# EN3143: Electronic Control Systems

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## Problem 1

$$G(s) = \frac{10}{3s^2 + 8s + 24} \tag{1}$$

```
clc;
clear all;
close all;

%Problem1
p1 = [1 (4/3)+(2.49444*1i)];
p2 = [1 (4/3)-(2.49444*1i)];
den = conv(p1, p2)

omegan = sqrt(den(3)/den(1))
zeta = (den(2)/den(1))/(2*omegan)
Ts = 4/(zeta*omegan)
Tp = pi/(omegan*sqrt(1-zeta^2))
pos = 100*exp(-zeta*pi/sqrt(1-zeta^2))

num1 = 10;
den1 = [3 8 24];
func1 = tf(num1, den1);
stepplot(func1);
```

Figure 1: MATLAB implementation for Problem 1

```
den =

    1.0000    2.6667    8.0000


omegan =

    2.8284


zeta =

    0.4714


Ts =

     3


Tp =

    1.2594


pos =

   18.6514
```

Figure 2: MATLAB calculated outputs for Problem 1

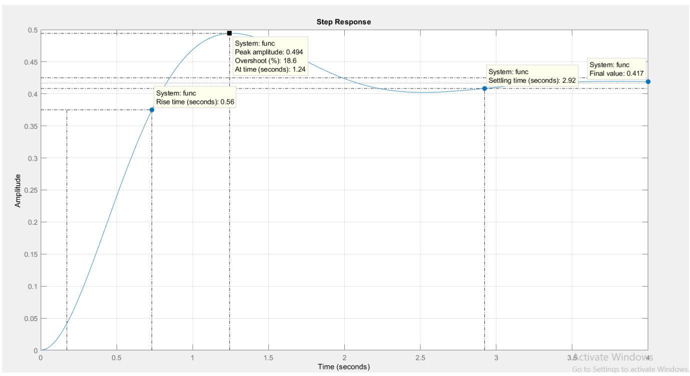



Figure 3: MATLAB plot obtained for Problem 1 using *tf* and *stepplot*

Values obtained through MATLAB calculations are:

$$\omega_n = 2.8284 rad/s, \;\; \zeta = 0.4714, \;\; T_s = 3s, \;\; T_p = 1.2594s, \;\; \%OS = 18.6514\% \tag{2}$$

Values obtained through MATLAB plot are:

$$\omega_n = 2.8738 rad/s, \;\; \zeta = 0.472, \;\; T_s = 2.92s, \;\; T_p = 1.24s, \;\; \%OS = 18.6\% \tag{3}$$

Numerical difference between each corresponding characteristic value is slight and therefore, the values are comparable.

**Problem 2**

$$G(s) = \frac{b}{s^2 + 8s + b} \tag{4}$$

In order to change the value of $\zeta$ from 0.2 to 2, the value of beta has been modified accordingly (the calculation is attached in the latter part of the document).

```
%Problem2
num2 = 400;
den2 = [1 8 400];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 100;
den2 = [1 8 100];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 44.444;
den2 = [1 8 44.444];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 25;
den2 = [1 8 25];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 16;
den2 = [1 8 16];
func2 = tf(num2, den2);
stepplot(func2)
hold on
```

Figure 4: MATLAB implementation for Problem 2 - I

```
num2 = 11.111;
den2 = [1 8 11.111];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 8.1633;
den2 = [1 8 8.1633];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 6.25;
den2 = [1 8 6.25];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 4.9383;
den2 = [1 8 4.9383];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 4;
den2 = [1 8 4];
func2 = tf(num2, den2);
stepplot(func2)
hold off
legend('\xi = 0.2','\xi = 0.4','\xi = 0.6','\xi = 0.8','\xi = 1.0','\xi = 1.2','\xi = 1.4','\xi = 1.6','\xi = 1.8','\xi =
```

Figure 5: MATLAB implementation for Problem 2 - II

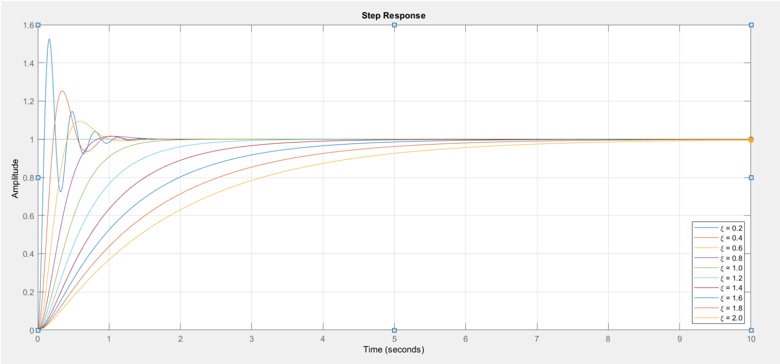



Figure 6: Obtained MATLAB plot for Problem 2

## Problem 3

```
%Problem3
num2 = 16;
den2 = [1 8 16];
func2 = tf(num2, den2);
stepplot(func2);
hold on
num2 = 100;
den2 = [1 8 100];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 25;
den2 = [1 8 25];
func2 = tf(num2, den2);
stepplot(func2)
hold on
num2 = 16;
den2 = [1 0 16];
func2 = tf(num2, den2);
stepplot(func2);
hold off
legend('\xi = 0','\xi = 0.4','\xi = 0.8','\xi = 1','Location','SouthEast');
```

Figure 7: MATLAB implementation for Problem 3 with several defined $\zeta$ values

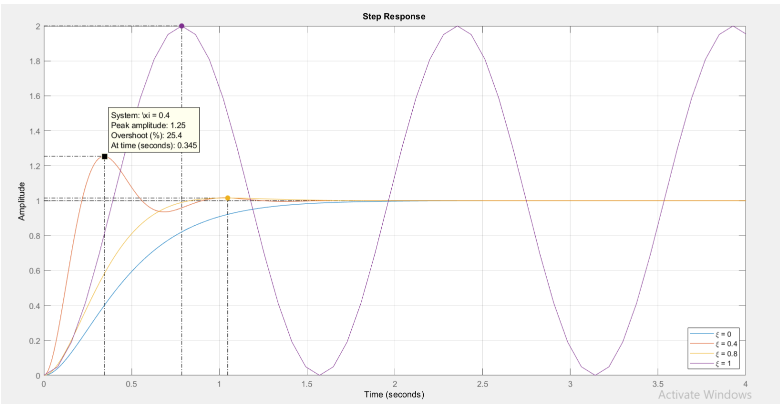



Figure 8: MATLAB plot for Problem 3 with $\zeta = 0.4$

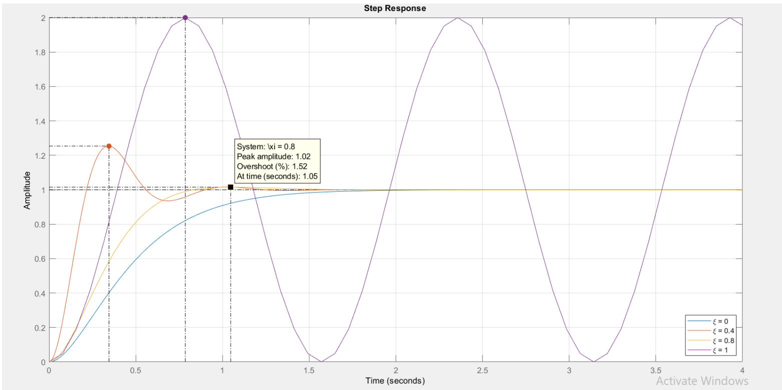
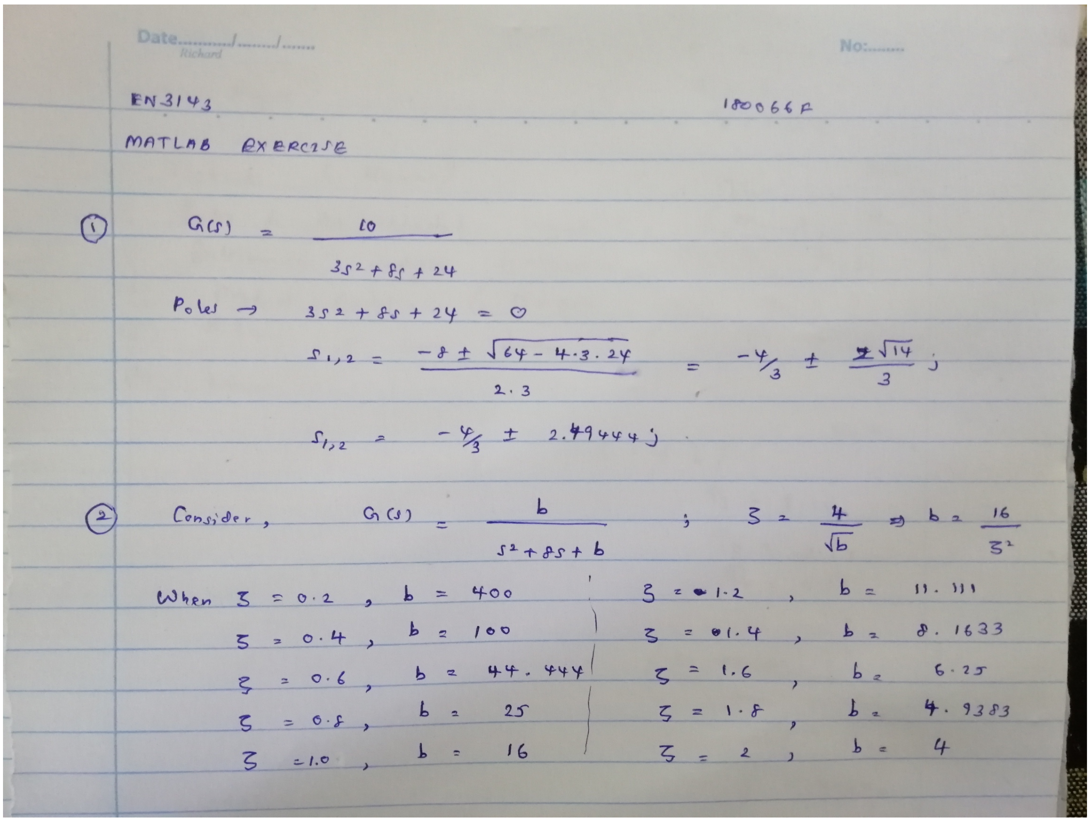



Figure 9: MATLAB plot for Problem 3 with $\zeta = 0.8$

Since when $\zeta = 0$, the output of a $2^{nd}$ order system is purely sinusoidal (i.e. undamped), $\%OS$ cannot be defined whereas when $\zeta = 1$ the system is critically damped, the value of $\%OS$ must be 0 since $c_{max} = c_{final}$. But when $\zeta$ is increased from 0 to 1 (between 0 and 1), the value of $\%OS$ should be from $\simeq 100\%$ to $\simeq 0\%$ without having $100\%$ or $0\%$ (i.e. open interval).

**Appendix: Calculations**



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MATLAB EXERCISE

(1) $G(s) = \dfrac{10}{3s^2 + 8s + 24}$

Poles $\rightarrow$ $3s^2 + 8s + 24 = 0$

$s_{1,2} = \dfrac{-8 \pm \sqrt{64 - 4 \cdot 3 \cdot 24}}{2 \cdot 3} = -4/3 \pm \dfrac{2\sqrt{14}}{3} j$

$s_{1,2} = -4/3 \pm 2.49444 j$

(2) Consider, $G(s) = \dfrac{b}{s^2 + 8s + b}$ ; $\zeta = \dfrac{4}{\sqrt{b}} \Rightarrow b = \dfrac{16}{\zeta^2}$

When $\zeta = 0.2$, $b = 400$ | $\zeta = 1.2$, $b = 11.111$

$\zeta = 0.4$, $b = 100$ | $\zeta = 1.4$, $b = 8.1633$

$\zeta = 0.6$, $b = 44.444$ | $\zeta = 1.6$, $b = 6.25$

$\zeta = 0.8$, $b = 25$ | $\zeta = 1.8$, $b = 4.9383$

$\zeta = 1.0$, $b = 16$ | $\zeta = 2$, $b = 4$

Figure 10: Calculations for Problem 1 and Problem 2