)

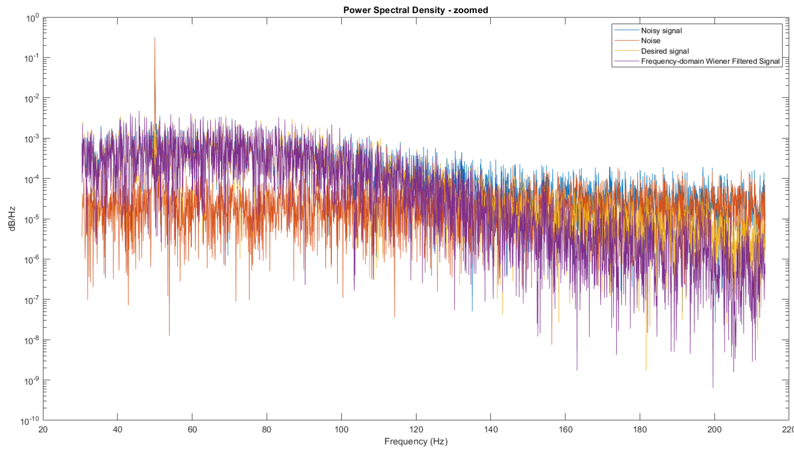


Figure 25: Power spectral density (zoomed)

With the comparison of PSD plots, it is obvious that the frequency-domain implementation has more potential to remove both high and low-frequency noise from the noisy ECG signal (with less computations in comparison to the previous approach in section 1.1) and more similar to the desired ECG signal in terms of the PSD and time-series signal.

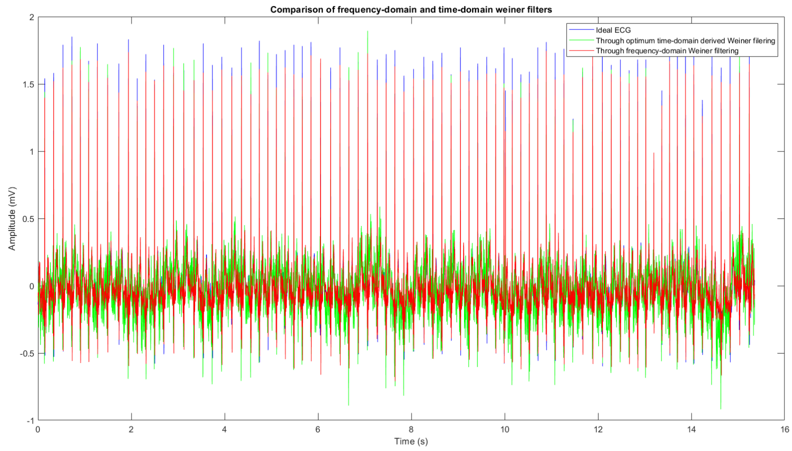


Figure 26: Comparison of frequency-domain and time-domain Wiener filters in time-series representation

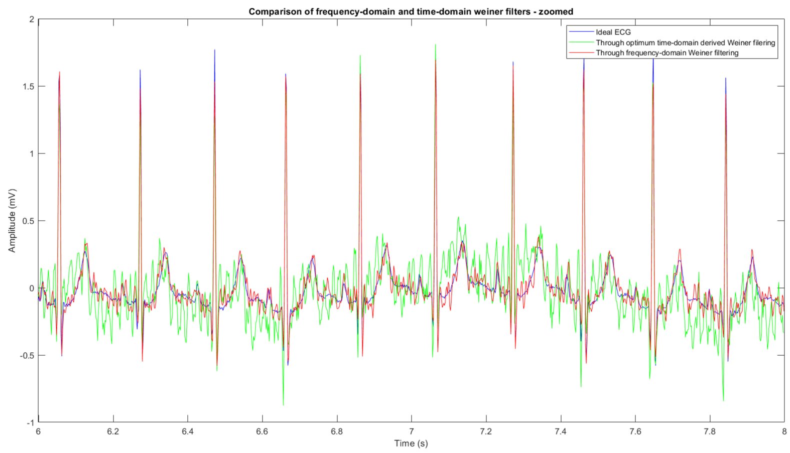


Figure 27: Comparison of frequency-domain and time-domain Wiener filters in time-series representation (zoomed)

1.3 Effect on non-stationary noise on the Wiener filtering

To add the non-stationary noise to the signal, $50Hz$ sinusoidal noise is added to the first half of the signal while $100Hz$ sinusoidal noise is added to the later part of the signal.

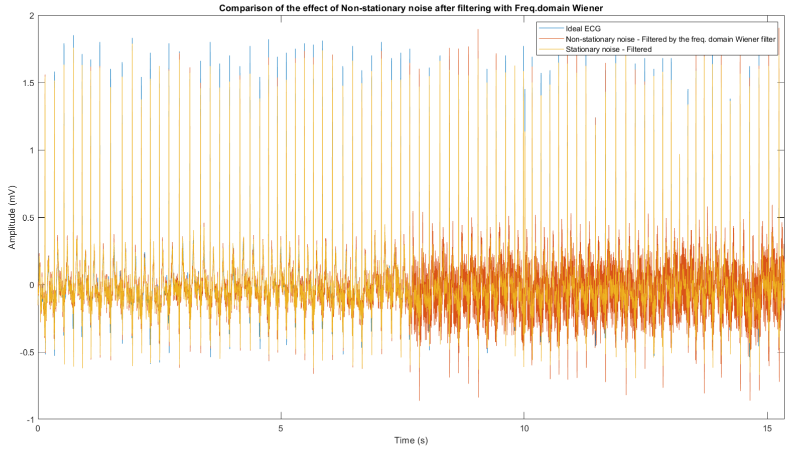


Figure 28: Comparison of the effect of non-stationary noise after filtering with frequency-domain filter

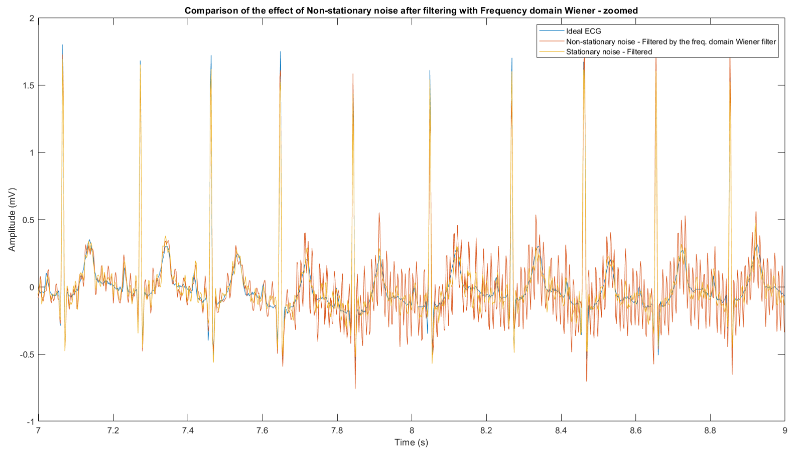


Figure 29: Comparison of the effect of non-stationary noise after filtering with frequency-domain filter (zoomed)

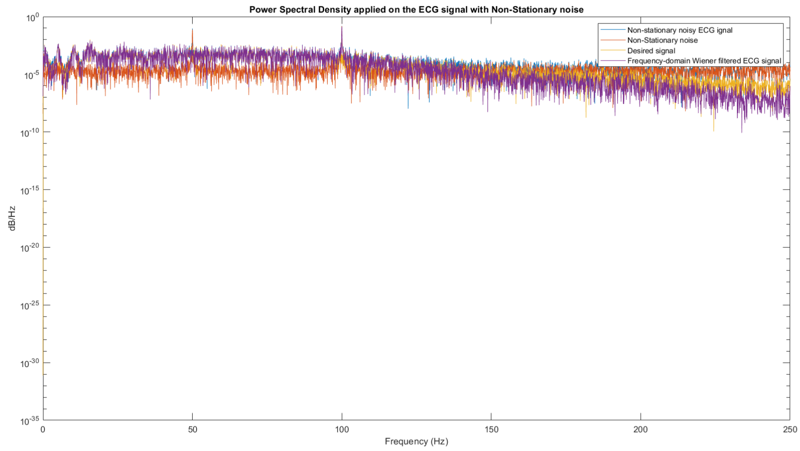


Figure 30: PSD applied on the ECG signal with non-stationary noise

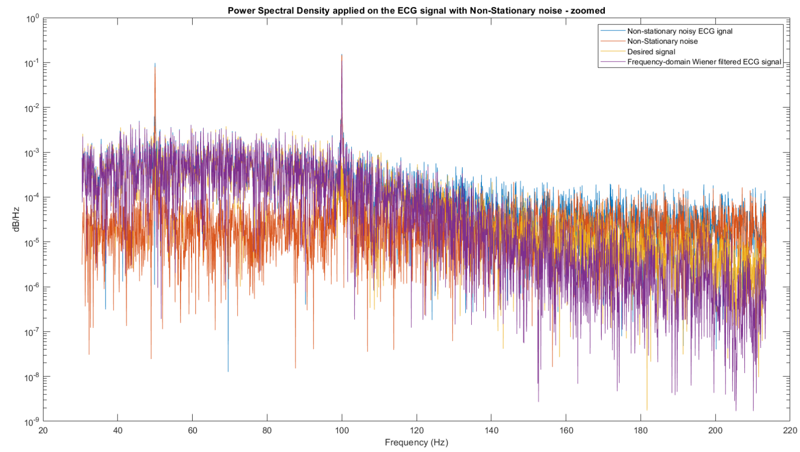


Figure 31: PSD applied on the ECG signal with non-stationary noise (zoomed)

Through the filtered results, it could be perceived that the Wiener filter is only able to sufficiently attenuate low-frequency noise, but is unable to remove the added high-frequency noise. This could be further verified by visualizing the PSD variations of the signals in which the $100Hz$ peak remained in the filtered signal even though $50Hz$ noise is removed by the Wiener filter.

2 Adaptive filtering

Adaptive filters are supposed to adjust their coefficients in accordance to the changes in the signal and noise characteristics while achieving the optimum filter. Therefore, the adaptive filter does not assume the stationary nature of either signal or noise.

Since the matrix inversion in the Wiener-Hopf equation is tedious especially when the filter order is high, and therefore, the convergence speed of the algorithm is considerably slow, the adaptive filters utilize several other algorithms, such as least mean square (LMS) and recursive least square (RLS) as alternatives to matrix inversion.

In this section, the LMS and RLS algorithms are tested with a developed sawtooth wave with a width of 0.5. The noisy signal is obtained by adding Gaussian white noise and sinusoidal noise to the sawtooth signal.

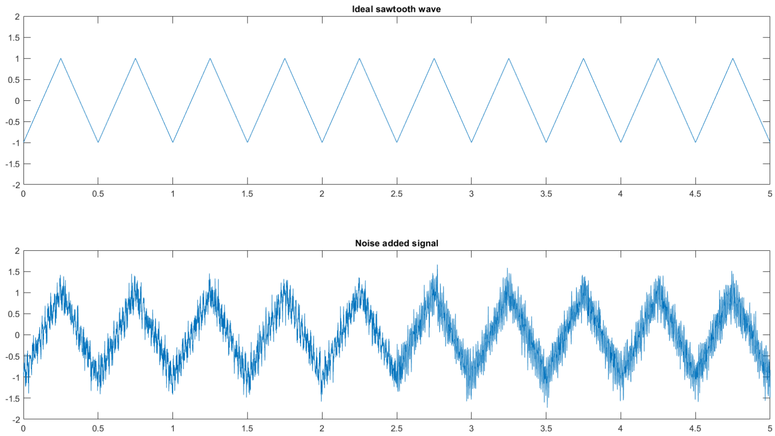


Figure 32: Ideal and noise added sawtooth signals

2.1 Least mean square algorithm

The following arbitrary constants are utilized in the implementation: $a = 1.61$, $\Phi_1 = 1/6$ and $\Phi_2 = 0.5$. Since different optimum filters could be obtained by adjusting the convergence factor μ and filter order M for a given $r(n)$.

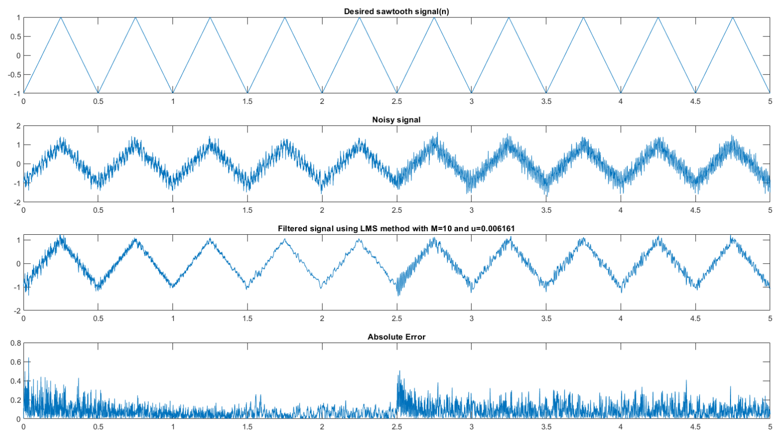


Figure 33: Adaptive filtering using LMS algorithm with order of 10

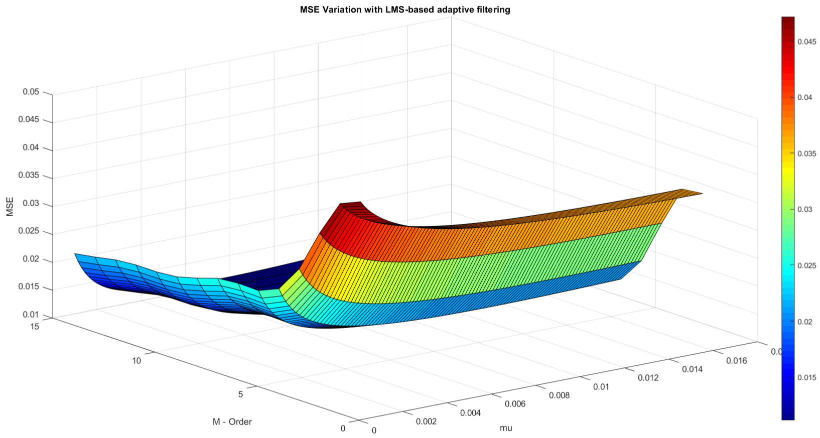


Figure 34: MSE variation with respect to filter order and μ in LMS algorithm

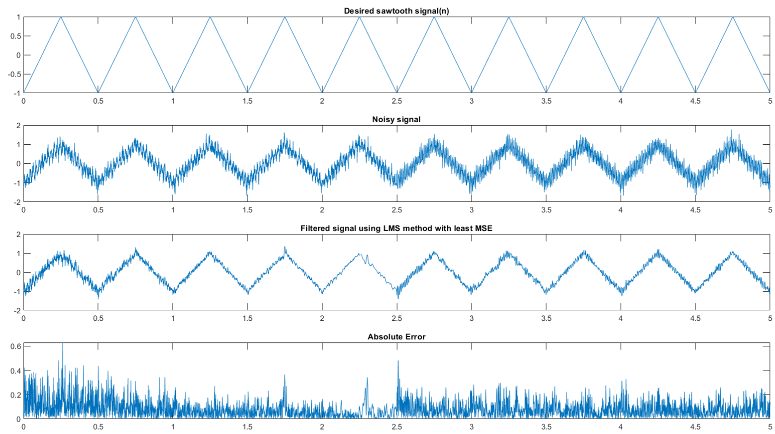


Figure 35: Adaptive filtering using LMS algorithm with least MSE

Through the results, it could be deduced that the minimum MSE of 0.0090688 could be obtained with order 15 and $\mu = 0.010876$. Further, it could be observed that for very small values of μ, the adaptive filter with LMS algorithm takes a considerate time to converge with significant error. However, the MSE reduces gradually with increasing order up to the optimum level (in this case, 15) and then increases again. Furthermore, for considerably larger values for μ, the filter could not converge (with no fine adjustments) at all with a large MSE.

2.2 Recursive least square algorithm

In this algorithm, the characteristics of the previous data are utilized to estimate the current output given the fact that the initial values for the matrices are provided.

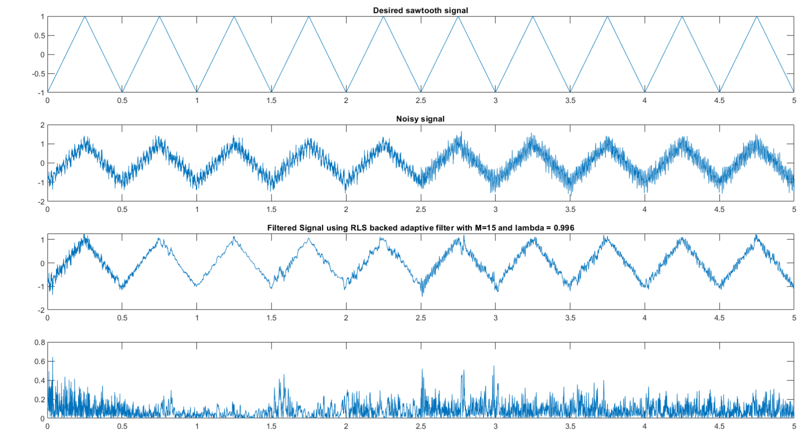


Figure 36: Adaptive filtering using RLS algorithm with order of 15

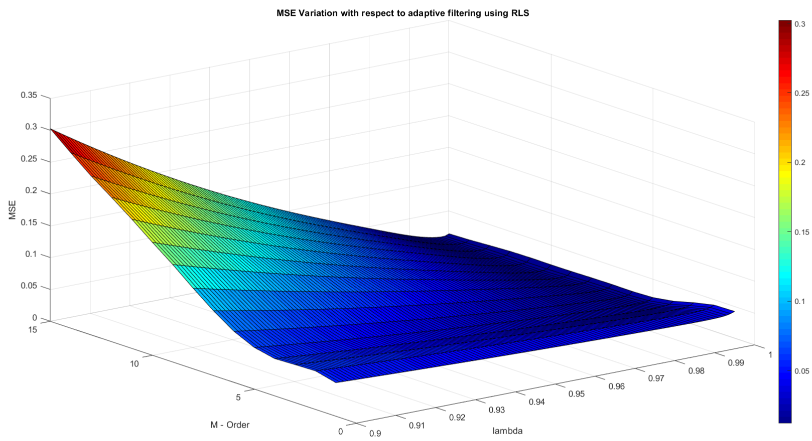


Figure 37: MSE variation with respect to filter order and μ in RLS algorithm

Through the results, it could be perceived that the minimum MSE of 0.010229 is obtained with the filter order equal to 15 and $\lambda = 0.9098$. It should also be noted that the value of λ should be nearly equal to 1 in order to have a considerable contribution from the previous repetitions. Further, it could be deduced from the plots that, for larger filter orders: the value of λ should be as close to 1 to reduce the error from the previous repeated samples.

Performance comparison between LMS and RLS algorithms Through the below plot of absolute error, it is obvious that the RLS algorithm converges (i.e. reduce its error) faster than the LMS algorithm.

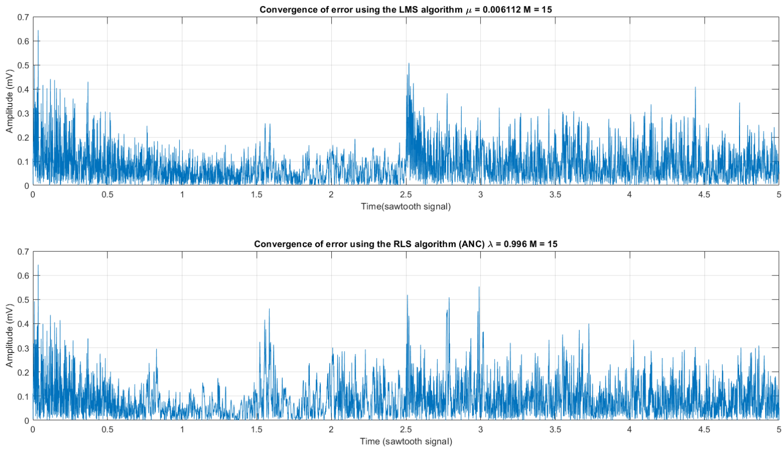


Figure 38: Performance comparison between LMS and RLS algorithms

Adaptive filter implementation on the ECG with non-stationary noise Through the below plots, it could be identified that even though both algorithms are capable of attenuating the low-frequency noise components in the noisy signal, the RLS algorithm performs well in suppressing the high-frequency noise in comparison to the LMS algorithm.

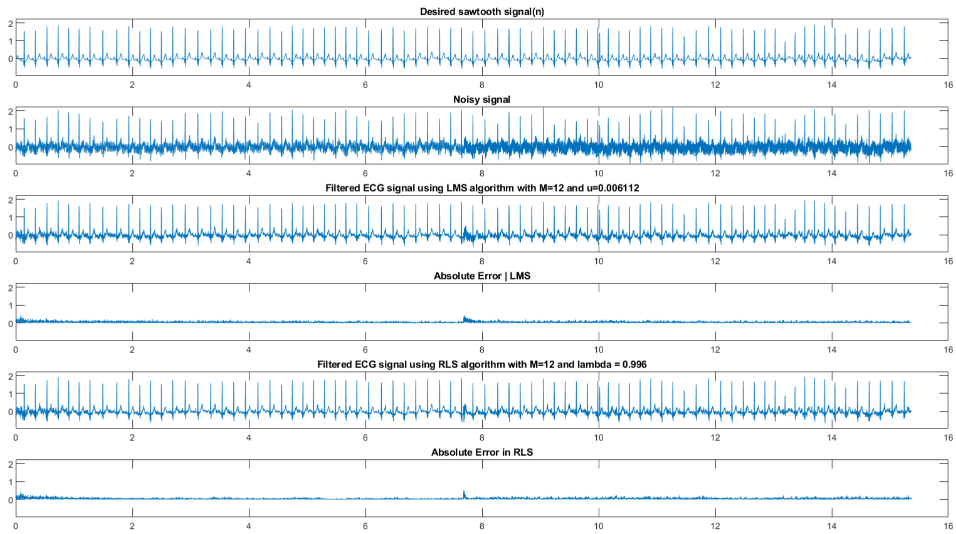


Figure 39: ECG signal filtering using two algorithms (LMS and RLS) in adaptive filtering