Stellar Evolution and Galaxies

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Problem 1

(a) Both the pp-chain and the CNO-cycle produce energy via a set of reactions (which we studied in the lectures). Both of these reactions are important during the main-sequence evolution of stars. What is practically the difference between the pp-chain and the CNO-cycle?
Answer:
The CNO cycle differs from the pp-chain since,

- CNO-cycle needs carbon to be present to act as a catalyst
- CNO-cycle is more temperature dependant than pp-chain to overcome the strong Coulomb barrier
- pp-chain has three simultaneous branches while CNO-cycle has two branches each with six reactions

(b) When investigating the stellar structure, which parameters are important?
Answer:

- (Initial) mass
- Composition (especially the metallicity)
- Rotation rate

(c) If the sun were to collapse into a neutron star with a radius of 8 km, what would be its density? (Assume no mass loss)
Answer:
Since there is no mass loss and assuming the sun and the neutron star as perfect symmetrical spheres,

$$\frac{4\pi R_\odot^3 \rho_\odot(r)}{3} = \frac{4\pi R_\star^3 \rho_\star(r)}{3} \tag{1}$$

$$\rho_\star(r) = \frac{R_\odot^3 \rho_\odot(r)}{R_\star^3} = \frac{696340^3 \times 1400}{8^3} = 9.233 \times 10^{17} kgm^{-3} \tag{2}$$

(d) The escape velocity of an object is given as $v_{esc} = \sqrt{\frac{2GM}{R}}$. If the sun were to turn into a black hole (without any mass loss), how small would it need to be?
Answer:
If the sun were to turn into a black hole, then the escape velocity should be at least the velocity of light (in a vacuum). Therefore,

$$c = \sqrt{\frac{2GM}{R_S}} \rightarrow R_S = \frac{2GM}{c^2} \tag{3}$$

$$R_S = \frac{2 \times 6.67 \times 10^{-11} \times 1.989 \times 10^{30}}{2.998^2 \times 10^{16}} = 2.952km \tag{4}$$

Problem 2

(a) Calculate the total energy produced per kilogram in the Sun by the pp-chain, CNO-cycle and the triple-alpha process.

Answer:

The total energy produced per second and per kg of gas from a gas with a certain density and temperature (through Taylor expansion),

$$\epsilon_{AB} = \epsilon_{0,reac} X_A X_B \rho^\alpha T^\beta \tag{5}$$

Therefore, the energy produced per second and per kg of gas for the full pp-chain,

$$\epsilon_{pp} = \epsilon_{0,pp} X_H^2 \rho T_6^4 = 1.08 \times 10^{-12} \times 0.33^2 \times 1.5 \times 10^5 \times 15.7^4 = 1.072 \times 10^{-3} \tag{6}$$

The energy produced per second and per kg of gas for the full CNO-cycle,

$$\epsilon_{CNO} = \epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20} = 8.24 \times 10^{-31} \times 0.33 \times 0.01 \times 1.5 \times 10^5 \times 15.7^{20} \tag{7}$$

$$\epsilon_{CNO} = 3.377 \times 10^{-4} \tag{8}$$

The energy produced per second and per kg of gas for the full triple-α process,

$$\epsilon_{3\alpha} = \epsilon_{0,3\alpha} X_{He}^3 \rho^2 T_8^{41} = 3.86 \times 10^{-18} \times 0.65^3 \times (1.5 \times 10^5)^2 \times 0.157^{41} \tag{9}$$

$$\epsilon_{3\alpha} = 2.567 \times 10^{-41} \cong 0 \tag{10}$$

(b) Calculate the ratio between the energy produced from the pp-chain and the CNO-cycle, and between the pp-chain and the triple- process. The energy produced by the CNO cycle is only around 1%–5% of the total energy production of the Sun.

If your answer for the ratio between the energy produced by the pp-chain and the CNO-cycle is very different from the expected 1%–5% for the sun, can you explain the possible reasons for the difference? What would you need to change to obtain a more correct answer?

Answer:

$$\frac{\epsilon_{pp}}{\epsilon_{CNO}} = \frac{1.072 \times 10^{-3}}{3.377 \times 10^{-4}} = 3.174 \tag{11}$$

$$\frac{\epsilon_{pp}}{\epsilon_{3\alpha}} = \frac{1.072 \times 10^{-3}}{2.567 \times 10^{-41}} = 4.176 \times 10^{37} \tag{12}$$

Since of the answer in (11), it is clear that the energy produced by the CNO-cycle is significantly deviated from the expectation (and the energy produced by the triple α is negligible from (12)). This is because of the assumption that the temperature is constant in the core, which is not true (the high temperatures are only evident in the center of the core). This means that most of the energy is produced at lower temperatures, where the pp-chain dominates in the energy production.

(c) Now repeat the previous calculation using a mean core temperature of about $T = 13 \times 10^6 K$. Use this temperature for the rest of this exercise.

Answer:

$$\frac{\epsilon_{pp}}{\epsilon_{CNO}} = \frac{\epsilon_{0,pp} X_H^2 \rho T_6^4 = 1.08 \times 10^{-12} \times 0.33^2 \times 1.5 \times 10^5 \times 13^4}{\epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20} = 8.24 \times 10^{-31} \times 0.33 \times 0.01 \times 1.5 \times 10^5 \times 13^{20}} \tag{13}$$

$$\frac{\epsilon_{pp}}{\epsilon_{CNO}} = \frac{5.039 \times 10^{-4}}{7.752 \times 10^{-6}} = 65.003 \tag{14}$$

$$\frac{\epsilon_{pp}}{\epsilon_{3\alpha}} = \frac{\epsilon_{0,pp} X_H^2 \rho T_6^4 = 1.08 \times 10^{-12} \times 0.33^2 \times 1.5 \times 10^5 \times 13^4}{\epsilon_{0,3\alpha} X_{He}^3 \rho^2 T_8^{41} = 3.86 \times 10^{-18} \times 0.65^3 \times (1.5 \times 10^5)^2 \times 0.13^{41}} \tag{15}$$

$$\frac{\epsilon_{pp}}{\epsilon_{CNO}} = \frac{5.039 \times 10^{-4}}{1.112 \times 10^{-44}} = 4.531 \times 10^{40} \tag{16}$$

(d) At what temperature does the CNO-cycle starts to dominate?
Answer:
When,

$$\epsilon_{pp} = \epsilon_{CNO} \rightarrow \epsilon_{0,pp} X_H^2 \rho T_6^4 = \epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20} \tag{17}$$

Therefore,

$$T_6 = \sqrt[16]{\frac{\epsilon_{0,pp} X_H}{\epsilon_{0,CNO} X_{CNO}}} = 16.875 \tag{18}$$

The temperature of the core should be at least 16.875 million K (if the temperature distribution is homogeneous).

(e) Use your calculations from above and the solar luminosity $L_\odot = 3.8 \times 10^{26} W$ to find the size of this radius R_E within which all the energy production takes place. Express the result in solar radii $R_\odot \cong 7 \times 10^8 m$. The solar core extends to about $0.2 R_\odot$. How well did your estimate of R_E agree with the radius of the solar core?
Answer:

$$L_\odot = \frac{4\pi R_E^3 \rho \epsilon}{3} \rightarrow R_E = \sqrt[3]{\frac{3L_\odot}{4\pi\rho\epsilon}} = \sqrt[3]{\frac{3 \times 3.8 \times 10^{26}}{4\pi \times 1.5 \times 10^5 \times 5.039 \times 10^{-4}}} \tag{19}$$

$$R_E = 0.152 \times 7 \times 10^8 = 0.152 R_\odot < 0.2 R_\odot \tag{20}$$

Therefore the estimation agrees with the radius of the solar core.

(f) Now assume that the CNO-cycle alone is responsible for the total energy production of the Sun. What would the radius R_E in this case? (again express the result in solar radii)
Answer:

$$L_\odot = \frac{4\pi R_E^3 \rho \epsilon}{3} \rightarrow R_E = \sqrt[3]{\frac{3L_\odot}{4\pi\rho\epsilon}} = \sqrt[3]{\frac{3 \times 3.8 \times 10^{26}}{4\pi \times 1.5 \times 10^5 \times 7.752 \times 10^{-6}}} \tag{21}$$

$$R_E = 0.610 \times 7 \times 10^8 = 0.610 R_\odot > 0.2 R_\odot \tag{22}$$

Problem 3

(a) Assume that the core temperature of a certain star, which we are going to call Star-B, on the main sequence is $T = 18 \times 10^6 K$ and its density is $\rho = 1.7 \times 10^5 kgm^{-3}$. Use the expressions for the nuclear energy production rates from the previous question (Question 2) to find out whether it is the pp-chain or the CNO cycle that dominates the energy production in Star-B while it is on the main sequence. Assume $X_H = 0.5$ and $X_{CNO} = 0.01$.

Answer:

The energy produced per second and per kg of gas for the full pp-chain,

$$\epsilon_{pp} = \epsilon_{0,pp} X_H^2 \rho T_6^4 = 1.08 \times 10^{-12} \times 0.5^2 \times 1.7 \times 10^5 \times 18^4 = 4.818 \times 10^{-3} \tag{23}$$

The energy produced per second and per kg of gas for the full CNO-cycle,

$$\epsilon_{CNO} = \epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20} = 8.24 \times 10^{-31} \times 0.5 \times 0.01 \times 1.7 \times 10^5 \times 18^{20} \tag{24}$$

$$\epsilon_{CNO} = 8.929 \times 10^{-3} \tag{25}$$

From (23) and (25), CNO-cycle dominates the energy production.

(b) Now assume that Star-B evolves off of the main-sequence and enters the Horizontal Branch. Again use the expressions for nuclear energy production (in Question 2) to find at which temperature the energy production rate of the triple-α process equals the energy production Star-B had on the main sequence (use your answer to the first part of Question 3).

Answer:

The energy produced per second and per kg of gas for the full triple-α process,

$$\epsilon_{3\alpha} = \epsilon_{0,3\alpha} X_{He}^3 \rho^2 T_8^{41} \tag{26}$$

But, in order for the triple-α process to occur, the core must be helium abundant. Therefore, $X_{He} = 1$. Thus,

$$\epsilon_{0,3\alpha} X_{He}^3 \rho^2 (\frac{T_6}{100})^{41} = \epsilon_{0,CNO} X_H X_{CNO} \rho T_6^{20} \tag{27}$$

$$T_6 = \frac{8.929 \times 10^{-3} \times 100^{41}}{3.86 \times 10^{-18} \times 1.7^2 \times 10^{10} \times 1} = 131.702 \tag{28}$$

Therefore, the required temperature will be 131.702 million K

Problem 4 Imagine that you have to explain the evolution of a star from the main sequence to the super-giant stage to another student who is completely new to this topic. So read through the lecture notes carefully, and think about the best way to explain the stellar evolution to a novice.

Then take an A4 sheet. You are allowed to make some simple drawings and write brief, clear explanations, but you are not allowed to write essays on different evolutionary phases – don't bore the new student! – and try only to use one A4 sheet. Make the drawings and brief explanations such that you can use them to be able to tell the new student how a star goes from the main sequence to the super-giant stage, describing the logic of how the core contracts/expands and how the star moves in the HR-diagram depending on temperature, means of energy transport and nuclear reactions.

Answer:

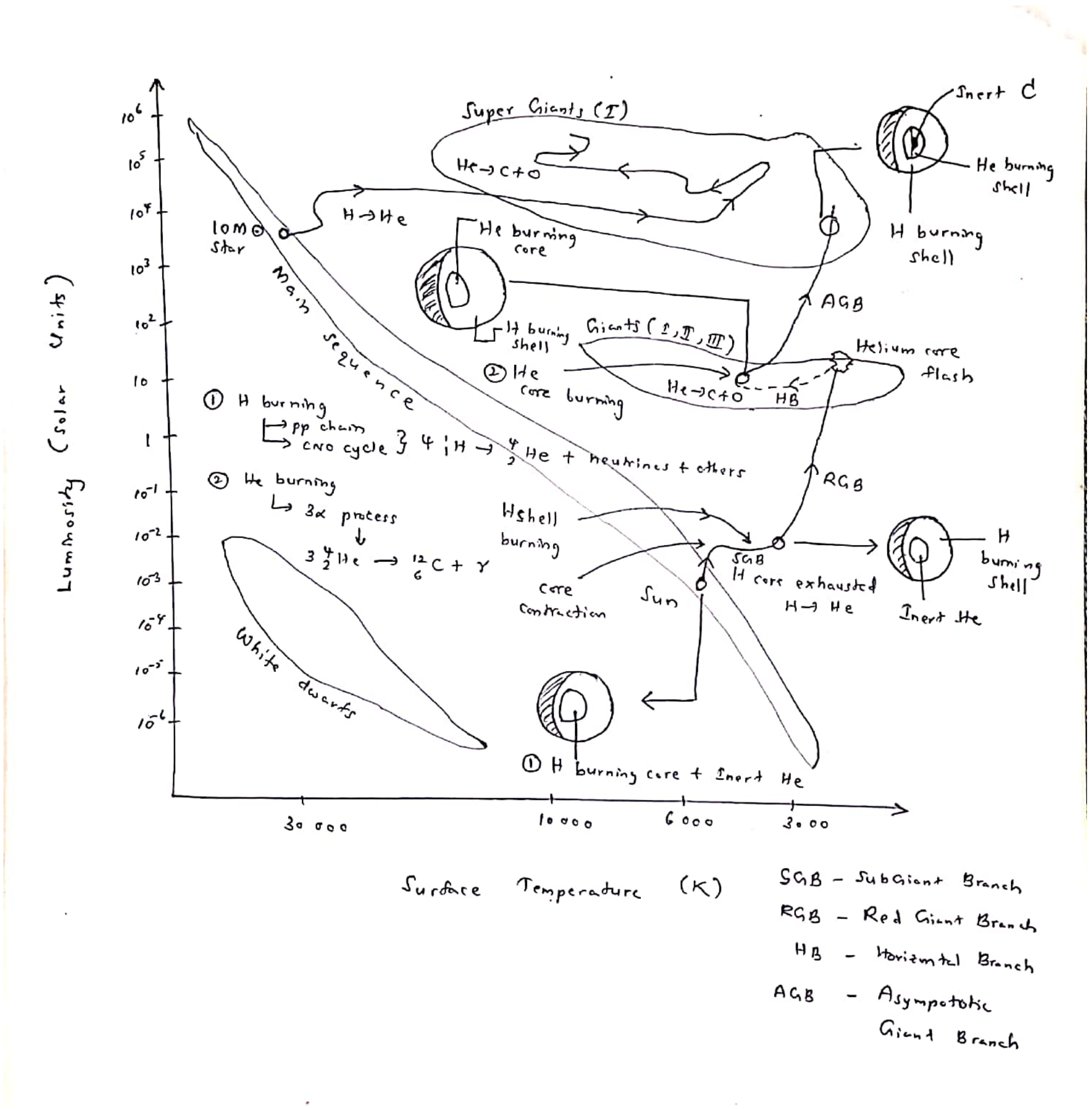

Figure 1: The simplified evolution of a star from the main sequence to the super-giant stage

Problem 5

(a) Where does the large amounts of energy released in supernova explosions originally come from? From nuclear processes or other processes?
Answer:
Gravitational energy is the crucial source of energy.

(b) Briefly explain the conditions required to produce a core-collapse supernova explosion
Answer:

- Core-collapse supernovae: Magnificent explosions that indicate the cataclysmic deaths of (defined) big stars
- Reason and the process: Occurs when the iron core (iron has the highest binding-energy per nucleon, which manipulate a growing iron-nickel core under high gravitational pressure) of a massive star collapses due to the force of gravity. Once the density in the core exceeds that of nuclear matter, the core rebounds generating pressure waves that propagate outward. A shock reheating/re-energizing mechanism is needed to explode the star
- Conditions: The mass of the core must exceed the Chandrasekhar limit of $1.4M_\odot$. The released 'thermal' neutrinos must form as neutrino-antineutrino pairs of all flavors, and total several times the number of electron-capture neutrinos. The two neutrino production mechanisms must convert the gravitational potential energy of the collapse into a ten-second neutrino burst. The mass of the progenitor star should be below $50M_\odot$ (if not it will collapse directly into a black hole without forming a supernova explosion).

Problem 6

(a) Our Sun rotates about its axis once every 25 days. Assume that the Sun is a solid sphere and use the law of the conservation of angular momentum to find the rotation period if the Sun is compressed to the typical size of a white dwarf with a radius of 6000 km.
Answer:
Using the conservation of angular momentum,

$$L_{before} = L_{after} \rightarrow I_1\omega_1 = I_2\omega_2 \rightarrow \frac{2M_\odot R_\odot^2 \times 2\pi}{5 \times P_\odot} = \frac{2M_\star R_\star^2 \times 2\pi}{5 \times P_\star} \tag{29}$$

Assuming no mass loss,

$$P_\star = \frac{R_\star^2 \times P_\odot}{R_\odot^2} = \frac{6000^2 \times 25}{696340^2} = 1.856 \times 10^{-3} days \tag{30}$$

(b) Repeat the above calculation if the Sun is compressed to the typical size of a neutron star with a radius of 8 km.
Answer:
Assuming no mass loss,

$$P_\star = \frac{R_\star^2 \times P_\odot}{R_\odot^2} = \frac{8^2 \times 25}{696340^2} = 3.300 \times 10^{-9} days \tag{31}$$

(c) A millisecond pulsar has period of 1 ms. What is the maximum radius that this pulsar can have before any objects on its surface start moving faster than the speed of light?
Answer:

$$\omega = \frac{2\pi}{P} = \frac{2\pi}{10^{-3}} = 2000\pi rad/s \tag{32}$$

Therefore,

$$R_{max} = \frac{2.998 \times 10^8}{2000\pi} = 47.715km \tag{33}$$