**Department of Electronic and Telecommunication Engineering**
**University of Moratuwa**

BM2101 – Analysis of Physiological Systems

Assignment 2

# Branch Cylinders: Dendritic Tree Approximations

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Contents

1 Question 1

As per the modeling of the single order branch passive electrical properties of axonal and dendritic trees, the membrane neuronal potential of each branch will be at steady state,

$$\frac{dV^2}{dX^2} = V \dots\dots\dots\dots\dots\dots\dots\dots [1]$$

Thus, the general solution will be as,

$$V_1(x) = A_1e^{-x} + B_1e^{x} \quad 0 \le X \le L_1 \dots\dots\dots\dots\dots\dots\dots\dots\dots [2.1]$$
$$V_{21}(x) = A_{21}e^{-x} + B_{21}e^{x} \quad L_1 \le X \le L_{21} \dots\dots\dots\dots\dots\dots\dots\dots [2.2]$$
$$V_{22}(x) = A_{22}e^{-x} + B_{22}e^{x} \quad L_1 \le X \le L_{22} \dots\dots\dots\dots\dots\dots\dots\dots [2.3]$$

Considering the boundary conditions at X=0,

$$\left.\frac{dV_1}{dX}\right|_{X=0} = -(r_i\lambda_c)_1 I_{app} \dots\dots\dots\dots\dots\dots\dots\dots [3]$$

From [2.1] and [3],

$$(-A_1) + B_1 = -(r_i\lambda_c)_1 I_{app} \dots\dots\dots\dots\dots\dots\dots\dots [7.1]$$

By considering the terminal ends of the daughter branches held at rest,

$$V_{21}(L_{21}) = V_{22}(L_{22}) = 0 \dots\dots\dots\dots\dots\dots\dots\dots [4]$$

From [2.2] and [4],

$$A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0 \dots\dots\dots\dots\dots\dots\dots\dots [7.2]$$

From [2.3] and [4],

$$A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0 \dots\dots\dots\dots\dots\dots\dots\dots [7.3]$$

Therefore,

$$A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$

Since membrane potential must be continuous at the nodes,

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1) \dots\dots\dots\dots\dots\dots\dots\dots [5]$$

From [2.1], [2.2], [2.3] and [5],

$$A_1e^{-L_1} + B_1e^{L_1} = A_{21}e^{-L_1} + B_{21}e^{L_1} = A_{22}e^{-L_1} + B_{22}e^{L_1}$$

Therefore,

$$A_1e^{-L_1} + B_1e^{L_1} - (A_{21}e^{-L_1} + B_{21}e^{L_1}) = 0 \dots\dots\dots\dots\dots\dots\dots\dots [7.4]$$
$$A_{21}e^{-L_1} + B_{21}e^{L_1} - (A_{22}e^{-L_1} + B_{22}e^{L_1}) = 0 \dots\dots\dots\dots\dots\dots\dots [7.5]$$

Since the current must be conserved at the nodes,

$$\frac{-1}{(r_i\lambda_c)_1}\left.\frac{dV_1}{dX}\right|_{X=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\left.\frac{dV_{21}}{dX}\right|_{X=L_1} + \frac{-1}{(r_i\lambda_c)_{22}}\left.\frac{dV_{22}}{dX}\right|_{X=L_1} \dots\dots\dots [6]$$

From [2.1],

$$\frac{-1}{(r_i\lambda_c)_1}\left.\frac{dV_1}{dX}\right|_{X=L_1} = \frac{A_1e^{-L_1} - B_1e^{L_1}}{(r_i\lambda_c)_1}$$

From [2.2],

$$\frac{-1}{(r_i\lambda_c)_{21}}\left.\frac{dV_{21}}{dX}\right|_{X=L_1} = \frac{A_{21}e^{-L_1} - B_{21}e^{L_1}}{(r_i\lambda_c)_{21}}$$

From [2.3],

$$\frac{-1}{(r_i\lambda_c)_{22}}\left.\frac{dV_{22}}{dX}\right|_{X=L_1} = \frac{A_{22}e^{-L_1} - B_{22}e^{L_1}}{(r_i\lambda_c)_{22}}$$

Therefore and from [6],

$$\frac{A_1e^{-L_1} - B_1e^{L_1}}{(r_i\lambda_c)_1} = \frac{A_{21}e^{-L_1} - B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_1} - B_{22}e^{L_1}}{(r_i\lambda_c)_{22}}$$
$$\frac{-A_1e^{-L_1} + B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21}e^{-L_1} - B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_1} - B_{22}e^{L_1}}{(r_i\lambda_c)_{22}} = 0$$
$$\frac{-A_1e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i\lambda_c)_{22}} = 0 \ldots\ldots\ldots [7.6]$$

## 2 Question 2

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{pmatrix}$$

$$x = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} \; and \; b = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Since $Ax = b$, from matrix multiplication and the properties of matrices, the following equations ([7.1], [7.2], [7.3], [7.4], [7.5] and [7.6]) could be obtained.

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app}$$
$$A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0$$
$$A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$
$$A_1e^{-L_1} + B_1e^{L_1} - (A_{21}e^{-L_1} + B_{21}e^{L_1}) = 0$$
$$A_{21}e^{-L_1} + B_{21}e^{L_1} - (A_{22}e^{-L_1} + B_{22}e^{L_1}) = 0$$
$$\frac{-A_1e^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i\lambda_c)_{21}} + \frac{-B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i\lambda_c)_{22}} + \frac{-B_{22}e^{L_1}}{(r_i\lambda_c)_{22}} = 0$$

## 3 Question 3

$$Ax = b$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

By using MATLAB, the solution for x,

$$x = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} 7.3680 \; x \; 10^{-4} \\ 1.6546 \; x \; 10^{-5} \\ 1.1252 \; x \; 10^{-3} \\ -2.7890 \; x \; 10^{-6} \\ 1.1252 \; x \; 10^{-3} \\ -2.7890 \; x \; 10^{-6} \end{pmatrix}$$

The necessary changes, which are applied to the given code, are attached in Appendix.

# 4 Question 4

By using the coefficients found above in Question 3 and assuming that the coefficient array is stored in the variable $x$ in the ordered manner, the steady-state voltage profile of each branch,

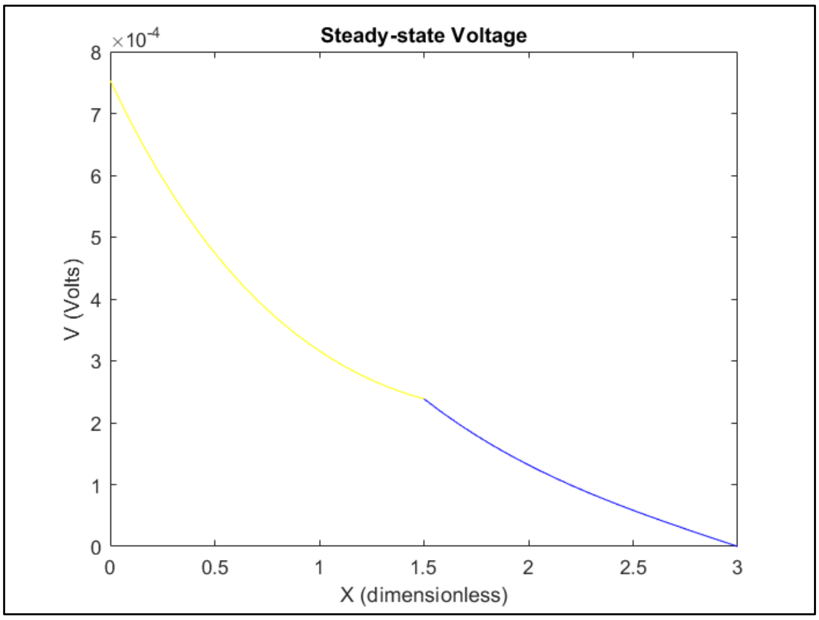


Figure 1: Steady-State Voltage Profile of each branch (Yellow: Parent branch, Red: Daughter branch 21, Blue: Daughter branch 22)

Both the voltage profiles of daughter branches have overlapped (In MATLAB graph, the red graph for the upper daughter branch - 21 has been over-drawn by the blue graph of the lower daughter branch - 22). This crucially denotes that the voltage profiles of both daughter branches are same.

# 5 Question 5

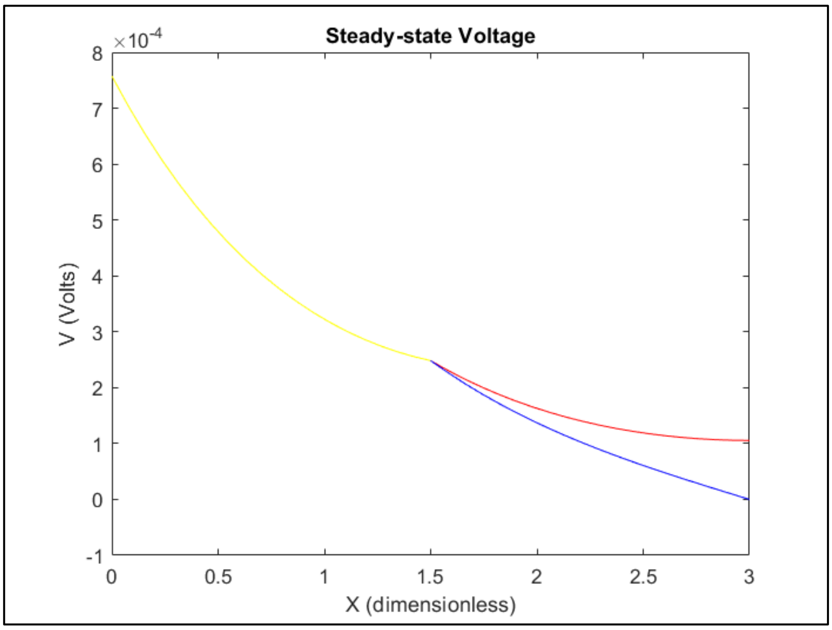


Figure 2: Steady-State Voltage Profile for 2(a) (Yellow: Parent branch, Red: Daughter branch 21, Blue: Daughter branch 22)

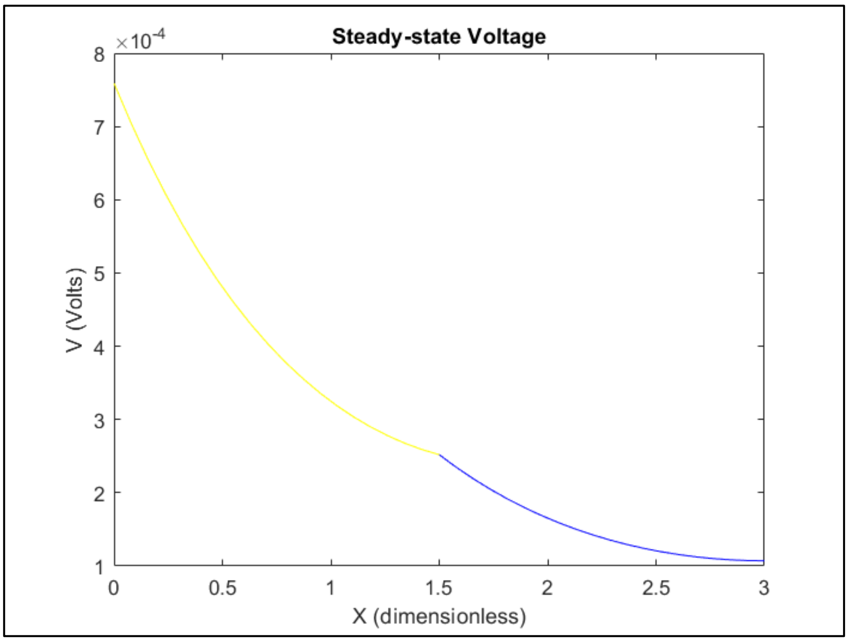


Figure 3: Steady-State Voltage Profile for 2(b) (Yellow: Parent branch, Red: Daughter branch 21 – overlapped with blue graph, Blue: Daughter branch 22)

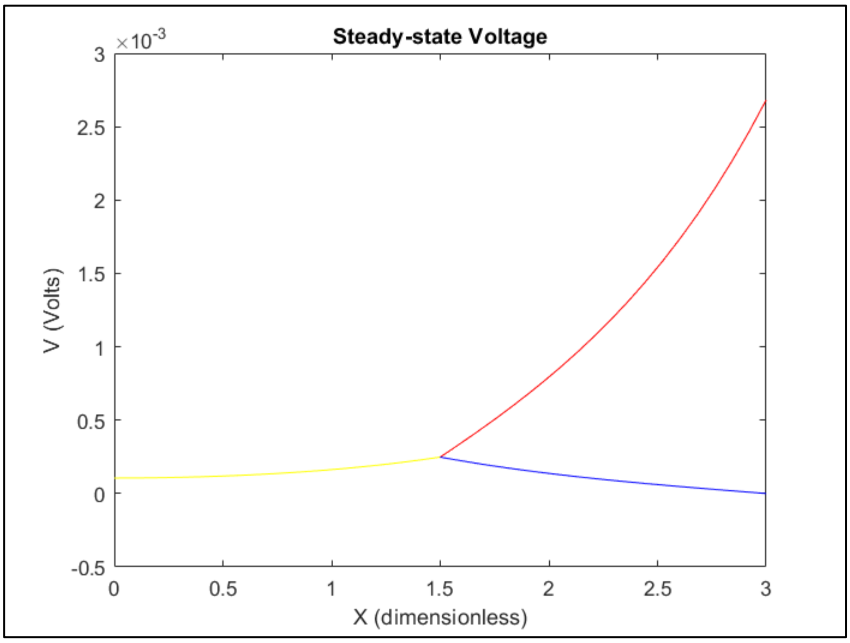

Figure 4: Steady-State Voltage Profile for 2(c) (Yellow: Parent branch, Red: Daughter branch 21, Blue: Daughter branch 22)

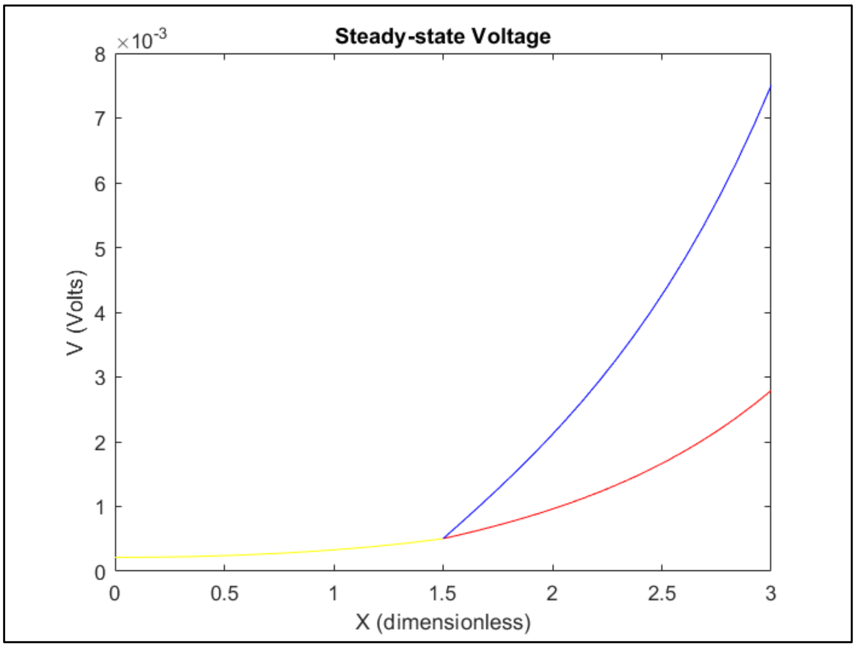

Figure 5: Steady-State Voltage Profile for 2(d) (Yellow: Parent branch, Red: Daughter branch 21, Blue: Daughter branch 22)

In 2(c), the boundary condition for first daughter branch (21) is,

$$\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}} = (r_i\lambda_c)_{21} I_{app} \dots\dots\dots\dots\dots\dots\dots\dots [8]$$

where $\frac{dV_{21}}{dX}$ denotes the gradient of membrane potential with respect to the defined electrotonic distance. The equation [8] could be rearranged as,

$$\frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}}{dX}\bigg|_{X=L_{21}} = -I_{app} \dots\dots\dots\dots\dots\dots\dots\dots . [8']$$

This equation [8'] indicates that a current of $I_{app}$ is injected into the inner conductor of the daughter branch (21) at $X = L_{21}$. Thus, as per equation [6], it could be concluded that a current of $I_{app}$ is leaving the daughter branch at the terminal $X = L_{21}$ (as per the right hand side of [6]).

Similarly, in 2(d), the boundary condition for daughter branch 21 exists as in 2(c) while a similar boundary condition is applied to the terminal end of the daughter branch 22. Thus,

$$\frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}}{dX}\bigg|_{X=L_{22}} = -I_{app} \dots\dots\dots\dots\dots\dots\dots\dots . [9]$$

Thus, following the same argument, the currents of $I_{app}$ are injected in both terminal ends of daughter branches (21 and 22) and hence, $I_{app}$ currents are leaving the daughter branches at $X = L_{21}$ and $X = L_{22}$ as per equation [6].

## 6 Question 6

For modified 2(b):

$$x = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} 7.2204 \times 10^{-4} \\ 1.7900 \times 10^{-6} \\ 7.2204 \times 10^{-4} \\ 1.7900 \times 10^{-6} \\ 7.2204 \times 10^{-4} \\ 1.7900 \times 10^{-6} \end{pmatrix}$$

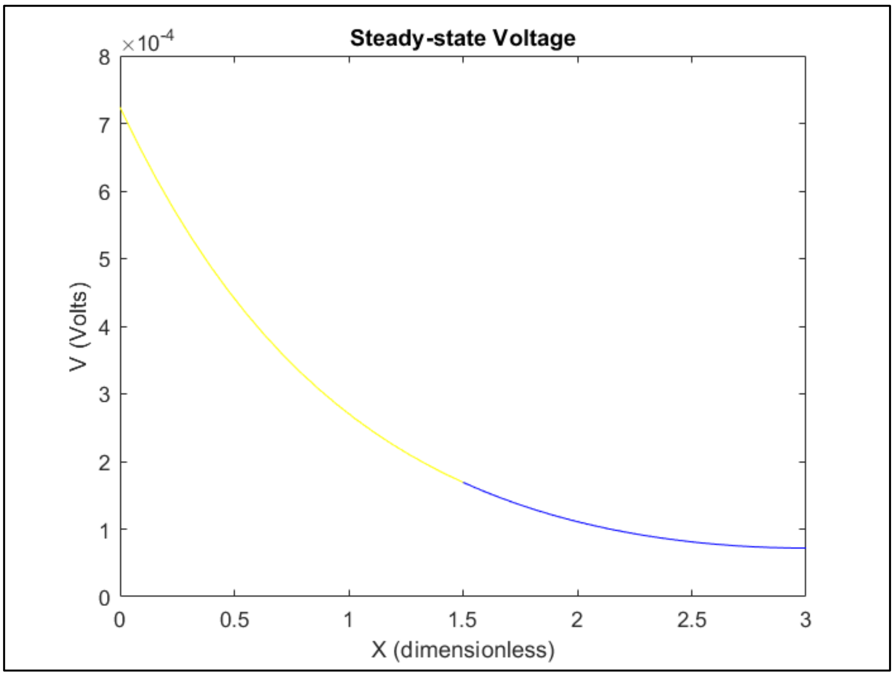


Figure 6: Modified Steady-State Voltage Profile for 2(b) (Yellow: Parent branch, Red: Daughter branch 21 (Overlapped with Blue graph in MATLAB plot), Blue: Daughter branch 22)

For modified 2(d):

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{A_1} \\ \boldsymbol{B_1} \\ \boldsymbol{A_{21}} \\ \boldsymbol{B_{21}} \\ \boldsymbol{A_{22}} \\ \boldsymbol{B_{22}} \end{pmatrix} = \begin{pmatrix} 7.1897 \times 10^{-5} \\ 7.1897 \times 10^{-5} \\ 7.1896 \times 10^{-5} \\ 7.1897 \times 10^{-5} \\ 7.1896 \times 10^{-5} \\ 7.1897 \times 10^{-5} \end{pmatrix}$$

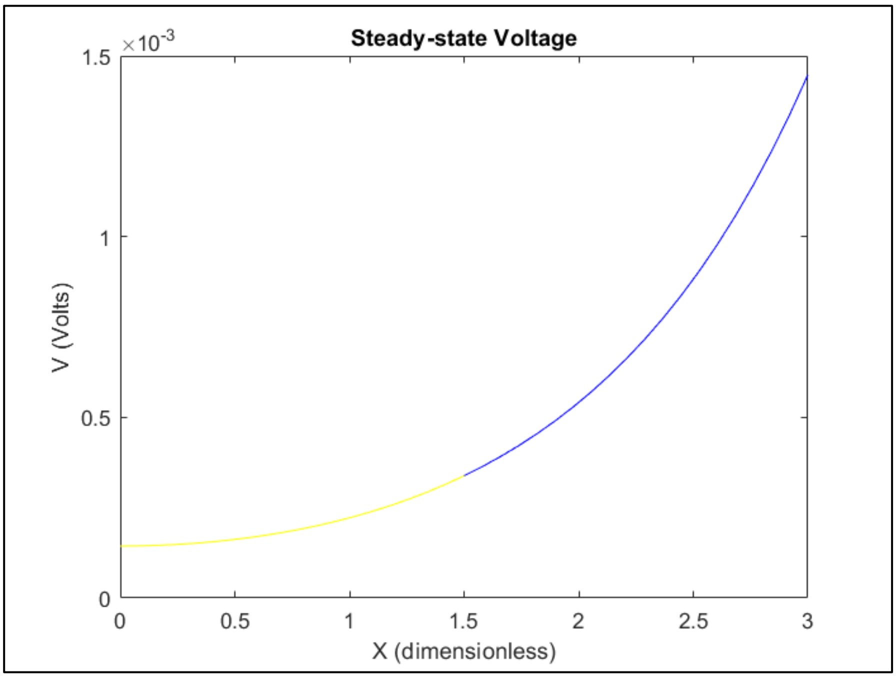


Figure 7: Modified Steady-State Voltage Profile for 2(d) (Yellow: Parent branch, Red: Daughter branch 21 (Overlapped with Blue graph in MATLAB plot), Blue: Daughter branch 22)

By evaluating the steady-state voltage profiles of modified 2(b) and 2(d) (Figure 6 and Figure 7 respectively), it could be concluded that the voltage profiles of daughter branches in each case are identical when $d_{21} = d_{22}$ (Only blue plot of branch 22 could be observed since it has been over-drawn on the red plot of branch 21).

Figure 3 (2(b)) and Figure 6 (Modified 2(b)) is comparable since both represent same boundary conditions for parent and daughter branches with different diameters for daughter branches such that:

- In figure 3, $d_{21} = 30 \times 10^{-4}\ cm$ and $d_{22} = 15 \times 10^{-4}\ cm$
- In figure 6, $d_{21} = d_{22} = 47.2470 \times 10^{-4}\ cm$

As it is observed, the decay of parent membrane potential with respect to the defined electrotonic distance is low in figure 3 ($d_{21} \neq d_{22}$) when the corresponding decay is compared with the figure 6 where $d_{21} = d_{22}$. In addition, it could be perceived that the decay of daughter membrane potential with respect to the defined electrotonic distance is slightly lower in figure 6 ($d_{21} = d_{22}$) when correspondingly compared with that of figure 3 ($d_{21} \neq d_{22}$).

Figure 5 (2(d)) and Figure 7 (Modified 2(d)) is also comparable since both represent same boundary conditions for respective branches with different diameters for daughter branches such that:

- In figure 5, $d_{21} = 30 \times 10^{-4}\ cm$ and $d_{22} = 15 \times 10^{-4}\ cm$
- In figure 7, $d_{21} = d_{22} = 47.2470 \times 10^{-4}\ cm$

As it is observed, the increment of parent membrane potential with respect to the defined electrotonic distance is low in figure 5 ($d_{21} \neq d_{22}$) when the corresponding increment is compared with the figure 7 where $d_{21} = d_{22}$. In addition, it could be perceived that the increment of daughter membrane potential with respect to the defined electrotonic distance is considerably lower in figure 7 ($d_{21} = d_{22}$) but in an identical manner in both daughter branches when correspondingly compared with that of figure 5 ($d_{21} \neq d_{22}$) where the increment is high but in different manners for two daughter branches.

# 7 Appendix

Question 4:

```
clc;
close all;
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;              % cm
d21 = 30e-4;             % cm
d22 = 15e-4;             % cm
%d21 = 47.2470e-4;        % E9 cm
%d22 = d21;                % E9 cm

l1 = 1.5;            % dimensionless
l21 = 3.0;           % dimensionless
l22 = 3.0;           % dimensionless

% Electrical properties of compartments

Rm = 6e3;            % Ohms cm^2
Rc = 90;             % Ohms cm
Rs = 1e6;            % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);      % Ohms
rl21 = c1*d21^(-3/2);        % Ohms
rl22 = c1*d22^(-3/2);        % Ohms

% Applied current
iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
l1)/rl22 -rl1*exp(l1)/rl22]; %Changed the sign of the index of exponential
term in 6x6 element

b = [rl1*iapp 0 0 0 0 0]';
x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (Volts)');
title('Steady-state Voltage');
```

Question 5:

2(a):

```
clc;
close all;
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;                % cm
d21 = 30e-4;               % cm
d22 = 15e-4;               % cm
%d21 = 47.2470e-4;          % E9 cm
%d22 = d21;                  % E9 cm

l1 = 1.5;           % dimensionless
l21 = 3.0;          % dimensionless
l22 = 3.0;          % dimensionless

% Electrical properties of compartments

Rm = 6e3;           % Ohms cm^2
Rc = 90;            % Ohms cm
Rs = 1e6;           % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);       % Ohms
rl21 = c1*d21^(-3/2);         % Ohms
rl22 = c1*d22^(-3/2);         % Ohms

% Applied current
iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 -exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
l1)/rl22 -rl1*exp(l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';
x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (Volts)');
title('Steady-state Voltage');
```

2(b):

```
clc;
close all;
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;             % cm
d21 = 30e-4;            % cm
d22 = 15e-4;            % cm
%d21 = 47.2470e-4;       % E9 cm
%d22 = d21;               % E9 cm

l1 = 1.5;           % dimensionless
l21 = 3.0;          % dimensionless
l22 = 3.0;          % dimensionless

% Electrical properties of compartments

Rm = 6e3;           % Ohms cm^2
Rc = 90;            % Ohms cm
Rs = 1e6;           % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);     % Ohms
rl21 = c1*d21^(-3/2);       % Ohms
rl22 = c1*d22^(-3/2);       % Ohms

% Applied current

iapp = 1e-9;    % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 -exp(-l21) exp(l21) 0 0;
     0 0 0 0 -exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
l1)/rl22 -rl1*exp(l1)/rl22]; %Changed the sign of the index of exponential
term in 6x6 element

b = [rl1*iapp 0 0 0 0 0]';
x = A\b;


y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (Volts)');
title('Steady-state Voltage');
```

2(c):

```
clc;
close all;
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;              % cm
d21 = 30e-4;             % cm
d22 = 15e-4;             % cm
%d21 = 47.2470e-4;        % E9 cm
%d22 = d21;                % E9 cm

l1 = 1.5;           % dimensionless
l21 = 3.0;          % dimensionless
l22 = 3.0;          % dimensionless

% Electrical properties of compartments

Rm = 6e3;           % Ohms cm^2
Rc = 90;            % Ohms cm
Rs = 1e6;           % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);      % Ohms
rl21 = c1*d21^(-3/2);        % Ohms
rl22 = c1*d22^(-3/2);        % Ohms

% Applied current

iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 -exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
l1)/rl22 -rl1*exp(l1)/rl22]; %Changed the sign of the index of exponential
term in 6x6 element

b = [0 rl21*iapp 0 0 0 0]';
x = A\b;


y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (Volts)');
title('Steady-state Voltage');
```

2(d):

```
clc;
close all;
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;              % cm
d21 = 30e-4;             % cm
d22 = 15e-4;             % cm
%d21 = 47.2470e-4;        % E9 cm
%d22 = d21;                % E9 cm

l1 = 1.5;            % dimensionless
l21 = 3.0;           % dimensionless
l22 = 3.0;           % dimensionless

% Electrical properties of compartments

Rm = 6e3;            % Ohms cm^2
Rc = 90;             % Ohms cm
Rs = 1e6;            % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);      % Ohms
rl21 = c1*d21^(-3/2);        % Ohms
rl22 = c1*d22^(-3/2);        % Ohms

% Applied current

iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 -exp(-l21) exp(l21) 0 0;
     0 0 0 0 -exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
l1)/rl22 -rl1*exp(l1)/rl22]; %Changed the sign of the index of exponential
term in 6x6 element

b = [0 rl21*iapp rl22*iapp 0 0 0]';
x = A\b;


y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (Volts)');
title('Steady-state Voltage');
```

Question 6

Modified 2(b):

```
clc;
close all;
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;                 % cm
%d21 = 30e-4;               % cm
%d22 = 15e-4;               % cm
d21 = 47.2470e-4;           % E9 cm
d22 = d21;                   % E9 cm

l1 = 1.5;            % dimensionless
l21 = 3.0;           % dimensionless
l22 = 3.0;           % dimensionless

% Electrical properties of compartments

Rm = 6e3;            % Ohms cm^2
Rc = 90;             % Ohms cm
Rs = 1e6;            % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);         % Ohms
rl21 = c1*d21^(-3/2);           % Ohms
rl22 = c1*d22^(-3/2);           % Ohms

% Applied current
iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 -exp(-l21) exp(l21) 0 0;
     0 0 0 0 -exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
l1)/rl22 -rl1*exp(l1)/rl22]; %Changed the sign of the index of exponential
term in 6x6 element

b = [rl1*iapp 0 0 0 0 0]';
x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (Volts)');
title('Steady-state Voltage');
```

Modified 2(d):

```
clc;
close all;
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;              % cm
%d21 = 30e-4;            % cm
%d22 = 15e-4;            % cm
d21 = 47.2470e-4;        % E9 cm
d22 = d21;                % E9 cm

l1 = 1.5;            % dimensionless
l21 = 3.0;           % dimensionless
l22 = 3.0;           % dimensionless

% Electrical properties of compartments

Rm = 6e3;            % Ohms cm^2
Rc = 90;             % Ohms cm
Rs = 1e6;            % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);      % Ohms
rl21 = c1*d21^(-3/2);        % Ohms
rl22 = c1*d22^(-3/2);        % Ohms

% Applied current

iapp = 1e-9;     % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 -exp(-l21) exp(l21) 0 0;
     0 0 0 0 -exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
l1)/rl22 -rl1*exp(l1)/rl22]; %Changed the sign of the index of exponential
term in 6x6 element

b = [0 rl21*iapp rl22*iapp 0 0 0]';
x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1)+x(2)*exp(y1);
v21 = x(3)*exp(-y21)+x(4)*exp(y21);
v22 = x(5)*exp(-y22)+x(6)*exp(y22);
figure;
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X (dimensionless)');
ylabel('V (Volts)');
title('Steady-state Voltage');
```